**To: Editor, Hydrology and Earth System Sciences**
**Subject:** Revised manuscript (#hess-2017-6)
**The Authors:** Sang Y.F., et al.
**Title:** A discrete wavelet spectrum approach to identifying non-monotonic trend pattern of hydroclimate data

**Response:**

The authors appreciate the Editor and Reviewers for helpful and constructive comments that improved our original manuscript submitted to HESS.

**Response to Reviwer#1's comments:**

*Comment 1: The method section seems to be the main contribution of this paper, but it is a bit terse and would be challenging for someone not familiar to wavelets to understand the approach. Wavelets are described in many papers and textbooks, but the use of wavelets to identify trends is not common in hydrology. It would be helpful to provide the reader with more background information so the reader can understand why certain decisions are being made here. That is, methods have few equations and have short statements of the assumptions that go into the choice of equations. The following comments identify specific locations of the text where the reader could use more information on the methods.*

**Reply:** Thanks very much for giving these valuable comments and suggestions, which is very helpful for improving the study results of our paper. The DWS approach proposed is the main contribution in our paper. Following the comment, we have rewritten and added many contents in Section 2, especially about the determination of proper decomposition level and the main technical steps of trend identification using the DWS approach, mainly to make the proposed approach more understandable. Besides, we also added some new references in the revised manuscript. More details can be found in the following point-to-point response.

*Comment 2: It would be helpful to the reader to provide background on decomposition M and why this is important for identifying a trend. Please consider adding some background on the decomposition level and why the largest level has a temporal scale that is L, the length as the time series. More specifically, why the largest level could be considered to be a trend. It could be noted that a smaller temporal scale could be important, and the decomposition level can be calculated as log2(T) if T is a temporal scale other than L, the length of the time series.*

**Reply:** Thanks very much for giving this valuable comment. The decomposition level $j$ just reflects the time scale (i.e., time scale $a_0^j$ in Eq. 2) for wavelet analysis, which is important to identify the trend pattern in a hydroclimate time series. Following the comment, we further explain the key point about the decomposition level by adding some contents in lines 121 and 124-125 and adding a new reference.

As generally considered, the variation of a time series at the biggest time scale (i.e., its data length) reflects its trend, thus in practice, the data length is usually chosen at the time scale for the trend identification. We also know that the time scale is an important factor for trend identification, and if those variations at smaller time scales are concerned, we can change the proper decomposition level in Eq. (2) and then get the trend pattern. Following the valuable comment, we added some contents as "*However, it should be noted that a meaningful trend closely depends on the temporal scale concerned. If the variability of series f(t) on certain smaller time scale K (K<L) is concerned, the proper decomposition level can be determined as* $log_2(K)$*; then, the sum of those sub-signals at the time scales bigger K can be the non-monotonic trend pattern identified.*" in lines 129-133 in the revised manuscript.

*Comment 3: Line 126: Can you indicate which wavelet is used in this analysis?*

**Reply:** Thanks. Following the helpful comment, in the revised manuscript we explained the approach for the choice of proper wavelet in lines 139-141. Further, the wavelet used for the analysis of time series in the study was added in line 208.

*Comment 4: Line 140: The text says that a spectrum is needed. Can you explain why E(j) is needed for each sub-signal?*

**Reply:** Thanks. Because we want to establish a reliable discrete wavelet spectrum for assessing the statistical significance of trend pattern in a time series, in the revised manuscript we defined the $E(j)$ as a spectrum value at each decomposition level $j$. As explained in lines 155-159, the discrete wavelet spectra $E_r(j)$ of various noise types strictly follow an exponentially decreasing rule with base 2 along with the decomposition level increase, which is obviously different from that of hydroclimate time series, thus we can take the former as a basis for establishing the discrete wavelet spectrum approach.

Following the favorable comment, we added some contents as: "*to establish the discrete wavelet spectrum (DWS) of time series, we need to specify a spectrum value E(j) for each sub-signal* $f_j(t)$ *(in Eq. 3), based on which we can quantitatively evaluate its importance and statistical significance.*" in lines 147-149 in the revised manuscript.

*Comment 5: Line 82: This is a good opportunity to add references to prior studies that document the DWT approach for trend estimation.*

**Reply:** Thanks. We have added five new references in lines 84-85 in the revised manuscript.

*Comment 6: Line 90: Can you add more description about which common practice is* *disobeyed?*

**Reply:** Thanks. Following the helpful comment, we rewrote the sentence as "*However, the practice of quadratic sum disobeys the common practice of computing variance in spectral analysis, and sometimes cannot reasonably assess the significance of non-monotonic trend*" in lines 93-95 in the revised manuscript.

*Comment 7: Line 135: Other studies have described similar approaches to identify a deterministic trend using DWT (e.g. Kallache et al., 2005). Could a stochastic component be added using the framework presented here?*

**Reply:** Thanks. In the revised manuscript we mainly proposed the DWS approach for identifying the trend pattern in a hydroclimate time series. To be specific, we used the discrete wavelet decomposition method to separate the trend pattern in a time series, and more importantly, we then established the discrete wavelet spectrum to assess its statistical significance, which is the novelty of this study and is different from previous studies.

Hydroclimate time series is generally composed of deterministic components, stochastic components and noise, and identification of the significance of a trend pattern is just to judge if it is a deterministic component. As explained above, in the revised manuscript we defined the $E(j)$ as a spectrum value at each decomposition level $j$, and found that the discrete wavelet spectra $E(j)$ of various noise types strictly follow an exponentially decreasing rule with base 2, which is obviously different from that of deterministic and stochastic components in hydroclimate time series, thus we took the former as a basis for establishing the discrete wavelet spectrum approach. That is, a stochastic component can be added using the framework presented in our study, and it would not influence the identification of trend pattern and its statistical significance.

*Comment 8: Line 137: This statement is subjective. Can you add references here to show why you are assuming that these methods are reliable and reasonable? What is your criteria for what is reliable and reasonable?*

**Reply:** Thanks. Following the favorable comment, these subjective statements were removed in the revised manuscript.

*Comment 9: Line 145: Please consider omitting the word "obviously." This is subjective and the result may not be obvious to everyone.*

**Reply:** Thanks. The subjective word "obviously" was removed in line 154 in the revised manuscript.

*Comment 10: Line 180: This statement has no supporting information. Please consider deleting this sentence.*

**Reply:** Thanks. These inaccurate statements were removed in the revised manuscript.

*Comment 11: Line 227: Can you provide an explanation of why the DWT approach has a different level of significance for different data lengths than the MK approach? The benefit of the DWT approach doesn't seem to be fully explained unless you describe the reason for it to be more stable than MK.*

**Reply:** Thanks very much for giving this valuable comment. In our opinion, the significance of a trend pattern is determined by both its own magnitude and its proportion in the original time series, and the significance of a trend pattern would change with data length, because the proportions of different components (including trend) in the original series vary with data length.

Following the comment, we added some new contents to more clearly explain why the significance level of a trend pattern varies with data length. To be specific, we explained that "*Generally, it would have more uncertainty when evaluating the statistical significance of trend pattern with a shorter length, corresponding to a bigger 95% confidence interval*" in lines 227-229 and "*the significance levels of the trend patterns do not consistently decrease with data length, but show some fluctuation, as the proportions of different components (including trend) in the original series vary with data length*" in lines 234-236.

*Comment 12: Various parts of the text say that a result is "interesting." Please try to omit this term, and let the reader decide which results are interesting.*

**Reply:** Thanks. The inaccurate word "interesting" was removed or changed as a more accurate word throughout the manuscript.

*Comment 13: Figure 1: Please consider adding the numbered steps from lines 159 to 178 to the flow chart. It may be difficult for some readers to relate the numbered steps to the steps in the flow chart. Why are the DWT equations shown at the top of the flow chart? These equations are already part of the first step on the upper left of the flow chart. Figure 3, can you provide more guidance on how to assess the significance at different data lengths? It appears that the DWS is significant when it plots above the 95% confidence bar in the blue lines. Can you provide more guidance?*

**Reply:** Thanks very much. Following the valuable comment, we divided the analysis process of trend identification using the DWS approach into five steps, and more details can be found in lines 168-190. It is just for making the Figure 1 presentable by putting the DWT equation on the top. Following this comment, the Figure 1 was carefully readjusted.

Moreover, following the valuable comment, we added some new contents to explained the results in Figure 3 as "*That is, if the red point at certain data length is above the 95% confidence bar, described by the blue line in Figure 3, it is thought that the trend pattern is significant at 95% confidence level*" in lines 219-221, and in its caption.

*Comment 14: References: Please add publication year to each reference.*

**Reply:** Thanks. We have checked all the references and make sure that all references have publication year.

**Response to Reviwer#2's comments:**

*Comment 1. DWS is a well established approach and has been widely applied, especially in signal analysis. I simply cannot see the novelty despite the authors stated they developed a new DWS approach. The novelty should be further elaborated and highlighted should the authors consider to revise and resubmit to another journal.*

**Reply:** Thanks for giving this helpful comment. We know that the discrete wavelet transform method has been widely used for trend identification; however, how to accurately assess the statistical significance of a trend pattern gotten from discrete wavelet decomposition results is a big challenge. To solve the problem, in the revised manuscript we mainly proposed the DWS approach for identifying the trend pattern in a hydroclimate time series. To be specific, we used the discrete wavelet decomposition method to separate the trend pattern in a time series, and more importantly, we then established the discrete wavelet spectrum to assess its statistical significance, which is the novelty of this study and is different from previous studies.

Following the helpful comments given by both the Reviewer 1 and 2, we have carefully rewritten and added many contents in the revised manuscript, and added some new references about the use of DWT for trend identification, mainly to more clearly explain the DWS approach proposed and emphasize the advantage of the approach.

*Comment 2. Application of DWS is limited on time series trend identification. Data interpretation is indeed important to understand the hydro-climate system. However, it would be much practically useful if the application can be extended to trend/data forecasting.*

**Reply:** Thanks for giving this favorable comment. Identification of trend is an important issue to understand the variability of hydroclimate process at long time scale, but it is not an easy task in practice. In the revised manuscript we mainly proposed the DWS approach for identifying the trend pattern in hydroclimate time series and assessing its significance.

Of course data simulation and forecasting is an important scientific issue and task to understand the future situations, and these have been many relevant studies, but in our opinion, accurate identification of the variability (including trend) of hydroclimate time series is the basic and primary task for the data interpretation and forecasting. Therefore, in the study we focused on the issue of trend identification, as an important basis of data forecasting at mid-to-long time scales.

*Comment 3. The analyzed hydro-climate data are averaged time series over 740 meteorological stations over China, if I understand correctly. By averaging, the features related to different climatic regimes. geological characteristics and geographical locations, etc. will be filtered out. To analyze the time series with different features would be of more interest and revealing than just to analyze the averaged data. Also, I don't think a time series with 53 annual value is long enough to detect the reliable trend*

**Reply:** Thanks very much. As explained above, in this study we mainly proposed the DWS approach for identifying the trend pattern in hydroclimate time series and assessing its statistical significance, but not investigating the variability of spatial-temporal variability of temperature and precipitation process over China. We used the TEM and PET time series mainly for verifying the applicability of proposed approach by analyzing the "warming" and the "warming hiatus" in temperature, and the reversed changes in potential evaporation, which cannot be described by a monotonic trend.

Besides, we consider that the identification of trend in a shorter time series would have more uncertainty, so in this study we proposed DWS approach just for assessing the statistical significance of trend pattern, which is described by a proper confidence interval in Figure 3 and 5. Those observed hydroclimate time series in China are usually about 50 years, by using which we can investigate their variability at decadal and multi-decadal time scales. Of course we know that a meaningful trend closely depends on the temporal scale concerned, and the proposed DWS approach can be used for identifying the trend pattern at different time scales. To clarify the point, we added some new contents in lines 129-133.

Thank you very much!

Best Regards!
Yan-Fang Sang

# A discrete wavelet spectrum approach to identifying non-monotonic trend pattern of hydroclimate data

Yan-Fang Sang[1,2], Fubao Sun[1], Vijay P. Singh[3], Ping Xie[4], Jian Sun[1]

1. Key Laboratory of Water Cycle & Related Land Surface Processes, Institute of Geographic Sciences and Natural Resources Research, Chinese Academy of Sciences, Beijing 100101, China;
2. Department of Atmospheric Sciences, University of Washington, Seattle 98195, Washington, USA;
3. Department of Biological and Agricultural Engineering & Zachry Department of Civil Engineering, Texas A and M University, 321 Scoates Hall, 2117 TAMU, College Station, Texas 77843-2117, U.S.A.
4. State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan 430072, China

**Corresponding author:**

Yan-Fang Sang: Tel/Fax: +86 10 6488 9310; E-Mail: sangyf@igsnrr.ac.cn, sunsangyf@gmail.com

Fubao Sun: E-Mail: sunfb@igsnrr.ac.cn

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**Abstract:** Hydroclimate system is changing non-monotonically and identifying its trend pattern is a great challenge. Building on the discrete wavelet transform theory, we develop a discrete wavelet spectrum (DWS) approach for identifying non-monotonic trend patterns in hydroclimate time series and evaluating their statistical significance. After validating the DWS approach using two typical synthetic time series, we examined the temperature and potential evaporation over China from 1961-2013, and found that the DWS approach detected both the "warming" and the "warming hiatus" in temperature, and the reversed changes in potential evaporation. ~~Interestingly, t~~Further, the identified trend patterns showed stable significance when the time series was longer than 30 years or so (i.e., the widely defined "climate" timescale). Our results suggest that non-monotonic trend patterns of hydroclimate time series and their significance should be carefully identified, and the DWS approach proposed has the potential for wide use in hydrological and climate sciences.

**Key words:** trend identification; discrete wavelet spectrum; decadal variability; statistical significance; Mann-Kendall test

## 1. Introduction

Climate and hydrological system are exhibiting great variability (Allen and Ingram, 2002; Trenberth et al., 2014). Quantitatively, identifying human-induced climate change signals in the usually changing hydroclimate system is of great socioeconomic significance (Diffenbaugh et al., 2008; IPCC, 2013), and remains a big challenge to both scientific and social communities. The simplest way to identify changes in the hydroclimate system would be to fit a monotonic (e.g., linear) trend at long time scales, at which a significance level would be assigned by a statistical test. Among the methods used for detection of trends, the Mann-Kendall non-parametric test is most widely used and has been successfully applied in climate change and its impact studies, when the time series is almost monotonic as required (Burn and Hag Elnur, 2002; Yue et al., 2002). However, due to its nonlinear and nonstationary nature, the hydroclimate system is changing and developing in a more complicated way rather than a monotonic trend way (Cohn and McMahon, 2005; Milly et al., 2008). For example, a debate on the recent change of global air temperature is receiving enormous public and scientific attention that the global air temperature increased during 1980-1998 passing most statistical significance tests and then stabilized afterwards till now, widely called “global warming hiatus” (Kosaka and Xie, 2013; Roberts et al., 2015; Medhaug et al., 2017). Another known example is “evaporation paradox” (Brutsaert and Parlange, 1998; Roderick and Farquhar, 2002) that potential evaporation has worldwide declined from the 1960s, again passing most statistical significance tests, but then reversed after the 1990s. In practice, for the hydroclimate time series, the non-monotonicity is more the rule rather than the exception (Dixon et al., 2006; Adam and Lettenmaier, 2008; Gong et al., 2010). Therefore, identifying the non-monotonic trend pattern hidden in those hydroclimate time series and assessing its statistical significance presents a significant research task for understanding hydroclimatic variability.

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Among those methods presently used in time series analysis, the wavelet method has the superior capability of handling the nonstationary characteristics of time series (Percival and Walden, 2000; Labat, 2005), so it may be more suitable for identifying non-monotonic trend patterns in hydroclimate time series. In a seminal work, Torrence and Compo (1998) placed the continuous wavelet transform in the framework of statistical analysis by formulating a significance test. Since then, the continuous wavelet method has become more applicable and rapidly developed to estimate the significance of variability in climate and hydrological studies. However, in the continuous wavelet results of time series, a known technical issue is the “data redundancy” (Gaucherel, 2002; Nourani et al., 2014), which is the redundant information across timescales leading to more uncertainty.

On the contrary, the other type of wavelet transform, i.e., the discrete wavelet method, has the potential to overcome that problem of data redundancy, in that those wavelets used for discrete wavelet transform must meet the orthogonal properties. Therefore, the discrete wavelet method can be more effective to identify the non-monotonic trend pattern in time series (Almasri et al., 2008; de Artigas et al., 2006; Kallache et al., 2005; Partal and Kucuk, 2006; Nalley et al., 2012). The discrete wavelet-aided identification of trend is usually influenced by some factors, such as choice of wavelet and decomposition level~~, and~~;

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moreover, the uncertainty evaluation of results should also be carefully considered. To overcome these problems, Sang et al. (2013) discussed the definition of trend, and further proposed a discrete wavelet energy function-based method for the identification of trend by comparing the difference of wavelet results between hydrological data and noise. The method used proper confidence interval to assess the statistical significance of the identified trend, in which the key equation for quantifying trend's significance is based on the concept of quadratic sum. However, ~~it~~ the practice of quadratic sum disobeys the common practice of computing variance in ~~of~~ spectral analysis, and sometimes cannot reasonably assess the significance of non-monotonic trend, because it neglects the big influence of trend's mean value. For instance, for those trends with small variation but big mean value, the quadratic sums are big values, based on which the statistical significance of trends would inevitably be over-assessed. Therefore, the evaluation of statistical significance of a non-monotonic trend in a time series should be based on its own variability but not other factors.

By combining the advantages of the discrete wavelet method and successful practice in the spectral analysis methods, this study aims at developing a practical but reliable discrete wavelet spectrum approach for identifying non-monotonic trend patterns in hydroclimate time series and quantifying their statistical significance, and further improving the understanding of non-monotonic trends by investigating their variation with data length increase. To do that, Section 2 presents the details of the newly developed approach building on the wavelet theory and spectrum analysis. In Section 3, we use both synthetic time series and annual time series of air temperature and potential evaporation over China as examples to investigate the applicability of the approach, which is followed by the discussion and conclusion in the final ~~S~~section~~ 4~~.

## 2. A discrete wavelet spectrum approach

Here we develop an approach, termed as "discrete wavelet spectrum approach," for identifying non-monotonic trend pattern in hydroclimate time series, in which the discrete wavelet transform (DWT) is used first to separate the trend pattern, and its statistical significance is then evaluated by using the discrete wavelet spectrum, whose confidence interval is quantified and described through the Monte-Carlo test.

Following the wavelet analysis theory (Percival and Walden, 2000), the discrete wavelet transform of a time series $f(t)$ can be expressed as:

$$W_f(j,k)=\int_{-\infty}^{+\infty} f(t)\psi_{j,k}^{*}(t)dt \quad with \quad \psi_{j,k}(t)=a_0^{-j/2}\psi(a_0^{-j}t-b_0k) \tag{1}$$

~~where~~ where $f(t)$~~$t$~~ is ~~the series to be analyzed with~~ a time order~~ $t$~~, and $\psi^{*}(t)$ is the complex conjugate of mother wavelet $\psi(t)$; $a_0$ and $b_0$ are constants, and integer $k$ is a time translation factor; $W_f(j,k)$ is the discrete wavelet coefficient under the decomposition level $j$ (i.e., time scale $a_0^{j}$). In practice, the dyadic DWT is used widely by assigning $a_0$=2 and $b_0$=1:

$$W_f(j,k)=\int_{-\infty}^{+\infty} f(t)\psi_{j,k}^{*}(t)dt \quad with \quad \psi_{j,k}(t)=2^{-j/2}\psi(2^{-j}t-k) \tag{2}$$

The highest decomposition level $M$ can be calculated as $log_2(L)$ (Foufoula-Georgiou and Kumar, 2014), where $L$ is the length of series $f(t)$. The sub-signal $f_j(t)$ in the original series $f(t)$ under each level $j$ ($j$ = 1, 2, …, $M$) can be reconstructed as:

$$f_j(t)=\sum_k W_f(j,k)\psi^{*}(2^{-j}t-k) \tag{3}$$

where the sub-signal $f_j(t)$ at the highest decomposition level (when $j$ = $M$)~~–~~ defines the non-monotonic trend pattern of the series $f(t)$, as generally understood. However, it should be

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noted that a meaningful trend closely depends on the temporal scale concerned. If the variability of series $f(t)$ on certain smaller time scale $K$ ($K<L$) is concerned, the proper decomposition level can be determined as $log_2(K)$; then, the sum of those sub-signals at the time scales bigger ~~$M$~~$K$ can be the non-monotonic trend pattern identified.~~.~~

Sang (2012) discussed the influence of wavelet and decomposition level choice and noise type on the discrete wavelet decomposition of time series, and further proposed some methods to solve them. By doing Monte-Carlo experiments, he found that those seven wavelet families used for DWT can be divided into three types, and recommended the first type, by which wavelet energy functions of various types of noise are stable and thus have little influence on the wavelet decomposition of time series. Specifically, one chooses an appropriate wavelet, according to the relationship of statistical characteristics among the original series, de-noised series and removed noise, chooses a proper decomposition level by analyzing the difference between energy function of the analyzed series and that of noise, and then identifies the deterministic components (including trend) by conducting significance testing of DWT. These methods are~~–~~ ~~based~~ built on the composition and variability of hydroclimate time series ~~itself~~at different time scales.~~, and thus are reliable and reasonable.~~ They were used here to accurately identify the non-monotonic trend pattern in a time series and assess its statistical significance.

Further, to establish the discrete wavelet spectrum (DWS) of time series, we need to specify a spectrum value $E(j)$ for each sub-signal $f_j(t)$ (in Eq. 3), based on which we can quantitatively evaluate its importance and statistical significance. Here we define $E(j)$ at the

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$j$th level by taking the variance of $f_j(t)$ following the general practice in conventional spectral analysis methods (Fourier transform, maximum entropy spectral analysis, *etc.*):

$$E(j) = \mathrm{var}(f_j(t)) \tag{4}$$

It can accurately quantify the intensity of variation of sub-signals (including trend) by eliminating the influence of their mean value, which is ~~obviously~~ different from the quadratic sum-based method proposed in Sang et al. (2013). For hydroclimate time series, both stochastic and deterministic components generally have different characteristics from purely noise components (Sang et al., 2012; Rajaram et al., 2015). ~~Interestingly, d~~Due to the grid of dyadic DWT (Partal and Cigizoglu, 2008), discrete wavelet spectra $E_r(j)$ of various noise types strictly follow an exponentially decreasing rule with base 2 (Sang 2012):

$$E_r(j) = 2^{-j} \tag{5}$$

The discrete wavelet spectra of deterministic components and that of noise are different. Hence, we define the DWS of noise as the “reference discrete wavelet spectrum (RDWS)~~;~~”, based on which we evaluate the statistical significance of the non-monotonic trend pattern of a time series.

To be specific, we design a technical flowchart to show how we develop the DWS approach for identifying the non-monotonic trend pattern of time series, and also for evaluating the statistical significance of that trend pattern (see the detail in Figure 1):

(1) For the series $f(t)$ normalized with length $L$, we analyze it using the DWT in Eq. (2) and (3), and calculate its discrete wavelet spectrum by Eq. (4);

(2) For the comparison purpose, we then use the Monte-Carlo method to generate the normalized noise data $N$ with the same length as the series $f(t)$, and determine its

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RDWS by Eq. (4). Considering that discrete wavelet spectra of various types of noise just consistently follow Eq. (5), here we generate the noise data following the standard normal distribution;

(3) We repeat the above ~~steps~~ step for 5000 times, and calculate the mean value and variance of the spectrum values (in Eq. 4) of the normalized noise data $N$ at each decomposition level $j$, based on which we can estimate an appropriate confidence interval of RDWS at the concerned confidence level. In this study we mainly considered the 95% confidence level;

(4) In comparing the discrete wavelet spectrum of the series $f(t)$ and the confidence interval generated by that of the noise (i.e., the RDWS), we can easily identify the deterministic components under the highest level as the non-monotonic trend pattern of the series, and determine whether it is significant. Specifically, if the spectrum value of the analyzed series' sub-signal under the highest level is above the confidence interval of RDWS, it is thought that the non-monotonic trend pattern is statistically significant; otherwise, if the spectrum value of the sub-signal under the highest level is in the confidence interval of RDWS, it is not statistically significant;

~~(4)~~(5) If a smaller time scale $K$ is concerned, we can use the decomposition level $log_2(K)$, instead of $M$, and then repeat the steps (1-4) to identify the non-monotonic trend pattern at that time scale.~~.~~

~~Because the DWS approach fits well the common idea of spectral analysis, and its superiority compared to the method in Sang et al. (2013) can be clearly understood, but they are not compared here.~~ In the following section, we mainly investigate the applicability of the

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DWS approach for identifying non-monotonic trend and its significance, and further investigate the variation of non-monotonic trend with data length increase to improve our understanding of trend.

**< Figure 1>**

## 3. Results

### 3.1 Synthetic series analysis

To test and verify the applicability of the developed discrete wavelet spectrum (DWS) approach for identifying non-monotonic trend pattern of a time series, we consider the general hydrological situations and use two synthetic data, generated with known signals and noise a priori. For investigating the variation of non-monotonic trend with data length increase, we set the length of the two series as 200, and the noise in them follows a standard normal distribution. The first synthetic series S1 consists of an exponentially increasing line and a periodic curve (the periodicity is 200) with some noise content (Figure 2, left panel); and the second synthetic series S2 is generated by including a hemi-sine curve, a periodic curve (the periodicity is 50) and some noise content (Figure 2, right panel). Using the MK test, series S1 shows a significant increase but the trend of series S2 is not significant.

In the DWS approach (Figure 1), we concern the time scale as data length, and use the Daubechies (db8) wavelet to decompose series S1 into seven (i.e., $<log_2 200$) sub-signals using Eq. (2) and Eq. (3)~~, and~~. Then, we take the sub-signals under the seventh level as the defined non-monotonic trend pattern. As shown in Figure 2 (left panel), the identified non-monotonic trend pattern in series S1 is similar to the true trend pattern. ~~Interestingly,~~However, the linear fitting curve (a monotonic curve) could not capture the detail

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of the trend pattern. The same approach applies to series S2 in Figure 2 (right panel) and the conclusion is not changed. Moreover, for series S2 with large variation at long time scales, the linear fitting curve or other monotonic curves may not be physically meaningful.

**< Figure 2>**

We compute the discrete wavelet spectra of the two synthetic series using Eq. (4), and use the reference discrete wavelet spectrum with 95% confidence interval to evaluate the statistical significance of their trend patterns. That is, if the red point at certain data length is above the 95% confidence bar, described by the blue line in Figure 3, it is thought that the trend pattern is significant at 95% confidence level. Using our DWS approach, the trend pattern of series S1, which is quasi-monotonic, is found significant (Figure 3a) as in the MK test (Figure 3c), but the non-monotonic series S2 shows a significant trend pattern (Figure 3b), which is different from the MK test (Figure 3d).

In Figure 3, we also present the significance of the identified trend patterns of the two series using both our DWS approach and the MK test and we change the length of the series to investigate the stability of the statistical significance. Generally, it would have more uncertainty when evaluating the statistical significance of trend pattern with a shorter length, corresponding to a bigger 95% confidence interval. Using our DWS approach, the 95% confidence interval (i.e., the height of blue bars in Figure 3) for evaluating the statistical significance of trend pattern generally decreases with the increase of data length~~ of data,~~ , as expected. However, in the MK test, the significance is always determined by the constant thresholds of +/-1.96, regardless of the data length.

In the DWS results in Figure 3, the significance levels of the trend patterns do not consistently decrease with data length, but show some fluctuation, ~~One would expect that~~ as the proportions of different components (including trend) in the original series vary with data length. Furthermore, one would expect that if the trend pattern of a series at a certain length is identified statistically significant, the trend pattern would extend with the increase of data length, thus its~~the~~ significance~~e–~~ may be more stable with a larger length of data considered. Using our DWS approach, the trend pattern of series S1 is significant when the data length is larger than 55 (Figure 3a), being similar to the result of the MK test (Figure 3c)~~. Interestingly, using our DWS approach,~~; the trend pattern of the series S2 is statistically significant when the data length is larger than 75 (Figure 3b~~),~~). However, ~~but~~ using the MK test, the monotonic trend pattern of series S2 is significant only when the data length is between 40 and 185 (Figure 3d). In summary, the significance of trend pattern identified by our DWS approach is more stable than that detected by the MK test, demonstrating the advantage of the DWS approach in dealing with non-monotonic hydroclimate time series.

**< Figure 3>**

### 3.2 Observed data analysis

We use the annual time series of mean air temperature (denoted as TEM) and potential evaporation (denoted as PET) over China to further verify the applicability of our developed DWS approach for identifying non-monotonic trend pattern of a time series. The two series were obtained from the hydroclimate data measured at 740 meteorological stations over China, with the same measurement years from 1961 to 2013. The data have been quality-checked to ensure their reliability for scientific ~~studies~~researches. The PET series was calculated from the Penman-Monteith approach (Chen et al., 2005).

Given the general nonstationary nature of observed hydroclimate time series, linear trends or more generally monotonic curves could not capture the trend pattern with large decadal variations and therefore are not particularly physically meaningful. In Figure 4 (left panel), we present the annual TEM time series visually showing nonstationary characteristics and non-monotonic variation. The TEM series decreases till the 1980s with fluctuations and then sharply rises till the 2000s, followed by a decreasing tendency. The large fluctuation of the mean air temperature after late 1990s is the known phenomenon of the "global warming hiatus" (Roberts et al., 2015). The linear fitting curve obviously missed out the more complicated trend pattern of the observed temperature time series. Using our DWS approach, we decompose the TEM series into five (i.e., <$log_2 53$) sub-signals using Eq. (2) and Eq. (3), and take the sub-signals under the fifth level as the trend pattern, which realistically presents the nonstationary variability of temperature over China (Figure 4, left panel).

We also apply this DWS approach to the annual PET time series. In the time series of PET (Figure 4, right panel), there was a decreasing trend for the period from 1961 to the 1990s, which is the known "evaporation paradox" leading to controversial interpretations continuing over the last decade on hydrological cycles (Brutsaert and Parlange, 1998; Roderick and Farquhar, 2002). That decreasing trend was then followed by an abrupt increase in around the 1990s, almost the same time when solar radiation was observed to be reversing its trend, widely termed as "global dimming to brightening" (Wild, 2009). ~~Interestingly~~Surprisingly, after the mid-2000s, PET starts to decrease again (Figure 4, right panel). Sometimes, one would propose to fit linear curves for separate periods. Again, linear curves could not capture the overall trend pattern of the PET series. Using the same DWS approach, we identify non-monotonic trend pattern of the PET series (Figure 4, right panel), which captures the two turning points of the changing trends in the 1990s and the 2000s.

**< Figure 4>**

The changes of trends in terms of magnitudes and signs for different periods lead to the difficulty in assessing and interpreting the significance of trends. For example, the PET time series shows a significant decrease using the MK test ($-3.76 < -1.96$) during 1961-1992 (Figure 5d). At that moment before the reversed trend reported, the significant decrease could be literally interpreted as that PET has significantly declined and might be declining in the future. However, the PET time series reversed after the 1990s and again in the 2000s, coming with an insignificant overall trend for the whole period of 1961-2013. For the more or less monotonic time series of the TEM series (1961-2013), the MK test detects a significant increase ($6.00 > 1.96$) (Figure 5c), which leads to the surprise when air temperature was reported to have stopped increasing after late the 1990s. In summary, it becomes vital to develop an approach for testing the significance of trend pattern, which is suitable for non-monotonic time series.

In this study, building on the discrete wavelet transform theory, we propose an operational approach, i.e., the DWS, for evaluating the significance of non-monotonic trend pattern in the TEM (Figure 5a) and PET (Figure 5b) series. For comparison purpose, we also conduct the significance test for the two time series using the MK test (Figure 5c and 5d). Similar to Figure 3, we change the data length to investigate the stability of statistical significance (Figure 5). Again, the result indicates that the 95% confidence interval for evaluating the statistical significance generally decreases with the data length, which is different from the constants +/-1.96 adopted in the MK test. The significance test using our DWS approach appears to be more stable with the data length than the MK test (Figure 5). ~~Interestingly, u~~Using our DWS approach the trend pattern in the TEM series becomes significant when the data length is 30 and the significance is more stable when it is greater than 35 (Figure 5a). For the case of the PET series, the trend pattern becomes statistically significant when the data length is larger than 25 (Figure 5b). The findings here have important implications for non-monotonic hydroclimate time series analysis, in that the timescale of defining *climate* and *climate change* by the World Meteorological Organization is usually 30 years (Arguez and Vose, 2011) and in hydrological practice it is between 25-30 years.

For the whole time series investigated here, whose length is larger than 30 years, we are able to examine the significance using the developed DWS approach. Combining the trend pattern in Figure 4 (left panel) and the significance test in Figure 5a, we can confirm that the trend pattern of the TEM time series from 1961-2013 identified in this study is significant at the 95% confidence interval. Similarly, the trend pattern in PET is also significant (Figure 4 right panel and Figure 5b). The significance test results suggest that the three main stages of the series (red lines, Figure 4) are detectable as the overall trend pattern from the variability of the series and are vital to understanding how the temperature and the PET series are changing. In particularly, the reversed changes in PET and its significance can be revealed by our DWS approach, which can provide more useful and physically meaningful information. Our results suggest that the non-monotonic trend pattern of hydroclimate time series and its significance should be carefully identified and evaluated.

**< Figure 5>**

## 4. Summary and Conclusion

Climate and hydrological system are changing non-monotonically. Identification of linear (or monotonic) trends in hydroclimate time series, as a common practice, cannot capture the detail of the trend pattern in the time series at long time scales, and then can lead to misinterpreting climatic and hydrological changes. Therefore, revealing the trend pattern of the time series and assessing its significance from the usually varying hydroclimate system remains a great challenge. To that end, we develop the discrete wavelet spectrum (DWS) approach for identifying the non-monotonic trend in hydroclimate time series, in which the discrete wavelet transform is used first to separate the trend pattern, and its statistical significance is then evaluated by using the discrete wavelet spectrum (Figure 1). Using two typical synthetic time series, we examine the developed DWS approach, and find that it can precisely identify non-monotonic trend pattern in the synthetic time series (Figure 2) and has the advantage in significance testing (Figure 3).

Using our DWS approach, we identify the trend pattern in the annual time series of average temperature and potential evaporation over China from 1961-2013 (Figure 4). The identified non-monotonic trend patterns precisely describe how temperature and PET are changing. Of particularly interest here is that the DWS approach can help detect both the "warming" and the "warming hiatus" in the temperature time series, and reveal the reversed changes and the latest decrease in the PET time series. The DWS approach can provide other aspects of information on the trend pattern in the time series, i.e., the significance test. Results show that the trend pattern becomes more significant and the significance test becomes more stable when the time series is longer than a certain period like 30 years or so, the widely defined “climate” time scale (Figure 5). Using the DWS approach, in both time series of mean air temperature and potential evaporation, the identified trend patterns are found significant (Figure 5).

In summary, our results suggest that the non-monotonic trend pattern of hydroclimate time series and its statistical significance should be carefully identified and evaluated, and the DWS approach developed in this study has the potential for wide use in hydrological and climate sciences.

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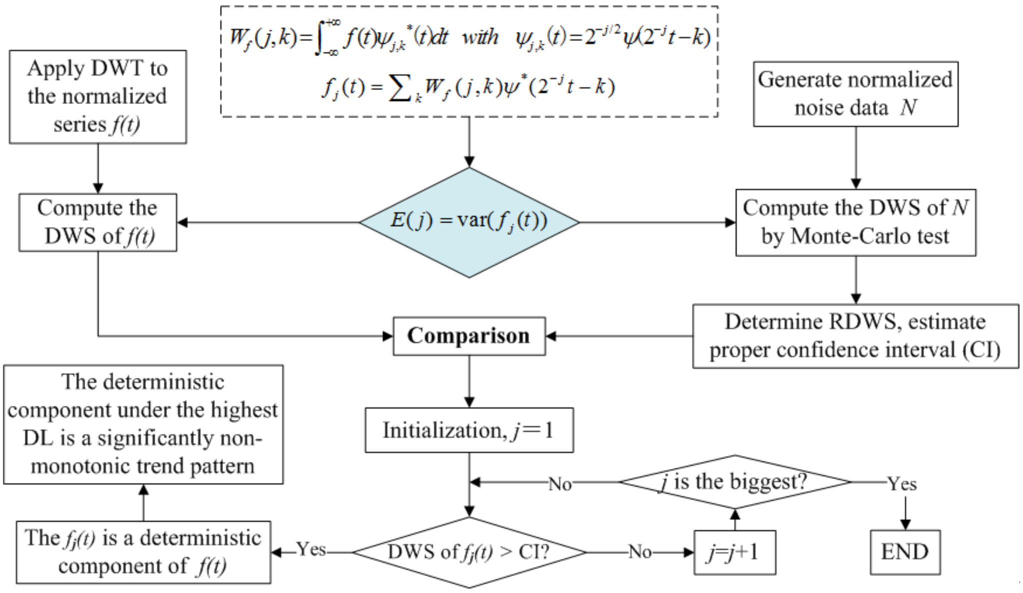
**Figures**



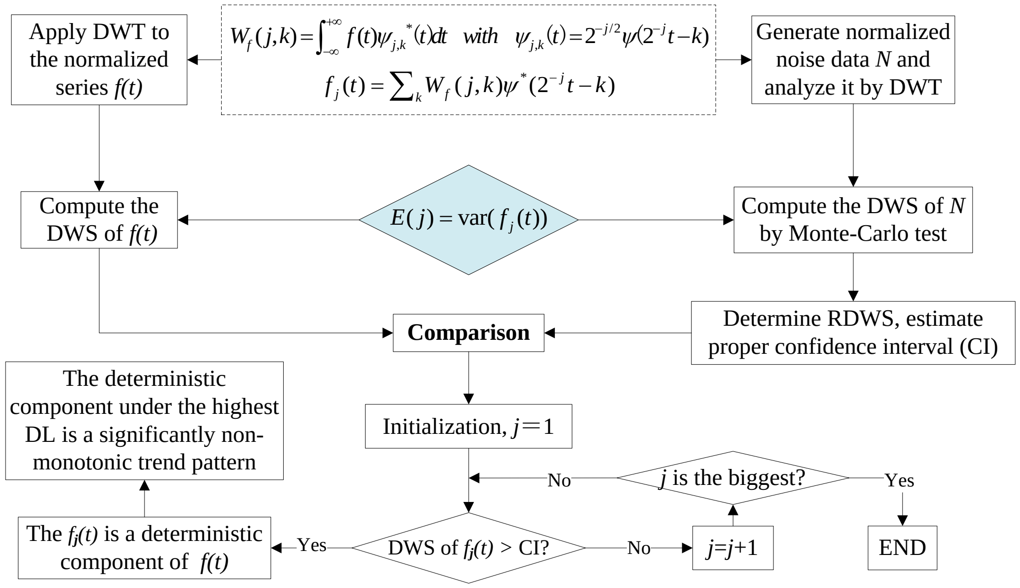




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**Figure 1.** Technical flowchart for identification of the non-monotonic trend pattern in a time series using the discrete wavelet spectrum approach developed. In the figure, “DWT” is the discrete wavelet transform, “DWS” is the discrete wavelet spectrum, “RDWS” is the reference discrete wavelet spectrum, “DL” is the decomposition level, and “CI” is the confidence interval.

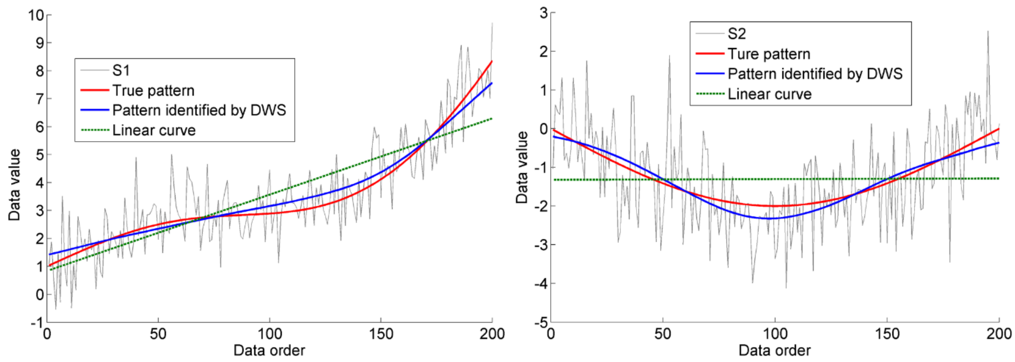


**Figure 2.** Non-monotonic trend patterns in the synthetic series S1 and S2 identified by the discrete wavelet spectrum (DWS) approach, and the linear trends in the two series. Synthetic series S1 is generated as: $S1=1.112^{0.1t}+0.8\times\sin(0.01\pi t)+\alpha$; and synthetic series S2 is generated as: $S2=\sin(0.04\pi t)+2\times\sin(\pi+0.005\pi t)+\alpha$, where $\alpha$ is a random process following the standard normal distribution.

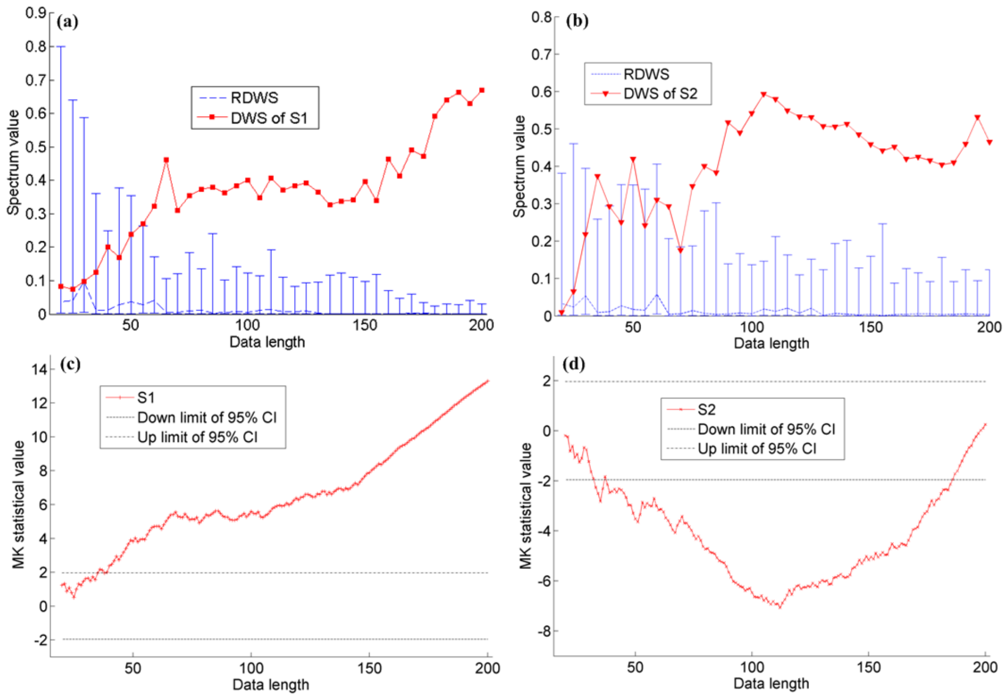


**Figure 3.** Evaluation of statistical significance of non-monotonic trend patterns in the synthetic series S1 (a) and S2 (b) with different data length by the discrete wavelet spectrum (DWS) approach, and the results by the Mann-Kendall (MK) test (c and d). In figure a and b, the blue line is the reference discrete wavelet spectrum (RDWS) with 95% confidence interval under each data length; if the red point at certain data length is above the blue bar, it is thought that the trend pattern is significant at 95% confidence level.~~; and~~ ~~i~~ In figure c and d, the two black dash lines indicate 95% confidence interval (CI) with the thresholds of +/- 1.96 in the MK test.

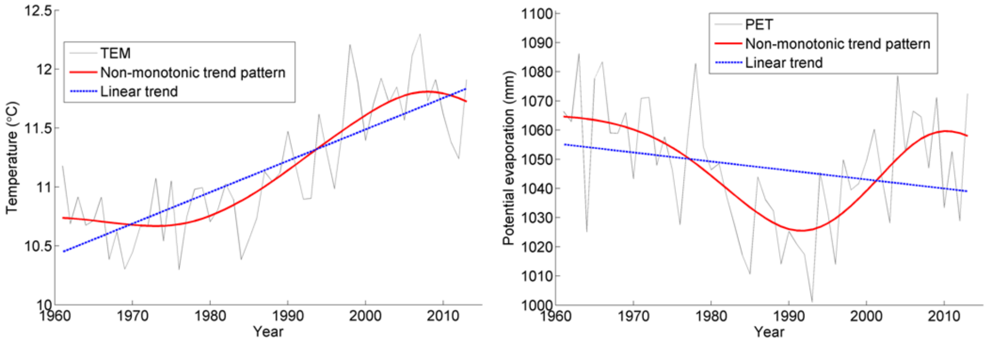


**Figure 4.** Non-monotonic trend patterns in the annual time series of the mean air temperature (TEM) and the potential evaporation (PET) over China from 1961-2013 identified by the discrete wavelet spectrum (DWS) approach, and the linear trends in the two series.

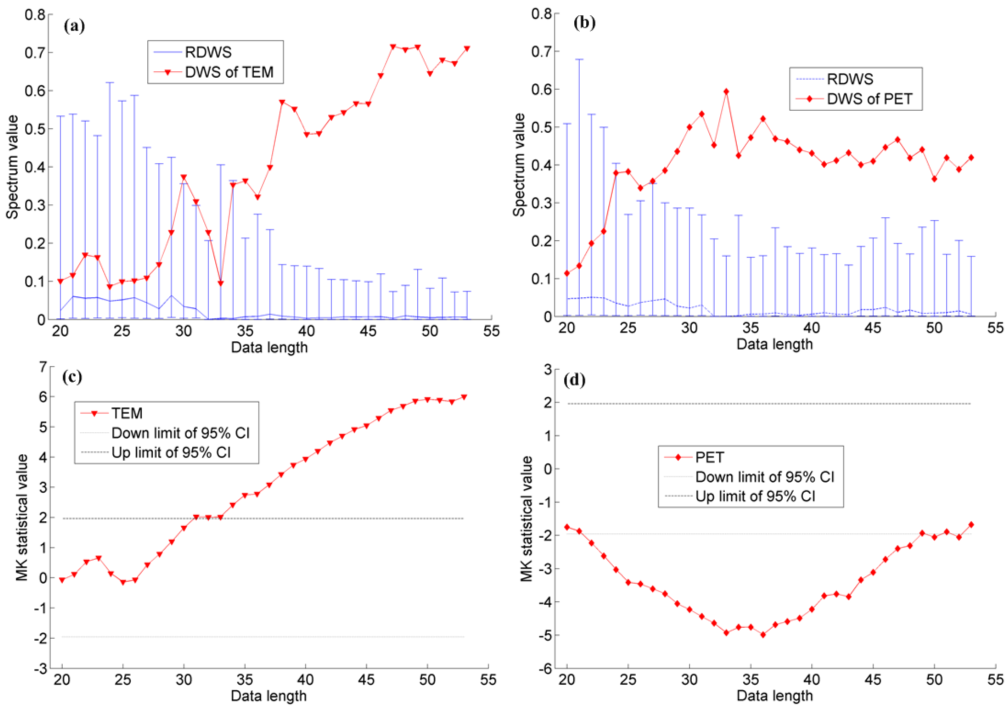


**Figure 5.** Evaluation of statistical significance of non-monotonic trend patterns in the annual time series of the mean air temperature (TEM, a) and the potential evaporation (PET, b) over China with different data length by the discrete wavelet spectrum (DWS) approach, and the results by the Mann-Kendall (MK) test (c and d). In figure a and b, The blue line is the reference discrete wavelet spectrum (RDWS) with 95% confidence interval under each data length; and in figure c and d, the two black dash lines indicate 95% confidence interval (CI) with the thresholds of +/- 1.96 in the MK test.