To: Editor, Hydrology and Earth System Sciences
Subject: Revised manuscript (#hess-2017-6)
The Authors: Sang Y.F., et al.
Title: A discrete wavelet spectrum approach to identifying non-monotonic trend pattern of hydroclimate data

Response:

The authors appreciate the Editor and Reviewers for helpful and constructive comments that improved our original manuscript submitted to HESS.

Response to Reviwer#1's comments:

Comment 1: The method section seems to be the main contribution of this paper, but it is a bit terse and would be challenging for someone not familiar to wavelets to understand the approach. Wavelets are described in many papers and textbooks, but the use of wavelets to identify trends is not common in hydrology. It would be helpful to provide the reader with more background information so the reader can understand why certain decisions are being made here. That is, methods have few equations and have short statements of the assumptions that go into the choice of equations. The following comments identify specific locations of the text where the reader could use more information on the methods.

Reply: Thanks very much for giving these valuable comments and suggestions, which is very helpful for improving the study results of our paper. The DWS approach proposed is the main contribution in our paper. Following the comment, we have rewritten and added many contents in Section 2, especially about the determination of proper decomposition level and the main technical steps of trend identification using the DWS approach, mainly to make the proposed approach more understandable. Besides, we also added some new references in the revised manuscript. More details can be found in the following point-to-point response.

Comment 2: It would be helpful to the reader to provide background on decomposition M and why this is important for identifying a trend. Please consider adding some background on the decomposition level and why the largest level has a temporal scale that is L, the length as the time series. More specifically, why the largest level could be considered to be a trend. It could be noted that a smaller temporal scale could be important, and the decomposition level can be calculated as log2(T) if T is a temporal scale other than L, the length of the time series.

Reply: Thanks very much for giving this valuable comment. The decomposition level j just reflects the time scale (i.e., time scale a_0^j in Eq. 2) for wavelet analysis, which is important to identify the trend pattern in a hydroclimate time series. Following the comment, we further explain the key point about the decomposition level by adding some contents in lines 121 and 124-125 and adding a new reference.

As generally considered, the variation of a time series at the biggest time scale (i.e., its data length) reflects its trend, thus in practice, the data length is usually chosen at the time scale for the trend identification. We also know that the time scale is an important factor for trend identification, and if those variations at smaller time scales are concerned, we can change the proper decomposition level in Eq. (2) and then get the trend pattern. Following the valuable comment, we added some contents as "*However, it should be noted that a meaningful trend closely depends on the temporal scale concerned. If the variability of series f(t) on certain smaller time scale K (K<L) is concerned, the proper decomposition level can be determined as $log_2(K)$; then, the sum of those sub-signals at the time scales bigger M can be the non-monotonic trend pattern identified.*" in lines 129-133 in the revised manuscript.

Comment 3: Line 126: Can you indicate which wavelet is used in this analysis?

Reply: Thanks. Following the helpful comment, in the revised manuscript we explained the approach for the choice of proper wavelet in lines 139-141. Further, the wavelet used for the analysis of time series in the study was added in line 208.

Comment 4: Line 140: The text says that a spectrum is needed. Can you explain why E(j) is needed for each sub-signal?

Reply: Thanks. Because we want to establish a reliable discrete wavelet spectrum for assessing the statistical significance of trend pattern in a time series, in the revised manuscript we defined the *E(j)* as a spectrum value at each decomposition level *j*. As explained in lines 155-159, the discrete wavelet spectra $E_r(j)$ of various noise types strictly follow an exponentially decreasing rule with base 2 along with the decomposition level increase, which is obviously different from that of hydroclimate time series, thus we can take the former as a basis for establishing the discrete wavelet spectrum approach.

Following the favorable comment, we added some contents as: "*to establish the discrete wavelet spectrum (DWS) of time series, we need to specify a spectrum value E(j) for each sub-signal $f_j(t)$ (in Eq. 3), based on which we can quantitatively evaluate its importance and statistical significance.*" in lines 147-149 in the revised manuscript.

Comment 5: Line 82: This is a good opportunity to add references to prior studies that document the DWT approach for trend estimation.

Reply: Thanks. We have added five new references in lines 84-85 in the revised manuscript.

Comment 6: Line 90: Can you add more description about which common practice is

Reply: Thanks. Following the helpful comment, we rewrote the sentence as "*However, the practice of quadratic sum disobeys the common practice of computing variance in spectral analysis, and sometimes cannot reasonably assess the significance of non-monotonic trend*" in lines 93-95 in the revised manuscript.

Comment 7: Line 135: Other studies have described similar approaches to identify a deterministic trend using DWT (e.g. Kallache et al., 2005). Could a stochastic component be added using the framework presented here?

Reply: Thanks. In the revised manuscript we mainly proposed the DWS approach for identifying the trend pattern in a hydroclimate time series. To be specific, we used the discrete wavelet decomposition method to separate the trend pattern in a time series, and more importantly, we then established the discrete wavelet spectrum to assess its statistical significance, which is the novelty of this study and is different from previous studies.

Hydroclimate time series is generally composed of deterministic components, stochastic components and noise, and identification of the significance of a trend pattern is just to judge if it is a deterministic component. As explained above, in the revised manuscript we defined the $E(j)$ as a spectrum value at each decomposition level j, and found that the discrete wavelet spectra $E(j)$ of various noise types strictly follow an exponentially decreasing rule with base 2, which is obviously different from that of deterministic and stochastic components in hydroclimate time series, thus we took the former as a basis for establishing the discrete wavelet spectrum approach. That is, a stochastic component can be added using the framework presented in our study, and it would not influence the identification of trend pattern and its statistical significance.

Comment 8: Line 137: This statement is subjective. Can you add references here to show why you are assuming that these methods are reliable and reasonable? What is your criteria for what is reliable and reasonable?

Reply: Thanks. Following the favorable comment, these subjective statements were removed in the revised manuscript.

Comment 9: Line 145: Please consider omitting the word "obviously." This is subjective and the result may not be obvious to everyone.

Reply: Thanks. The subjective word "obviously" was removed in line 154 in the revised manuscript.

Reply: Thanks. These inaccurate statements were removed in the revised manuscript.

Reply: Thanks very much for giving this valuable comment. In our opinion, the significance of a trend pattern is determined by both its own magnitude and its proportion in the original time series, and the significance of a trend pattern would change with data length, because the proportions of different components (including trend) in the original series vary with data length.

Following the comment, we added some new contents to more clearly explain why the significance level of a trend pattern varies with data length. To be specific, we explained that "*Generally, it would have more uncertainty when evaluating the statistical significance of trend pattern with a shorter length, corresponding to a bigger 95% confidence interval*" in lines 227-229 and "*the significance levels of the trend patterns do not consistently decrease with data length, but show some fluctuation, as the proportions of different components (including trend) in the original series vary with data length*" in lines 234-236.

Reply: Thanks. The inaccurate word "interesting" was removed or changed as a more accurate word throughout the manuscript.

Reply: Thanks very much. Following the valuable comment, we divided the analysis process of trend identification using the DWS approach into five steps, and more details can be found

in lines 168-190. It is just for making the Figure 1 presentable by putting the DWT equation on the top. Following this comment, the Figure 1 was carefully readjusted.

Moreover, following the valuable comment, we added some new contents to explained the results in Figure 3 as "*That is, if the red point at certain data length is above the 95% confidence bar, described by the blue line in Figure 3, it is thought that the trend pattern is significant at 95% confidence level*" in lines 219-221, and in its caption.

Comment 14: References: Please add publication year to each reference.

Reply: Thanks. We have checked all the references and make sure that all references have publication year.

Thank you very much!

Best Regards!
Yan-Fang Sang