General Comments:
This paper uses an idealized simulation with prescribed soil moisture gradients to derive a simplified algorithm that represents the amount of precipitation generated by local evaporation and advection terms. The authors note that previous studies have qualitatively shown how soil moisture gradients and atmospheric profile influence precipitation, and state that their goal is to quantitatively isolate the primary drivers of precipitation. I believe their methods, i.e. using an idealized model with prescribed soil moisture gradients, are sound, and their results are relevant. Overall, I find that the paper convincingly demonstrates the relative important of soil moisture gradients over the absolute magnitude of soil moisture, which makes sense physically, but it glosses over some other important points that deserve more explanation, such as the importance of the atmospheric profiles. Also, the derivation of the algorithm they use seems fine, but needs some clarification in order for the reader to be able to completely recreate their results. The second stated goal of the paper is to determine "what is the relative role of the atmosphere, or in other words the efficiency in converting these potential moisture sources into precipitation." Terms that represent the efficiency of advection and evaporation are derived, but there is no discussion of how the actual atmospheric profile impacts those terms, which then detracts from the significance of these findings. Also, while the authors cite publications that use the two atmospheric profiles utilized in the model simulations, they do not display them in a figure or discuss them in any way. This leaves the reader wondering what the difference is between them, what the profiles are like, and how these profiles could affect the results. For example, a profile that is more unstable could increase convection and strengthen the circulation, however there is no context like this provided in the paper. Also, I looked up the two profiles in the cited publications and found it difficult to compare them because they are presented in different formats. Because of these oversights, the reader is left unsure why the authors included two different profiles in the first place, and how the atmospheric profile impacts the authors' findings.
Response:
We thank the reviewer for his/her comments which helped us to revise the parts of the manuscript which were not clear. As suggested by the reviewer we clarified why the particular soundings employed in the study were chosen and decided to add a figure with two skew-t diagrams relative to the different atmospheric soundings. Furthermore, we elaborated more on the physical meaning of the efficiencies and why the atmospheric profiles have different ones. In the following we present the responses to the reviewer specific comments and technical corrections, which also answer the questions presented in the general comments.

Specific Comments:
1. page 3 line 10-12: "the change of precipitation with soil moisture does not depend on the soil moisture content itself and that the most efficient way to increase precipitation consists in increasing the surface wetness gradient.", but page 1 line 8-9: "these changes surprisingly do not depend on soil moisture itself but instead purely on parameters that describe the atmospheric initial state." — is it the atmospheric state or the soil moisture gradient that is most important? Also, see my other comments about the importance of addressing the atmospheric state more thoroughly in the paper.
Response: we agree with the reviewer that the presence of both sentences was misleading. For this reason we modified them in the manuscript and added some clarification notes on the dependency of precipitation on both soil moisture and the atmospheric state. We revised section 4.3 and stressed the conclusion that, although the derivative of precipitation does not depend on soil moisture but just on the atmospheric state through the efficiencies and the B parameter, the absolute value of precipitation does depend on soil moisture as shown in Eq. 15.

2. Page 10, Line 11: "In order to test the validity of the theory proposed in section 2" is confusing. This is stated in section 2, and I'm not sure what the theory is. Suggest repeating what the theory is or otherwise clarifying here.

Response: We apologise for the wrong reference: it should have been Section 3 instead. We corrected this in the manuscript.

3. Page 4: Please clarify why the "dry-soil advantage profile of Findell and Eltahir 2003" is used and why it is appropriate for this investigation.
Response: This particular sounding was observed on 23 July 1999 in Lincoln, Illinois (USA) and was chosen as a typical example by Findell and Eltahir 2003 for cases when a strong heating of the surface forces the triggering of convection. Given that on the dry patch low soil moisture availability causes strong sensible heat fluxes to heat the air above we though that using this sounding would produce the strongest response in the atmosphere.

4. Please include an additional figure with the two atmospheric profiles (from Findell and Eltahir 2003 and Schlemmer et al. 2012).
Response: we added an additional figure in the manuscript with a skew-t diagram of the two soundings (see Fig.1 of this document).

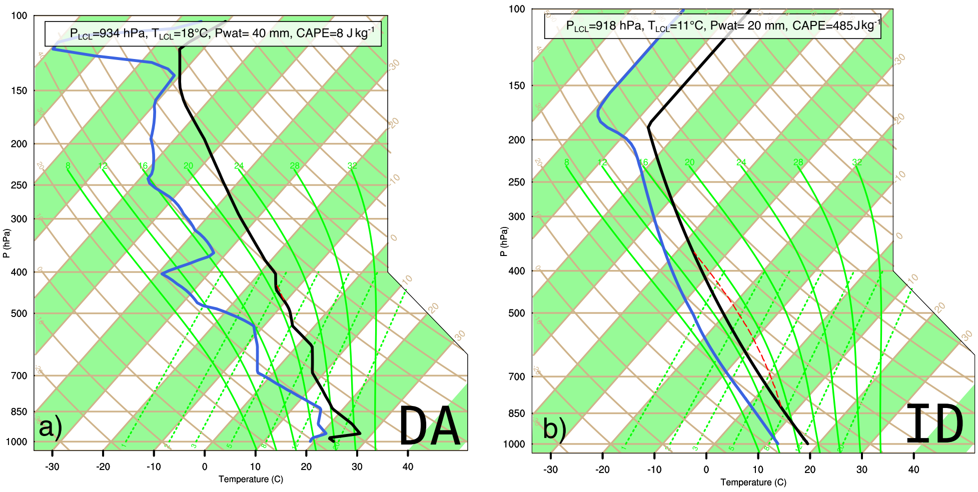

Figure 1: Skew-t diagrams for the sounding used in the simulations.

5. Page 4: Please clarify why the Schlemmer et al. (2012) profile is used over a different one, what question is answered by including it in the study, and how it differs from the profile from Findell and Eltahir 2003.
Response: we used the profile of Schlemmer et al. (2012) for one main reasons, namely that it greatly differs from the sounding of Findell and Eltahir (2003). In particular it has a lower surface temperature and lower integrated water vapour content although having a larger initial instability. This allows us to test the idealized model and show that efficiencies and the B parameter do depend on the atmospheric state. We performed additional simulations using the second sounding presented in Findell and Eltahir 2003 but, as the results were similar to the DA case, we didn't include those in the manuscript. We added a few sentences in the manuscript to justify our choice.

6. Figure 2: This figure takes some time and effort to interpret. It would be easier for the reader if vectors were used in place of windspeed contours and if the "dry" and "wet" sides are labeled. Also, please add a sentence to the text explicitly stating which side in Figure 2 is warmer (and why) and which direction the front is propagating. This all may seem obvious, and is stated more explicitly later in the text, but to the first-time reader it takes time to put it all together while examining figure 2.
Response: We changed Fig. 2 in order to ease its interpretation. We used vectors instead of contours to indicate zonal winds and used explicit labels for the dry and wet patches. We think now it is clear

where the patches are located so that there is no need to add another sentence to say which side of Figure 2 is warmer and in which direction the front is propagating.

7. Page 8 Line 24: Clarify what "the fact that one efficiency doesn't match well" means. Which efficiency? And it doesn't match well with what?
Response: We meant that one efficiency is not enough to describe the variations of precipitation. As shown in Fig. 6 when using a single efficiency the decrease of precipitation with increasing value of soil moisture on the dry patch cannot be captured. We rephrase this sentence in the manuscript.

8. Page 8 Line 29: which sounding is "another sounding?". Also see previous comments about soundings. This would be a good place to spend some time discussing what it is about the two profiles that result in efficiencies that are higher than with the first sounding.
Response: We were referring to the sounding of Schlemmer et al. (2012) so we added this explicit reference in the text. We think that the higher efficiencies obtained with the Schlemmer et al. (2012) sounding are due to a combination of different effects. One of those is the different convection triggering. With the sounding of Schlemmer et al. (2012) convection is triggered almost 1 hour before than with the sounding of Findell and Eltahir (2003). This allows the atmosphere to fully exploit the instability caused by the morning heating and to develop a stronger front propagation, as shown in Fig. 6 of the manuscript. Although the maximum advection of moisture over the dry patch in the ID cases is smaller than the one of the DA cases, the atmosphere is able to efficiently convert it into precipitation, thus leading to larger efficiencies. This is also evident in Fig. 2 of this

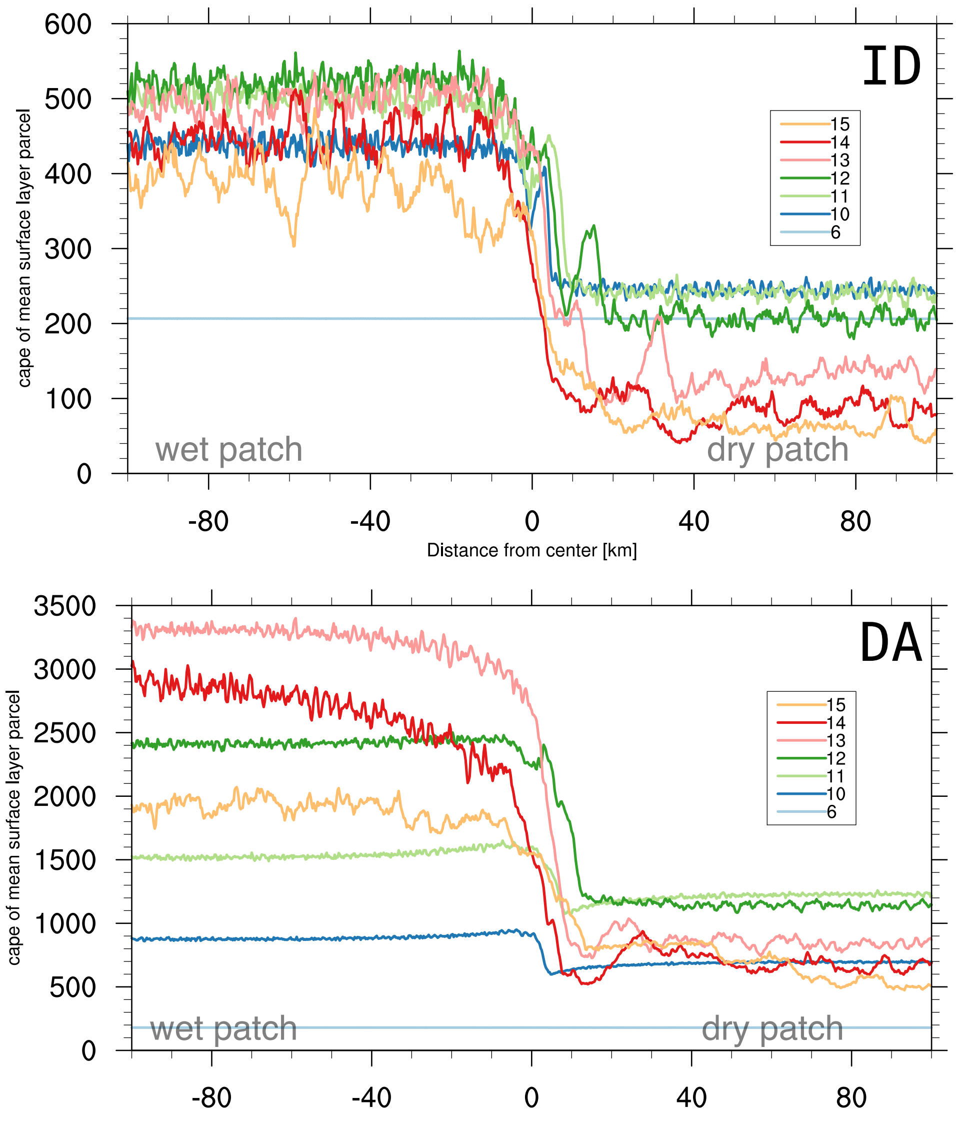

Figure 2: meridional average of convective available potential energy (CAPE, J/kg) for the simulation with the ID sounding (upper panel) and the DA sounding (lower panel). For both simulations the extreme case with a dry patch at 20% saturation is used. The lines indicate the value at different hours during the day (6 LST, initialization time, to 15 LST).

document where the value of CAPE is plotted as a function of time and x-dimension only. When using the DA sounding it can be seen that convective potential instability at 15 LST is larger than the one at the initial time over both patches, while in the ID sounding the opposite happens. Furthermore, the Findell and Eltahir (2003) sounding was not prone to the development of intense rain events, as shown in Cioni & Hohenegger (2017), thus the smaller efficiencies. We added this brief discussion to the manuscript.

9. Page 9 Line 2: "a weaker sensitivity of that particular atmospheric state" . . ..see above comments about the atmospheric profiles. This reference is too vague, and needs more explanation.
Response: We meant that with this sounding precipitation amounts seem not to depend much on the advected moisture, as the estimated B parameter is smaller. We corrected the reference on the manuscript.

10. Page 10 Lines 18-19. This is the first point that the soil type is referenced. The data and methods should include a sentence stating the soil type used in the simulation, the reason why it is used, and its field capacity.
Response: We added a sentence stating the soil type used (5, loam) and the reason why it was chosen, specifically because it is the most frequent soil type over Germany. Also the field capacity and the wilting point were added to the manuscript.

11. Page 12: The derivation of beta needs some more explanation. Was it derived using a best fit method from Figure 3? I'm not sure.
Response: as explained in Lines 11-20 the parameter B is obtained through a best fit of the values of advection and evaporation, where evaporation is approximated through Eq. 4. We further clarified this aspect in the manuscript.

12. As a reader, it was difficult to get through sections 4.2 and 4.3. There were some jumps in the logic between equations that were hard to follow, and not all terms were defined (see above). I think if the authors revisit these sections and provide more explicit explanations even where they think the transitions should be obvious, it will help the reader finish the paper.
Response: We agree with the reviewer. For this reason we revised section 4.2 and 4.3 by expanding all the steps used in deriving Eq. 8 and 9 from Eq. 5 and Eq. 15. We added a definition for all the missing variables and further expanded the explanations in the text.

13. Page 15, Line 20: "these parameters depend solely on the atmospheric state." See above comments.
Response: see answers above.

14. Figures 9 and 10: These are important figures. More explanation of these figures is needed, particularly the significance of n_a < n_b (and visa versa) and of beta, and what that means physically. As a reader, I found myself quite bogged down by this point and it was difficult to extract what the authors were hoping to convey with these figures.
Response: We agree with the reviewer: these are the most important figures of the paper and deserve more explanation. We think that the significance of n_a < n_e (and vice-versa) is related to the different way advection and evaporation sources are used by the atmosphere to produce precipitation. In our simulations we always find that the efficiency of advection is larger than the efficiency of evaporation which would mean that the atmosphere is somehow able to use more of the advected moisture than of the evaporated one to produce precipitation. The B parameter, instead, appears to be an additional parameter which describes the importance of advection. We think that this is related to the strength of cold pools, which enhance the advection processes. We revised the explanation of these figures.

Technical Corrections:
Page 1 Line 1-2: For clarity, I suggest rewording the first sentence of the abstract to read “Soil moisture heterogeneities influence the onset of convection and subsequent evolution of thunderstorms producing heavy precipitation through the triggering of mesoscale circulations.”
Response: Corrected.

Page 1 Line 6: Suggest rewording to read “A key element of the model is the representation of precipitation as a weighted sum”
Response: Corrected.

Page 1 Line 18: Suggest rewording to read “and which can then affect the distribution of precipitation.”
Response: corrected

Page 2 Line 17: Please clarify what is meant by “a negative spatial coupling coexists together with a positive temporal coupling.”
Response: We added a clarification sentence: " That is, areas drier than their surrounding (spatial component) but wetter than the climatological value (temporal component) may receive more precipitation than other ones"

Page 3 Section 2.1 heading: Is the subheading “2.1 Experimental Design” needed here? There are no other subsections in Section 2.
Response: corrected.

Page 3 Line 3: “overt” should be “over”
Response: corrected.

Page 6 Lines 6-8: This sentence is difficult to understand. I suggest rewording it.
Response: We added further clarification when describing the algorithm: "More specifically, at the first two time instants the maximum is searched over the entire dry patch while from the third time step onward the maximum search is performed in a box centered on a first guess obtained from a simple linear extrapolation of the previous time instants. "

Page 7 Line 11: “It is immediate to verify” is awkward. I suggest rewording.
Response: we changed the sentence to "it can be verified".

Page 8 Line 5: “firstly” should be “first”
Response: corrected.

Page 8 Line 23: The text states n_a = 0.15 and n_b = 0.10, but Figure 5 states that they are 0.16 and 0.11, respectively.
Response: This was not a typo and deserve some explanation. Both efficiencies can be estimated either from the extreme case DA_20_100 and DA_100_100, assuming that evaporation or advection, respectively, are negligible, or from fitting Eq. 2 to the value obtained in the simulations (as done in Fig.5). The estimate of the efficiencies obtained with these two different methods are just slightly different (0.15 vs. 0.16 and 0.10 vs. 0.11), thus the confusion. We clarified this aspect in the manuscript to make clear that one could use both methods to estimate the efficiencies.

Page 10 Line 19: what is “the expected one”? Please clarify.
Response: we meant the wilting point of the particular soil employed in the simulations. This is now clarified in the manuscript, also in light of the previous comment about the soil characteristics.

Equation 6: I couldn’t find a definition for L_front anywhere in the text. Please include a definition here.
Response: L_front represents the penetration lenght of the front. We added a definition for L_front in section 4.2.

General Comments
This study aims to identify the most important parameters that impact rainfall variations over spatially drier patches. Relying on idealized simulations and a simplified model, the authors concluded that precipitation changes over a heterogeneous surface do not depend on soil moisture, but the initial atmospheric state. The research is interesting. However, the manuscript needs to be substantially clarified and the formulation of the simplified model should be further justified.
Response: We thank the reviewer for his/her precious comments which helped us to revise many sections of the manuscript which were not clear. In the following we will answer all the major and minor comments of the reviewer.

Major Comments
1. The simulations
As described in Section 2, the simulations were conducted using an atmospheric model (ICON-LEM) coupled with a land surface model (TERRA-ML). Accordingly, it appears that soil moisture over both the dry and wet patches evolves as the model integrates forward and soil moisture in each experiment is only specified at the initial time. I am not completely sure about whether this is the case, because assumptions in the simplified model are more consistent with simulations using constant soil moisture values throughout the model run. Please clarify. Also, please briefly describe the purpose of reducing dynamic contributions of advection on precipitation when setting up the size of the simulation domain (Pg. 3, lines 26-28).
Response: The referee is right: soil moisture in our setup is prescribed at the initial time and then freely evolves during the day as a response to the atmospheric forcing (precipitation, evaporation...) and to the soil model. Thus, whereas in the formulation of evaporation of the simplified model (Eq. 4) we used the value of soil moisture at initialisation time and assumes it stays constant, soil moisture, and thus evaporation, will change in the simulations. However changes in soil moisture over one diurnal cycle are not expected to be so strong to significantly feed back on evaporation and precipitation. We investigate this by computing the daily average value of soil moisture in the simulations and comparing it to its initial value (see Tab. 1 of this document). Except in the

Case	Initial soil moisture (dry patch average) [$m^3\ m^{-3}$]	Diurnally-averaged soil moisture (dry patch average) [$m^3\ m^{-3}$]
DA_20_100	0.0908	0.109
DA_50_100	0.227	0.227
DA_65_100	0.2951	0.270
DA_100_100	0.454	0.291

Table 1: Prescribed initial value of volumetric soil moisture and diurnally-averaged value over the entire period of the simulation for different cases (first column). All values are considered averaged over the dry patch, although for the initial soil moisture we prescribe the same value everywhere over the dry patch (see manuscript).

DA_100_100 case, the values are fairly similar. The big difference between initial soil moisture and diurnally-averaged soil moisture in DA_100_100 is due to the fact that the soil cannot stay saturated and thus the soil model will produce an instantaneous runoff to bring back soil moisture to the field capacity. However this has no effect on the evaporation as it does not change for soil moisture values larger than the field capacity (see Fig.7 of the manuscript). We will clarify this aspect in the manuscript.
When deciding the domain size we wanted to reduce as much as possible the dynamical contribution of advection on precipitation, that is the spurious effect of the front collision on precipitation. As showed in Fig. 6 the collision of the two fronts in the middle of the dry patch,

because of the periodic boundary condition, enhances convergence, uplift and thus precipitation. Since such effect can alter the interpretation of the results we wanted to delay the diurnal front collision as much as possible, while keeping the computation costs affordable for running several sensitivity experiments. That's why we settled on a domain which is 400x100 km^2 big.

2. The simplified model

1) **Assumptions:** According to Section 4, the authors assumed that “Ewet does not depend on Ï ¸Tdry” (Pg. 13, line 1) and “evaporation over the dry patch does not depend on the soil moisture of the wet patch” (Pg. 14, line 1). These two assumptions are needed to get the key results (Eqs. 10, and 15), but not clearly justified. When either Ï ¸Tdry or Ï ¸Twet varies, should not precipitation over the wet or dry patches change, which in turn impact Ewet or Edry through the impact on soil moisture therein? On Pg. 11 (lines 2-7), it is assumed that the advection of water vapor and hydrometeors is mainly constrained in the boundary layer. As shown in Fig. 2, however, the return flow at ~1-3 km is not negligible. Could you please justify this assumption further?

Response: There is no dependency of E_{wet} on ϕ_{dry} because in Eq. 4, and more generally in the Budyko formulation, evaporation over a surface depends on the local soil moisture, in this case ϕ_{wet}. Furthermore, our formulation of the evaporation considers a soil moisture constant in time, as explained in the answer to the previous comments. This is well justified as we only consider one diurnal cycle: over this period precipitation is expected to change soil moisture only marginally and thus not to change evaporation appreciably. Regarding the return flow it should be noted that in our simulation this branch of the circulation has a much weaker intensity (in terms of zonal velocities at least 50% less) and lasts just for a few hours. For these reasons we consider it as negligible when developing the idealized model.

2) **Derivations:** Please provide more details on how to approximate Eq. 5 to get Eq. 6, and how Eq. 7 is obtained. To get Eq. 8, it seems that one has to assume the vertical extent of moistening process due to latent heat flux, Hmoist, is the same over the dry and wet patches. Is Hmoist related to turbulent eddies? If so, this assumption can be problematic because low level temperature differences are up to ~4 K between the dry and wet patches (Fig. 2), where sensible heat flux differences can reach 280 W/m2 (Pg. 5, lines 5-6).

Response: We decided to revise section 4.1 and 4.2 and specifically to remove Eq. 6 since it contained an ambiguous notation. Instead we decided to include the approximation of specific humidity (eq. 7) and then proceed to explain how to obtain Eq. 8. The latter equation assumes that the vertical extent of the moistening process H_{moist} is the same as the vertical extent of the breeze circulation H_{front}. We agree with the reviewer that the two are not exactly the same. To check the validity of this assumption we computed these two heights from the simulations. H_{moist} was computed as the height of the PBL over the wet patch, diagnosed with the bulk Richardson number method (see Seibert et al., 2000). H_{front} instead was computed as the height at which the zonal pressure anomaly ahead of the front reaches 0 (see Rochetin et al., 2017). Although slight differences up to 300-500 m were present at some time instants, the two variables showed similar values. As the goal is to develop a simplified model that only retains the main drivers of precipitation variability, we think that the assumption is well justified. Furthermore, it should be noted that, since H_{moist} does not depend on ϕ_{dry}, including its effect won't change the results, i.e. the fact that the derivative of precipitation does not depend on ϕ_{dry}. We included this information in section 4 of the manuscript.

3) **Comparison to Lintner et al. (2013)** As noted in the article (Pg. 10, lines 26-30), feedbacks between the land-surface and atmosphere are neglected in the simplified model after taking evaporation as Eq. 4. Consequently, it is not unexpected that soil moisture can be irrelevant to

precipitation change in the simplified model. If the formulation of evaporation in Lintner et al. (2013), where land-atmosphere interactions are considered, is used in the derivation, will the theoretical model proposed here still be valid?
Response: We have to disagree with the reviewer. Within the framework of our simplified model precipitation changes are independent of soil moisture as the derivative of precipitation with respect to soil moisture does not depend on the latter. As explained in the manuscript this is an effect not only of the linear dependency of evaporation on soil moisture but also of the constant front velocity. Note that the idealized model does not consider explicit land-surface interactions but is in good agreement with the results of the simulations which are coming from a coupled land-atmosphere model. Thus, we don't think that the results would differ much in case the land-atmosphere interactions would be considered as long as one diurnal cycle is considered (see also our response to comment 1 above).

4) **Precipitation efficiency associated with evaporation and advection** Could you please elaborate further on why precipitation efficiency is independent of soil moisture? Although the authors showed that precipitation efficiency associated with advection is independent of evaporation using the extreme case DA_20_100, where Edry is negligible, it is not clear on why precipitation efficiency associated with evaporation is independent of soil moisture. Overall, it is hard to evaluate the simplified model according to how it is presented. The conclusion that precipitation change over a heterogenous surface is independent of soil moisture can be an artifact that land-atmosphere interactions are eliminated in the theoretical model.
Response: From the physical point of view, soil moisture controls directly evaporation but not precipitation. The control of soil moisture on evaporation is already included in Eq. 4. The efficiencies then describe the processes that take place in the atmosphere which convert moisture sources (advection and evaporation) into precipitation. They are used to represent the fact that two different atmospheric states will produce different precipitation amount, even though the soil moisture and evaporation can be identical. For this reason, the efficiencies should not be defined as functions of soil moisture.
The conclusion that precipitation changes over a heterogeneous surface are independent of soil moisture is related to the assumptions made in the idealized model: as long as the front propagation velocity does not depend on soil moisture and the evaporation is a linear function in soil moisture our results won't be affected. Moreover, as shown in Figs. 8 and 9 of the manuscript, our assumptions are justified as our model, despite its simplicity, is able to reproduce the simulation results fairly well. We clarified these aspects in the conclusions and in section 4.

3. Writing
The manuscript requires an editorial revision to correct wording issues. Some sentences are either awkward or redundant. For example, ". . ., in a nutshell, . . ." (Pg. 6, line 4), ".. thanks to the previous section . . ." (Pg. 12, line 12) and etc. can be removed.
Response: we removed these ambiguous sentences.

Minor Comments
1.Are equations 5 and A2 written correctly as advection? It is also unusual to have dot product between a scalar (qtot) and a vector (ufront or v).
Response: We modified the equations by removing the dot and using vector notation. Otherwise the equations are correctly written.

2.Pg. 2, lines 16-17: Please clarify further on Guillod et al. (2015), what does ". . . a negative spatial coupling coexists together with a positive temporal coupling" mean and indicate?
Response: We added a clarification sentence: " That is, areas drier than their surrounding (spatial component) but wetter than the climatological value (temporal component) may receive more precipitation than other ones"

3.Pg. 3, line 25: Why a rectangular domain can limit computational cost?
Response: This sentence was meant to represent a comparison between a squared domain (which in our case would be 400x400 km^2 big) and our chosen rectangular domain (400x100 km^2) which contains less grid points and is thus less expensive to run simulations on.

4.Pg. 9, lines 6-7: Please provide relevant evidences on “. . . several secondary events develop due to the waves propagating away from the collision”.
Response: We agree that using the term"wave" was not appropriate so we rephrased this part to read "In the ID_20_100 case strong precipitation events with local maxima of 10 mm h^{-1} are produced in the center of the patch after the fronts’ collision and several secondary events develop due to the fronts propagating again away from the collision. "

5.Fig. 2: It can be better to show wind as vectors, rather than contours.
Response: We revised Fig. 2 to ease its interpretation. Now winds are displayed through vectors and not contours.

6.Fig. 9: Change “∂Pdry/Ï ¸Tdry” as “∂Pdry/∂Ï ¸Tdry”.
Response: We thank the reviewer for spotting this typo, which we promptly corrected.

7.Table 1: Change “Name” as “Experiment”.
Response: Corrected

A simplified model of precipitation enhancement over a heterogeneous surface

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Abstract. Soil moisture heterogeneities ~~through the triggering of mesoscale circulations~~ influence the onset of convection and subsequent evolution of ~~thunderstorms producing heavy precipitation. However~~precipitating systems through the triggering of meso-scale circulations. However, local evaporation also plays a role in determining precipitation amounts. Here we aim at disentangling the effect of advection and evaporation on precipitation over the course of a diurnal cycle by formulating a simple conceptual model. The derivation of the model is inspired from the results of simulations performed with a high-resolution (250 m) Large-Eddy Simulation model over a surface with varying degrees of heterogeneity. ~~Key~~ A key element of the conceptual model is the representation of precipitation as weighted sum of advection and evaporation, each weighted by its own efficiency. The model is then used to isolate the main parameters that control the variations of precipitation over ~~spatially drier patches~~a spatially drier patch. It is found that these changes surprisingly do not depend on soil moisture itself but instead purely on parameters that describe the atmospheric initial state. The likelihood for enhanced precipitation over drier soils is discussed based on these parameters. Additional experiments are used to test the validity of the model.

1 Introduction

Will ~~more soil moisture~~ a wetter soil lead to more or less precipitation? This apparently simple question inspired many studies over the course of the last 50 years. Over a homogeneous surface precipitation is expected to increase with surface evaporation, and thus with soil moisture in a soil moisture-limited regime (Manabe, 1969; Budyko, 1974), regardless of the atmospheric state (Cioni and Hohenegger, 2017) as long as convection can be triggered on both dry or wet surfaces (Findell and Eltahir, 2003). However, the real world is far from being homogeneous. The presence of heterogeneity in surface soil moisture induces thermally-driven meso-scale circulations (Segal and Arritt, 1992) which transport moist air from spatially wetter patches to spatially drier patches, acting against the initial perturbation of soil moisture, and which can then affect the distribution of precipitation.

Many idealized studies have investigated the effect of such ~~circulation~~ circulations on convection and ensuing precipitation. Avissar and Liu (1996) found that the land-surface wetness heterogeneity (i.e. spatial gradients of soil moisture) controls the transition from a randomly scattered state of convection to a more organized one where clouds form ~~at the front of~~ ahead of the front associated with the meso-scale circulation. The presence of such circulations also tend to enhance the precipitation amount. Further analyses have shown that this basic response can be modified by many environmental factors.

Yan and Anthes (1988) found that accumulated precipitation is maximized over spatially dry patches ~~is maximized~~ when the patch length is comparable to the local Rossby radius of deformation (~ 100 km in mid-latitudes), a result that was later confirmed by Chen and Avissar (1994) and Lynn et al. (1998). Robinson et al. (2008) proposed an alternative explanation by which the effect of surface hot spots is maximized for wavelength of roughly 50 km, that is when the aspect ratio of the applied heating matches the ratio of vertical and horizontal wavenumbers demanded by the dispersion relation for buoyancy (gravity) waves.

Froidevaux et al. (2014) explored the interaction between horizontal soil moisture variations, wind and precipitation. They found that, only when winds are too weak to control the propagation of thunderstorms, more precipitation is observed over drier surfaces. Finally, the response of precipitation also depends upon the background atmospheric profile. Chen and Avissar (1994) found that the presence of a moist atmospheric profile over a spatially drier surface reduces the precipitation advantage as the surface ~~fluxes of moisture~~ heat fluxes, which drive the surface heating and thus the circulation, are reduced. Hence, from such studies, an increase of precipitation over spatially drier patches is ~~expected~~ maximized when the gradient of surface wetness is high, the soil moisture heterogeneity length-scale is around 50-100 km and no background wind is present.

These same mechanisms can be observed in some areas of the world, the so-called hot spots of land-atmosphere interactions (Koster et al., 2004). Several observational studies (e.g. Taylor et al. (2012)) showed that in the Sahel region thunderstorms occur preferably over regions drier than their surroundings. In other areas of the world the synoptic forcing is usually so strong that a robust relationship of causality between soil moisture and precipitation ~~can not~~ cannot be found (Tuttle and Salvucci, 2017). Instead of speaking of heterogeneous or homogeneous conditions, Guillod et al. (2015) have indicated that over most areas of the world, except the Sahel, a negative spatial coupling coexists together with a positive temporal coupling. That is, areas drier than their surrounding (spatial component) but wetter than the climatological value (temporal component) may receive more precipitation than other ones.

Although the aforementioned studies have qualitatively shown how precipitation is influenced by soil moisture, soil moisture gradients and by the atmospheric environment, here we aim at developing a simplified conceptual model to ~~isolate under which conditions an increase in precipitationis more likely. We focus on the~~ formally isolate the control of soil moisture on precipitation. In particular we aim at developing a mathematical expression for the derivative of precipitation with respect to soil moisture in the case of a heterogeneous surface to understand the response of precipitation to soil moisture changes. In this case, precipitation is not only affected by the advection of moisture due to the meso-scale circulation but also by local evaporation (Wei et al., 2016). These two factors depend differently on soil moisture.

The meso-scale circulation triggered by the surface wetness heterogeneity strengthens with decreasing ~~local soil moisture~~ soil moisture of the dry patch, as this gives a larger spatial gradient of surface heat fluxes and thus of surface pressure. Instead local evaporation is limited with reduced local soil moisture. The superposition of local evaporation and remote moisture advection eventually contribute to the observed precipitation, with the atmosphere being the medium that weights these two different contributions.

~~We thus need to quantify two different effects. First, how local evaporation as well as moisture advection depend on soil~~

~~moisture. Second, what is the relative role of the atmosphere, or in other words the efficiency in converting these potential moisture sources into precipitation. The results obtained by the aforementioned studies seem to suggest that a local increase of local soil moisture will lead to a negative variation of precipitation, i.e. the derivative of precipitation with respect to soil moisture is negative. Our goal is to find a simple formulation for the derivative of precipitation as function of soil moisture in case of an heterogeneous surface.~~Lintner et al. (2013) already derived an equation for the derivative of precipitation with respect to soil moisture based on a model of intermediate-level complexity of the tropical atmosphere (Quasi-equilibrium Tropical Circulation Model 1 (QTCM1), Neelin and Zeng (2000)). Inspired by their work, we develop a theoretical model which is based on similar assumptions but ~~greatly~~ simplifies the formulation of moisture advection and evaporation~~which are all expressed in terms of linearized features. Also~~ . In particular the fact that we consider the specific case of advection by a thermally-induced meso-scale circulation, and not by the large-scale flow, ~~as in Lintner et al. (2013),~~ will allow us to greatly simplify the idealized framework.

~~We aim here at a minimal representation that allows us to isolate what are the fundamental quantities that cause variations of precipitation with soil moisture and to determine which is the most efficient way to increase precipitation with local soil moisture. To this aim in section **??** we build an idealized framework that allow~~ Section 2 describes the model and experimental set-up that allows us to simulate the evolution of convective clouds and precipitation over ~~land~~ a heterogeneous land-surface during a diurnal period. After a brief analysis of the features of the convective diurnal cycle in section 3.1 we estimate the various terms of the moisture balance and in particular the efficiencies of the conversion of evaporation and advection into precipitation in section 3.2. ~~By deriving~~ These results are used in section 4 to derive a simple conceptual model ~~that agrees with the results of model simulations in section 4 we~~ of how precipitation responds to soil moisture changes over a heterogeneous surface. We will show that, at least to a first order, the change of precipitation with soil moisture does not depend on the soil moisture content itself ~~and that the most efficient way to increase precipitation consists in increasing the surface wetness gradient~~but only on the atmospheric state. The results are concluded in section 5.

2 ~~Method~~ Methods

2.1 ~~Experimental design~~

~~The modelling~~

(caption) Package caption Error: outside floatSee the caption package documentation for explanation.If you do not understand this error, please take a closer lookat the documentation of the ‘caption’ package, especially thesection about errors.Try typing <return> to proceed.If that doesn’t work, type X <return> to quit.blueSkew-T diagrams of the two soundings used to initialize the atmosphere in the simulations. Panel (a) shows the dry soil advantage sounding of findell2003atmospheric0pt, blueDAblue, while panel (b) shows the idealized sounding of schlemmer2012diurnal0pt, blueIDblue. The upper inset in both panels show the value of pressure at the lcl0pt, temperature at the lcl0pt, precipitable water and cape0pt.

This sounding, indicated throughout the manuscript as DA, was observed on 23 July 1999 in Lincoln ~~,~~ (Illinois, USA~~. However, we set winds to zero all over the atmospheric column to simplify the analysis~~) and was chosen as a typical example by Findell and Eltahir (2003) for cases when a strong heating of a homogeneous surface favors the triggering of convection. ~~We~~ To study the response of precipitation to variations in soil moisture, we perform a set of experiments by setting at the initial time ϕ_{wet} to the saturation value and varying ϕ_{dry}, with values ranging from the saturation to 20% of the saturation value~~, which~~ . The latter value is below the wilting point for the chosen soil type ~~.~~ (loam). More details about the soil type can be found in Cioni and Hohenegger (2017) and Doms et al. (2011). The upper part of Tab. 1 summarizes the simulations performed with this basic configuration.

In order to test the validity of the theory proposed in section ~~2,~~ 4 based on this set of basic experiments ~~,~~ we perform

~~Name~~Experiment	Sounding	ϕ_{dry}	ϕ_{wet}
	Basic configuration		
`DA_20_100`	`DA`	20	100
`DA_30_100`		30	100
`DA_40_100`		40	100
`DA_50_100`		50	100
`DA_60_100`		60	100
`DA_65_100`		65	100
`DA_70_100`		70	100
`DA_80_100`		80	100
`DA_100_100`		100	100
	Not-saturated wet patch		
`DA_20_70`	`DA`	20	70
`DA_30_70`		30	70
`DA_40_70`		40	70
`DA_50_70`		50	70
`DA_60_70`		60	70
`DA_70_70`		70	70
	Idealized sounding		
`ID_20_100`	`ID`	20	100
$\vert$	$\vert$	$\vert$	$\vert$
`ID_100_100`	`ID`	100	100

Table 1. Overview of the performed simulations. The first column indicates the experiment name, ~~while~~ whereas the second column indicates the sounding used for initialization: `DA` for dry soil advantage, after Findell and Eltahir (2003), and `ID` for idealized, after Schlemmer et al. (2012). Third and fourth columns indicate the value of soil moisture over the dry ~~ϕ_{dry}~~ and wet ~~ϕ_{wet}~~ patches, respectively, in percentage of the saturation ~~values~~value. Naming convention for the experiments follows `SOUNDING`_ϕ_{dry}_ϕ_{wet}. The vertical lines that characterize the `ID` cases are used to omit the repetition of the same experiments description, i.e. `ID_30_100`, `ID_40_100`, etc. .

further sensitivity experiments. First, we decrease the initial value of ϕ_{wet} to 70 % of the saturation value. Second, we change the initial atmospheric profile~~to the one used in Schlemmer et al. (2012)~~. We tested the wet soil advantage sounding of Findell and Eltahir (2003) where, in contrast to the dry soil advantage sounding, convection triggering requires a strong moistening of the boundary layer. We also tested the sounding of Schlemmer et al. (2012), indicated as `ID`, which represents an idealization of the typical atmospheric state prone to convection in Europe ~~. Results of these perturbed experiments will be briefly described throughout the paper when needed.~~ (see Fig. **??** b). This sounding thus greatly differs from the conditions as observed in Lincoln. It has a lower surface temperature, a lower integrated water vapour content but a larger initial instability. As the use of the wet soil advantage sounding of Findell and Eltahir (2003) yields very similar results as in `DA`, which is not the case when using `ID`, we only report here on the `ID` simulations.

3 Results

3.1 General features of convection

Here, we describe the general features of the extreme case, `DA_20_100`, which reproduces the features expected from this kind of simulations. The differential heating of the two patches, caused by the heterogeneity in soil moisture, manifests itself in a gradient of both sensible and latent heat fluxes. ~~Here, we describe the general features of the extreme case, `DA_20_100`, which, in a nutshell, reproduces the features expected from this kind of simulations.~~ At 12 Local Standard Time (LST) the difference in sensible heat ~~fluxes~~ flux between the two patches reaches almost 280 W m^{-2}. This results in a difference in near-surface virtual potential temperature of about 4 K at the same time ~~.~~ (see the colored contours in Fig. 1). As a consequence, a pressure gradient of about 1 hPa develops close to the surface, which supports a thermally-driven circulation (Segal and Arritt, 1992). The circulation is constituted by a front of moist air moving inland over the dry patch at lower levels (from the surface up to 1 km) and a return flow between 1 and 3 km, as shown by the wind ~~contours~~ vectors in Fig. 1. As a result of the circulation, and as found in past studies, convection preferentially develops over the dry patch and in particular ~~on the border between the wet and the dry patch~~at the edge of the front associated with the meso-scale circulation.

In order to track the front associated with the meso-scale circulation we use an algorithm designed to follow ~~the front only~~ one of the front moving over the dry patch. The algorithm is based on the y-averaged zonal wind speed at ~~approximately 100~~ 150 m of height~~and~~. It is triggered when the ~~velocity~~ wind speed in the middle of the domain reaches 1 m s^{-1} and automatically ~~interrupted~~ stops when the opposite fronts collide in the center of the dry patch. At every ~~time instant~~ output time step (15 minutes) a search of the maximum value of zonal wind speed is performed in a box ~~centered on a first guess based on a linear extrapolation of the previous time instants. This is necessary~~ which is suitably chosen in order to maintain the focus of the tracking algorithm on the ~~edge of the front~~front. More specifically, at the first two time instants the maximum is searched over the entire dry patch while from the third time step onward the maximum search is performed in a box centered on a first guess obtained from a simple linear extrapolation of the previous time instants. The size of the box is the only parameter that needs to be tuned when tracking the front in different simulations. Otherwise, the algorithm is robust. As an example, in the `DA_20_100` case shown in Fig. 2, the box comprises 5 grid points, thus approximately 1.25 km.

Figure 2 (a) shows the Hovmöller diagram of the zonal wind and the tracked position of the front every 15 minutes with

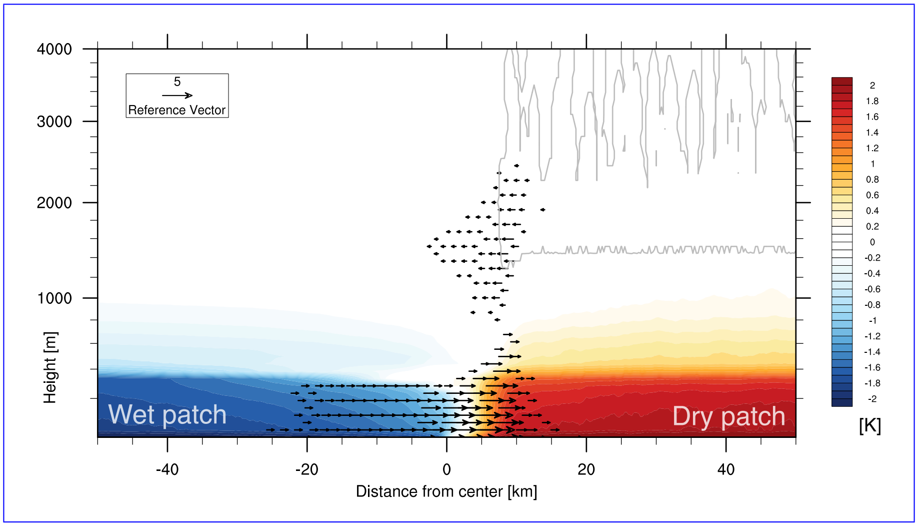

Figure 1. x-z diagram at 1200 LST of y-averaged quantities for the DA_20_100 case. ~~Temperature~~ Zonal temperature anomaly (color contours), zonal ~~winds~~ wind (~~contour lines from -4 to 4 every 0.5~~ vectors, values between -0.5 and 1 m s^{-1} ~~, positive values with green contours, negative values with purple ones~~are masked) and cloud water mixing ratio (grey contour, only 10^{-5} g kg^{-1} isoline). On the x-axis numbers indicate the distance from the center of the domain in km. ~~On the y-axis height from the surface is indicated in meters.~~

shaded circles for the case DA_20_100. In Fig. 2 (b) the position and speed of the front obtained with the aforementioned algorithm are displayed. The front starts to slowly propagate in the late morning with a velocity smaller than 2 m s^{-1} but is later accelerated by cold pools, in agreement with Rieck et al. (2015). The cold pools are formed after the first strong precipitation event between 12 and 13 LST. The speed of the front reaches values ~~larger than 8~~ of up to 7 m s^{-1} before the front collides with the opposing front coming from the outer boundary due to the periodic domain. When the soil moisture of the dry patch exceeds 70% of the saturation value no circulation forms ~~,~~ because the gradient in surface temperature is too weak to cause a pressure difference between the patches. In this case the convection transitions to a randomly scattered state (Avissar and Liu, 1996) and we define the speed of the front to be ~~0.~~0 m s^{-1}.

3.2 Local and remote sources of precipitation

The diurnal cycle of precipitation can be inspected and compared to the one of evaporation and advection, using the methodology introduced in appendix A. This is needed to later formally express precipitation as function of soil moisture (see section 4). Figure 3 shows the various components of the moisture balance computed every 5 minutes from the model output ~~every 5 minutes~~ and averaged over the dry patch as well as over the entire domain. It ~~is immediate to verify~~ can be verified that the advection term averaged over the entire domain is zero, as expected. Instead, when considering the residual averaged over the dry patch~~this term~~, A_{dry}, it is always positive, indicating a net transport of moisture from the wet to the dry patch.

The advection of moisture over the dry patch increases in the late morning as a result of the ~~increasing thermal gradient between the two patches~~ propagation of the front (see Fig. 2) and reaches a maximum at around 13 LST. This ~~behaviour~~ behavior is sim-

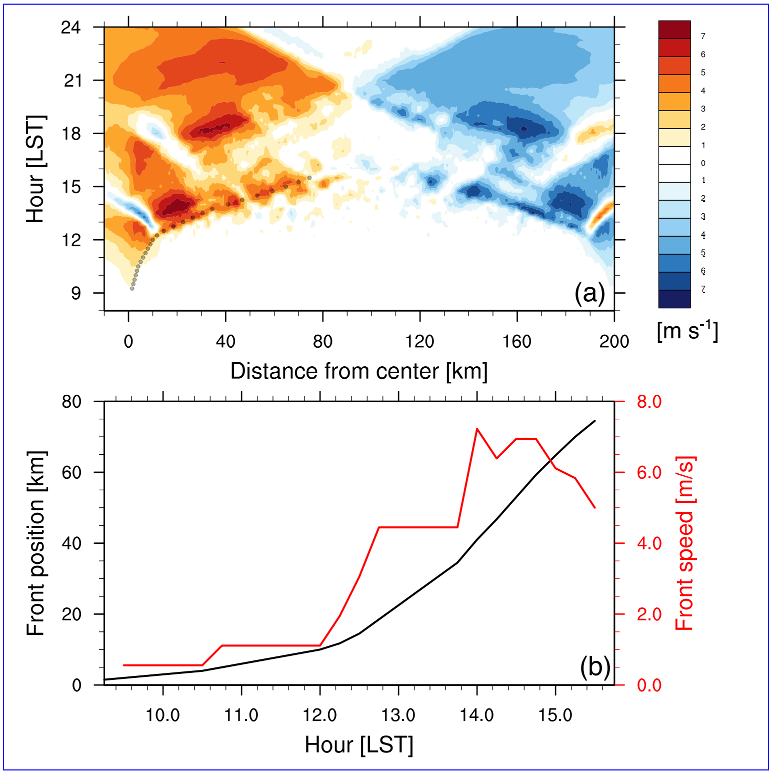

Figure 2. Tracking of the ~~breeze~~ front associated with the meso-scale circulation for the case `DA_20_100`. (a) Hovmöller diagram (distance from domain center vs. time) of the y-averaged zonal wind at a height of ~~approximately 100~~ 150 m above the surface. Dots indicate the position of the front tracked every 15 minutes (see text for details). (b) Front inland propagation (black line) with respect to the center of the domain [km] and front speed (red line) derived using finite differences [m s^{-1}].

ilar to the one observed by ~~Yan and Anthes (1988, Fig. 9)~~Yan and Anthes (1988, their Fig. 9). The first deep convection event in `DA_20_100` between 12 and 13 LST produces a strong cold pool which causes a strong surface divergence, ~~i.e.~~ explaining the minimum at about 14 LST in Fig. 3. Given that the maximum of precipitation associated with this event is located in the vicinity of the boundary between the wet and the dry patch, this induces a net negative effect on A_{dry}.

In order to study the variation of the moisture budget terms as function of ϕ_{dry} we conduct the same moisture balance analysis for every simulation and integrate the values over the entire diurnal cycle (18 hours). Results are reported in Tab. 2. As expected the advection term decreases with increasing local soil moisture whereas local evaporation increases. Overall the accumulated precipitation averaged over the dry patch decreases when the soil moisture increases, as shown also in Fig. 4. ~~This already suggests that advection~~ The sharp decrease of precipitation with increasing value of soil moisture seems to suggest that advection and evaporation are characterized by different weights when producing precipitation. In fact, if the contribution of these processes would be the same, we would expect to observe a flattening of the precipitation values (blue asterisks in Fig. 4) instead than a decrease. In other words, advection appears to be more efficient than evaporation in producing precipitation,

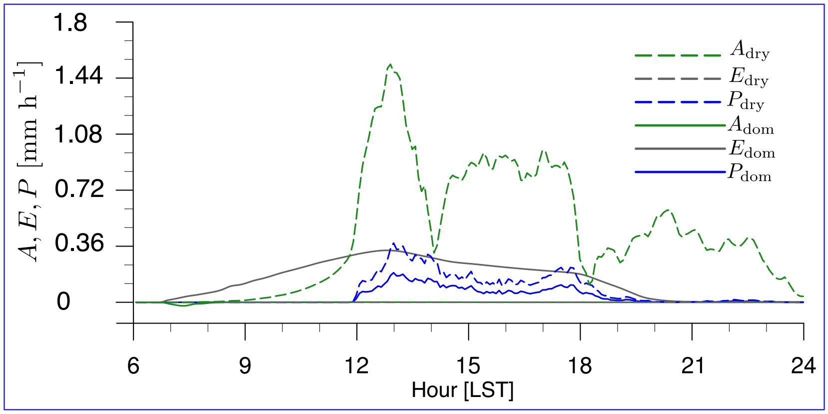

Figure 3. Different terms of the moisture balance (Eq. ~~??~~A3) computed for the entire domain (subscript ~~dom~~$_{\mathrm{dom}}$, solid lines) and for the dry patch (subscript dry, dashed lines) in the `DA_20_100` case. A indicates advection, E evaporation and P precipitation. Units are mm h^{-1}. Note that all variables in this figure are instantaneous. ~~See Appendix A for a comprehensive explanation of all the symbols.~~

Case	ϕ_{dry}	$A_{\mathbf{dry}}$	$E_{\mathbf{dry}}$	$P_{\mathbf{dry}}$	η
`DA_20_100`	0.0908	7.796	0.0008	1.255	0.161
`DA_30_100`	0.1362	7.467	0.0113	1.077	0.144
`DA_40_100`	0.1816	7.223	0.1140	1.005	0.137
`DA_50_100`	0.2270	6.673	0.6373	0.912	0.125
`DA_60_100`	0.2724	4.665	2.0271	0.888	0.133
`DA_65_100`	0.2951	3.222	3.0393	0.805	0.129
`DA_70_100`	0.3178	1.734	4.0920	0.708	0.122
`DA_80_100`	0.3632	-0.412	5.2770	0.533	0.094
`DA_100_100`	0.4540	0.031	5.0800	0.560	0.110

Table 2. Values of soil moisture [m^3 m^{-3}], advection [mm], evaporation [mm] and precipitation [mm] over the dry patch accumulated over the diurnal cycle. ~~All values have mm units.~~ The rightmost column shows the precipitation efficiency (dimensionless) computed as $\frac{P_{\mathrm{dry}}}{A_{\mathrm{dry}}+E_{\mathrm{dry}}}$.

as the increase of E_{dry} with soil moisture is ~~not followed by an increase in~~ followed by a sharp decrease of P_{dry}.

These qualitative observations can be formalized by defining precipitation efficiencies. This approach was ~~firstly~~ first proposed by Budyko (1974) and later adopted by many studies including the one of Schär et al. (1999). The overall assumption underlying the pioneering work of Budyko (1974) is that moisture coming from inside (local evaporation) or outside (remote advection) of some closed domain is well mixed. Under this assumption one can express the precipitation over a certain area as:

$$P_{\mathrm{area}} = \eta(A_{\mathrm{area}} + E_{\mathrm{area}}) \tag{1}$$

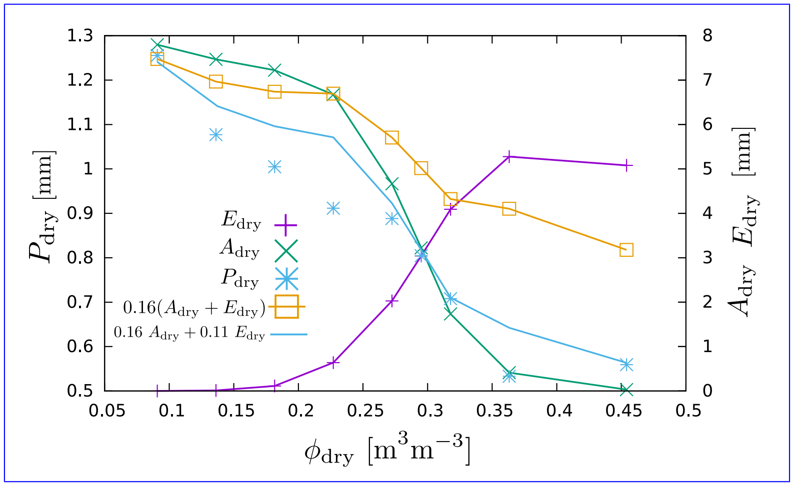

Figure 4. Values of advection (green line and crosses), evaporation (purple line and ~~crosses~~plus symbols) and precipitation (blue asterisks) ~~of~~ from Tab. 2 as function of soil moisture. The orange line represents an estimate of precipitation obtained as ~~weighted~~ a sum of advection and evaporation weighted by the same efficiency, i.e. $0.16(A_{\mathrm{dry}} + E_{\mathrm{dry}})$ while the ~~yellow~~ blue line represents a similar estimate ~~used~~ obtained by ~~obtaining~~ using two different efficiencies, i.e. $0.16\ A_{\mathrm{dry}} + 0.11\ E_{\mathrm{dry}}$.

where η is the precipitation efficiency. All the terms are considered as areal averages and integrated over a certain time period. The rightmost column of Tab. 2 shows the efficiency η computed according to Eq. 1. It can be seen that, in this case, convection is not so efficient in converting local and remote sources of moisture into precipitation as the values range from 16% to ~~11%. This range of values is interpreted as resulting from the different processes that contribute to precipitation over the dry patch~~9%. More importantly, the efficiency values vary by up to 7% depending on the initial ϕ_{dry}. In fact, in the case `DA_20_100`, evaporation over the dry patch is negligible, i.e. $E_{\mathrm{dry}} \simeq 0$, so that Eq. 1 applied to the dry patch reads $P_{\mathrm{dry}} \simeq \eta A_{\mathrm{dry}}$. Thus, the efficiency obtained in this case is representative of the advection process and can be interpreted as an *advection efficiency* $\cancel{\eta_{\mathrm{A}}}\eta_A$. On the other hand, in `DA_100_100` the advection is negligible so that in this case we obtain an *evaporation efficiency* $\cancel{\eta_{\mathrm{E}}}\eta_E$. Taking all these findings together, we rewrite Eq. 1 as:

$$P_{\mathrm{area}} = \eta_{\cancel{\mathrm{A}}A} \cdot A_{\mathrm{area}} + \eta_{\cancel{\mathrm{E}}E} \cdot E_{\mathrm{area}} \tag{2}$$

where now $\cancel{\eta_{\mathrm{A}} \neq \eta_{\mathrm{E}}.}$ $\eta_A \neq \eta_E$. The values estimated from Tab. 2 are $\eta_A = 0.16$ and $\eta_E = 0.11$.

Fig. 4 confirms that, regardless of the particular choice of ~~the~~ a single efficiency η, the decrease of precipitation over wetter soils cannot be captured~~when using a single efficiency. Instead~~. In contrast, using the two efficiencies η_A and η_E gives a much better match with the simulated value of P_{dry} (see ~~yellow~~ blue line in Fig. 4). Also, by using two efficiencies, the latter become independent of soil moisture. The ~~resulting efficiencies , obtained~~ efficiencies can be alternatively estimated through a fit of Eq. 2 ~~, are~~ using all the values of A_{dry} and E_{dry} in Tab. 2. In this case we obtain the values $\eta_A = 0.15$ and $\eta_E = 0.10$ which, as expected, do not differ much from the ones computed using the two extreme cases.

The fact that one efficiency ~~doesn't match well~~is not enough to describe the variations of precipitation, in contrast to previous studies, may be linked to the fact that we consider a small domain and a short time scale. The assumption of a well-mixed atmosphere ~~is likely to hold~~ likely holds better on a continental (e.g. Europe) and seasonal scale, as in Schär et al. (1999). Using two efficiencies nevertheless ~~as it~~ requires data from at least two simulations ~~to estimate them~~with different values of advection, evaporation and precipitation.

Initializing the atmosphere with a different sounding will likely lead to different efficiencies. This is illustrated with the ID_ cases (see Tab. 3), where ~~another sounding was~~ the idealized sounding of Schlemmer et al. (2012) is used to initialize the atmosphere (see section 2). For a given soil moisture, advection reaches smaller value that in the DA case. This is mainly an effect of larger precipitation amounts that fall on the wet patch which in turn prevents an efficient advection of moisture from the wet to the dry patch.

The efficiencies computed for this case range from 47% to ~~35~~31%, indicating that the atmosphere is more efficient at converting advection and evaporation into precipitation ~~. Nonetheless, also for this case $\eta_A > \eta_E$. Evaporation now reaches a maximum which is larger than~~ than in DA. The higher efficiencies obtained with the ID sounding are due to a combination of different effects. One of those is the different convection triggering. With the ID sounding convection is triggered almost 1 hour before than with the DA sounding (not shown). This allows the atmosphere to fully exploit the instability caused by the morning heating which manifests itself in a stronger enhancement of precipitation at the front, as shown in Fig. 5. This is also corroborated by the fact that Convective Available Potential Energy (CAPE) at 15 LST is larger than the one at the initial time over both patches in DA_20_100 whereas it is depleted over the dry patch in the ~~one attained by advection. This is mainly an effect of larger precipitation amounts that fall over~~ ID_20_100 case (not shown). Moreover, as indicated by Fig. **??**, the dew-point depression in the ID sounding is smaller than in the DA sounding throughout most of the ~~wet patch which in turn prevents an efficient advection of moisture from the wet to the dry patch.~~ atmospheric column. This suggests that, in the ID case, convective updrafts are less affected by the entrainment of environmental dry air. We verify this by computing the average difference in Moist Static Energy (MSE) between updrafts, defined as grid points with vertical velocity greater than 1 m s^{-1} and cloud water content greater than 10^{-4} kg kg^{-1}, and the environment (not shown). Results show that this difference in the ID case is less than 50 % the values observed in the DA case.

Case	$A_{\mathbf{dry}}$	$E_{\mathbf{dry}}$	$P_{\mathbf{dry}}$	η
ID_20_100	3.814	0.008	1.789	0.468
ID_30_100	3.861	0.028	1.912	0.492
ID_40_100	3.920	0.143	1.671	0.411
ID_50_100	3.557	0.659	1.740	0.413
ID_60_100	2.631	2.054	1.759	0.376
ID_70_100	0.865	4.080	1.542	0.312
ID_80_100	-0.068	4.884	1.652	0.334
ID_100_100	0.022	4.776	1.662	0.346

Table 3. As in Tab. 2 but for the ~~additional~~ ID sounding.

~~In the ID_ cases the decrease of precipitation observed over the dry patch with higher values of soil moisture~~Despite the differences, the ID case confirms that advection and evaporation exhibit distinct efficiencies and that precipitation decreases with increased local soil moisture. Here the decrease of precipitation is smaller than the one obtained in the DA_ cases. Although this could be related to a weaker sensitivity of ~~that particular~~ the ID atmospheric state to ~~the meso-scale circulation~~modifications in the land-surface heterogeneity, we note that the amount of precipitation ~~computed~~ strongly depends on the collision of the fronts. As shown in Fig. 5 the collision of the fronts in the center of the dry patch has different effects on precipitation depending on the atmospheric state. In the ID_20_100 case strong precipitation events with local maxima of 10 mm h^{-1} are produced in the center of the patch after the ~~front~~ fronts' collision and several secondary events develop due to the ~~waves~~ fronts propagating away from the collision. Instead, in the DA_20_100 case, no strong precipitation event is produced when the

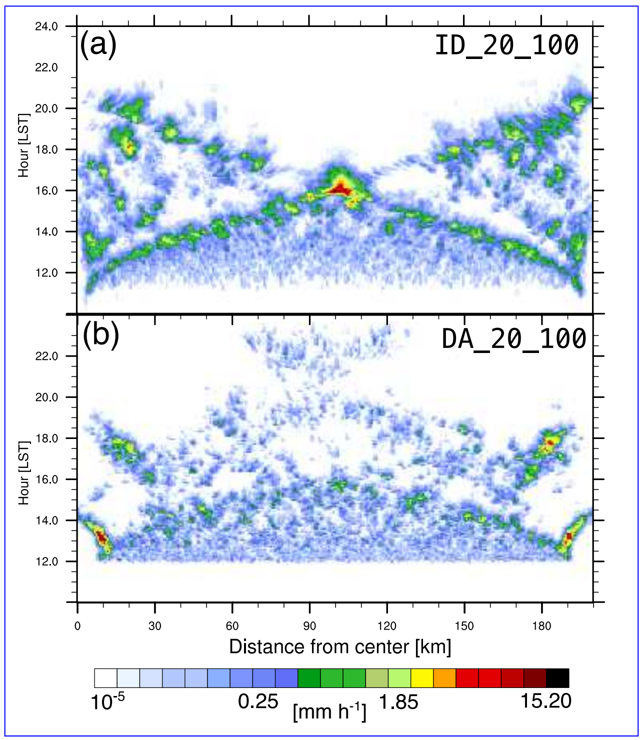

Figure 5. Hovmöller diagram of precipitation rate [mm h^{-1}] in case ID_20_100 and DA_20_100.

fronts collide.

4 Theoretical model

In section 3.2 we showed that precipitation can be expressed as a linear combination of advection and evaporation weighted by different efficiencies which are assumed independent of soil moisture ~~.~~ (Eq. 2). Knowing this we can now try to answer one of the first question that was posed in the introduction: What are the minimum parameters that control the variation of precipitation with soil moisture? In order to do so we first have to derive some functional forms of evaporation and advection in terms of soil moisture.

4.1 Surface evaporation

The simplest parametrization of evaporation (we will neglect the transpiration part given that our study ~~do not consider~~ does include plants) is the so-called *bucket model* introduced by Budyko (1961) and extended by Manabe (1969). Evaporation is defined as a potential term controlled by a limiting factor (also called stress factor). Here we use such a formulation to approximate first the surface latent heat flux LH ~~(ϕ, t) as~~ [mm h^{-1}] at a certain point in space and time as a function of soil moisture ϕ [m^3 m^{-3}]:

$$\mathrm{LH}(\phi, t) = \mathcal{A} Q_{\mathrm{net}}(t) \times \begin{cases} 0 & \text{for } \phi < \phi_{\mathrm{wp}} \\ \frac{\phi - \phi_{\mathrm{wp}}}{\phi_{\mathrm{crit}} - \phi_{\mathrm{wp}}} & \text{for } \phi_{\mathrm{wp}} \leq \phi \leq \phi_{\mathrm{crit}} \\ 1 & \text{for } \phi > \phi_{\mathrm{crit}} \end{cases} \tag{3}$$

where ~~$Q_{\mathrm{net}}(t)$~~ Q_{net} [mm h^{-1}] is the net incoming radiation at the surface (long-wave+short-wave), ϕ_{wp} ~~the soil~~ [m^3 m^{-3}] the soil moisture at the permanent wilting point and ϕ_{crit} [m^3 m^{-3}] is the critical ~~threshold~~ soil moisture at which evaporation does not increase any more with increasing soil moisture. As explained by Seneviratne et al. (2010) this does not usually correspond to the field capacity.~~The enthalpy of vaporization does not show up in Eq. 3 as units are already in mm h^{-1}.~~

$\mathcal{A}$ is a proportionality constant which needs to be introduced and specified given that, even in the extreme case of a saturated soil, non-zero sensible heat fluxes and ground heat flux ~~prevents~~ prevent the entire conversion of Q_{net} into ~~latent heat fluxes~~LH. The constant $\mathcal{A}$ clearly depends on the particular soil model employed as well as on the different parameters that characterize the ~~particular~~ soil type considered (e.g. albedo, heat capacity) and partially also on the atmosphere.

In order to link Eq. 3 to the accumulated evaporation E_{dry} ~~contained~~ needed in Eq 2 we ~~can consider the time average of Q_{net}, which we indicate as~~ average Eq. 3 over the dry patch and integrate it over the accumulation period τ. By doing so we assume a constant value for soil moisture and replace it with the value at the initialization time. Such assumption is motivated by the fact that changes in soil moisture over one diurnal cycle are not expected to be able to significantly feed back on evaporation and precipitation on such a short timescale. The assumption is also well justify as the daily average value of soil moisture remains similar to its initial value (not shown). This gives:

$$E_{\mathrm{dry}}(\phi_{\mathrm{dry}}) = \tau \mathcal{A} \langle Q_{\mathrm{net}} \rangle \times \begin{cases} 0 & \text{for } \phi_{\mathrm{dry}} < \phi_{\mathrm{wp}} \\ \frac{\phi_{\mathrm{dry}} - \phi_{\mathrm{wp}}}{\phi_{\mathrm{crit}} - \phi_{\mathrm{wp}}} & \text{for } \phi_{\mathrm{wp}} \leq \phi_{\mathrm{dry}} \leq \phi_{\mathrm{crit}} \\ 1 & \text{for } \phi_{\mathrm{dry}} > \phi_{\mathrm{crit}} \end{cases} \tag{4}$$

where now E_{dry} does not depend on time nor space. $\langle Q_{\mathrm{net}} \rangle$ ~~, and multiply for a time scale which is defined to be~~ denotes the net surface incoming radiation averaged over the dry patch and over the period τ~~=18 hours, that is the entire length of the~~

~~simulation (and of the resulting diurnal cycle). The resulting Eq. 4 has mm units.~~

$$E_{\mathrm{dry}}(\phi) = \tau \mathcal{A} \langle Q_{\mathrm{net}} \rangle \times \begin{cases} 0 & \text{for } \phi < \phi_{\mathrm{wp}} \\ \frac{\phi - \phi_{\mathrm{wp}}}{\phi_{\mathrm{crit}} - \phi_{\mathrm{wp}}} & \text{for } \phi_{\mathrm{wp}} < \phi < \phi_{\mathrm{crit}} \\ 1 & \text{for } \phi > \phi_{\mathrm{crit}} \end{cases}$$

~~Note that in Eq. 4 instead of using $\langle \phi \rangle$ we just consider the~~, whereas ϕ_{dry} corresponds to the initial value of soil moisture~~at the initialization time for the sake of simplicity.~~.

Equation 4 can now be used to fit the values of E_{dry} computed from the simulations (Tab. 2) ~~and~~ to obtain an unambiguous value for the parameters $\mathcal{A}$~~and $\phi_{\mathrm{wp,crit}}$~~, ϕ_{wp} and ϕ_{crit} (see Fig. 6). These are estimated to be ~~$\mathcal{A} \simeq 0.663$, $\phi_{\mathrm{wp}} \simeq 0.213$~~ $\mathcal{A} = 0.663$, $\phi_{\mathrm{wp}} = 0.213$ m^{3}m^{-3} and ~~$\phi_{\mathrm{crit}} \simeq 0.350$~~ $\phi_{\mathrm{crit}} = 0.350$ m^{3}m^{-3}. Note that the latter estimate is not far from the field capacity of this soil type, i.e. 0.340 m^{3}m^{-3}, while the estimated wilting point is almost double the expected one, i.e. 0.110 m^{3}m^{-3}. This is related to the fact that the employed bare soil evaporation scheme tends to shut down evaporation too early as noted by Schulz et al. (2016) and Cioni and Hohenegger (2017). Thus, both $\phi_{\mathrm{wp,crit}}$ depend not only on the ~~particular soil type employed~~ employed soil type but also on the soil model.

Figure 6 shows the fit of Eq. 4, together with the values obtained in the simulations. It reveals ~~a very good~~ an excellent

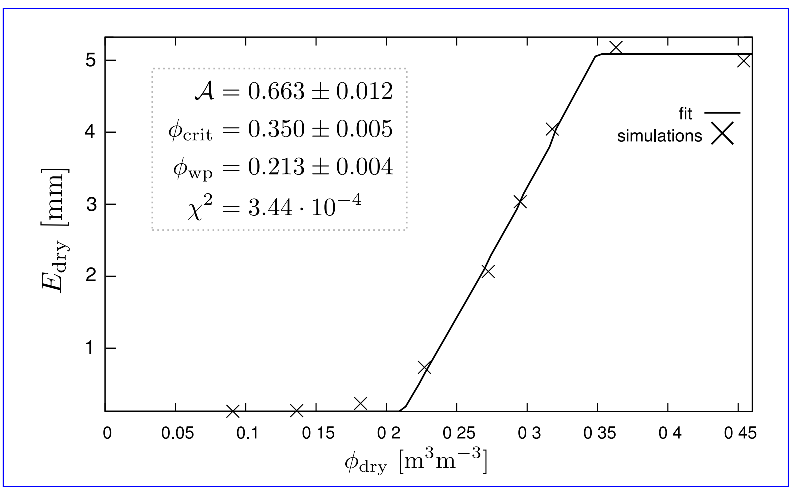

Figure 6. Fit of E_{dry} with values obtained from the simulations of the default configuration (`DA_20_100` to `DA_100_100`). Crosses indicate values obtained from simulations while line indicates the fit performed using ~~the function of~~ Eq. 4. Upper left inset ~~show~~ shows the values obtained by the fit together with absolute errors and the residual sum of squares χ^2~~value~~, i.e. the sum of the squared difference between the values predicted by the fit and the ones obtained in the simulations.

agreement between theory and simulations. The small discrepancies mainly come from the fact that we assume a constant value of $\langle Q_{\mathrm{net}} \rangle = 300$ W m^{-2} $= 0.43$ mm h^{-1} across the simulations, although the simulated value depends on soil moisture and varies by about 7%. This is due to different cloud regimes which alter the surface radiation balance (Cioni and Hohenegger, 2017, Sec. 4b).

We note that our formulation of evaporation differs from the one used in the model of Lintner et al. (2013) where potential evaporation was used in place of Q_{net}~~. Our approach in deriving the theoretical model directly neglects the feedback between the land-surface and the atmosphere, as evaporation does not implicitly depend on the near-surface atmospheric specific humidity but only on soil moisture~~, which is the main difference between the original framework of Budyko (1961) and the one of Manabe (1969).

4.2 Advection

Our goal is to find a formulation of A_{dry} ~~can be obtained directly from its mathematical definition~~ as a function of soil moisture. This can be achieved starting from the definition of Eq. A2 and assuming that the advection of every tracer is mainly due to the propagation of the ~~breeze front (see Appendix A for an exhaustive definitions of the symbols) :~~ front associated with the meso-scale circulation, hence $H = H_{\text{front}}$. In this case

$$A_{\text{dry}} = -\frac{1}{\rho_w}\cancel{\tau}\int_{\cancel{H_{\text{front}}}\,0}^{\tau} \int_0^{H_{\text{front}}} \mathbf{v}_{\text{front}} \cdot \nabla q_{\text{tot}}\Big|_{\text{dry}} \rho_a \,\mathrm{d}z\,\mathrm{d}t \simeq -\frac{1}{\rho_w}\int_0^{\tau}\int_0^{H_{\text{front}}} u_{\text{front}} \frac{\partial q_{\text{tot}}}{\partial x}\Big|_{\text{dry}} \rho_a \,\mathrm{d}z\,\mathrm{d}t \qquad (5)$$

where H_{front} is the height of the front associated with the meso-scale circulation or, equally, the Planetary Boundary Layer (PBL) height, ~~u_{front}~~ $\mathbf{v}_{\text{front}}$ its speed, ρ_a is the air density and ρ_w the water density. ~~A fundamental approximation is already applied when writing~~ Eq. 5 ~~and should be highlighted. The features that describe the front propagation, like its speed, were defined only up to the collision time (see e. g. Fig. 2). Thus in Eq. 5 we are implicitly neglecting the contributions that may arise after the front collision. We are aware that one may solve this issue by considering the variables E_{dry}, A_{dry} and P_{dry} accumulated only up to the front collision. However this would possibly introduce different errors in the evaluation of the aforementioned variables as the time of collision would need to be defined and is not always clearly visible (see again Fig. 5).~~ has been already approximated given that the front propagates mainly in the x direction (see Sec. 3.1), so that there is no y component of ∇q_{tot}. ~~Eq. 5 is further approximated by :~~

$$\cancel{A_{\text{dry}} \sim \tau \frac{\rho_a}{\rho_w} \frac{H_{\text{front}}}{L_{\text{front}}} \langle \Delta \overline{q}_{\text{tot}} \cdot u_{\text{front}} \rangle}$$

~~With $\Delta\overline{q}_{\text{tot}}$ we indicate the difference in the vertical integral of q_{tot} between the two patches. Note that, although we use the same notation as in Appendix A, this time $\overline{q}_{\text{tot}}$ is integrated only up to~~ The propagation speed of the front u_{front} and ρ_a can be seen as constants in the vertical within the height H_{front}~~, which is considered constant over the time when the front is active. Furthermore, in deriving Eq. ?? we assumed that the horizontal gradient~~, while q_{tot} remains a function of x, z and time t. The time integration can be replaced by considering the average over time multiplied by the time scale τ to obtain

$$A_{\text{dry}} = -\tau \frac{\rho_a}{\rho_w} \langle u_{\text{front}} \rangle_{\text{dry}} \langle \int_0^{H_{\text{front}}} \frac{\partial q_{\text{tot}}}{\partial x}\Big|_{\text{dry}} \mathrm{d}z \rangle = -\tau \frac{\rho_a}{\rho_w} \langle u_{\text{front}} \rangle_{\text{dry}} \langle \int_0^{H_{\text{front}}} \frac{\Delta q_{\text{tot}}}{L_{\text{front}}} \mathrm{d}z \rangle = -\tau \frac{\rho_a}{\rho_w} \langle u_{\text{front}} \rangle_{\text{dry}} \frac{H_{\text{front}}}{L_{\text{front}}} \langle \Delta q_{\text{tot}} \rangle \qquad (6)$$

In Eq. 6 we approximated the derivative of q_{tot} ~~depends only on the x-dimension and is constant in the vertical dimension between the surface and H_{front}.~~ as the difference between the two patches Δq_{tot} divided by the penetration length of the front, L_{front} (Crosman and Horel, 2010).

~~Because of its definition, $\Delta\overline{q}_{\text{tot}}$ entails the contributions of every tracer.~~To simplify the problem we assume ~~$\Delta\overline{q}_{\text{tot}} \simeq \Delta\overline{q}_v$~~ $\Delta q_{\text{tot}} \simeq \Delta q_v$, which is viewed as the difference in specific humidity ahead of the front and behind it. As in studies which have viewed sea breezes as gravity current (Robinson et al., 2013), we assume that this difference is not directly affected by the circulation, which gives an upper bound estimate. ~~Hence we can write:~~ The changes in q_v due to surface evaporation accumulated up to a certain time τ can then be written as

$$q_v(\cancel{t}\tau) = q_v(0) + \cancel{\frac{\rho_w \mathrm{LH}}{\rho_a H_{\text{moist}}}\tau_{\text{moist}}} \frac{\rho_w E}{\rho_a H_{\text{moist}}} \Rightarrow \Delta q_v = -\frac{\rho_w}{\rho_a H_{\text{moist}}} \underbrace{(E_{\text{wet}} - E_{\text{dry}})}_{\Delta E} \tag{7}$$

where ~~$q_v(0)$ is the specific humidity at the initial time,~~ H_{moist} is the vertical extent of the moistening process due to the ~~latent heat flux LH and τ_{moist} its time scale. In computing the difference in specific humidity between the two patches Δq_v the term~~ accumulated surface evaporation E and $q_v(0)$ ~~disappears, as the initial condition is homogeneous. Thus, by~~ is the specific humidity at the initial time. By assuming that the moistening is confined to the PBL~~and that $\tau_{\text{moist}} = \tau$ we can rewrite the advection as~~

$$\cancel{A_{\text{dry}} \sim \tau \frac{\tau \langle u_{\text{front}} \rangle}{L_{\text{front}}} \langle \Delta \mathrm{LH} \rangle}$$

~~Regarding the other termsof Eq. 8,~~ , so that $H_{\text{moist}} = H_{\text{front}}$ we can substitute Eq. 7 into Eq. 6 to obtain

$$A_{\text{dry}} = \frac{\tau \langle u_{\text{front}} \rangle}{L_{\text{front}}} \Delta E \tag{8}$$

Our analysis thus indicates that the advection only depends on four terms: τ, which is a constant, the difference in E between the two patches, which can be estimated from Eq. 4 and which depends on the soil moisture, as well as L_{front} and u_{front}. In all simulations the ~~results of the simulations show that the~~ front has a constant inland propagation of $L_{\text{front}} \simeq 100$ km, which corresponds to half of the patch size. More importantly, the front speed does not vary much with different surface heterogeneity gradients, against our initial expectations that motivated this study (see Introduction). For example, between the `DA_20_100` and the `DA_60_100` cases only a 3% relative decrease in the front speed is observed (not shown).

This ~~counterintuitive~~ counter intuitive behaviour is related to the fact that cold pools ~~cause the first~~ lead to a noticeable acceleration of the front, as seen in section 3.1. Although the front is initially triggered by the surface heterogeneity, and different surface heterogeneities may lead to different initial propagation velocities, the much faster cold pools end up determining the front velocity, thus masking the effect of the surface heterogeneity. This stands in agreement with what found by Rieck et al. (2015), and in particular with the thermodynamic contribution of cold pools to the propagation speed of the front (their Eq. 1). ~~In our case~~ Moreover, cold pools are distributed along the front and continuously fed by precipitation events~~behind of it.~~, similarly to what happens in squall-lines. Given this spatial organization, their strength and propagation ~~does~~ do not depend on the surface state~~as in Gentine et al. (2016) but mostly on the upper air state as in Peters and Hohenegger (2017), the latter one~~

~~not being greatly~~ , as in the case for isolated convection (Gentine et al., 2016). Instead they solely depend on the state of the mid- to upper-troposphere (Peters and Hohenegger, 2017) which is also not significantly modified by surface fluxes ~~during the~~ over the course of one diurnal cycle.

We can thus finally express advection simply as

$$A_{\text{dry}}(\phi_{\text{dry}}) = \cancel{\tau}\mathcal{B}\Delta\mathrm{E}(\phi_{\text{dry}}) \tag{9}$$

where $\mathcal{B} = \tau \langle u_{\text{front}} \rangle / L_{\text{front}}$ is a proportionality constant that does not depend on ~~the surface state. The difference $\langle \Delta \mathrm{LH} \rangle$ is known, thanks to the previous section, and can be used to fit the values of advection~~ soil moisture. Using the parameters $\mathcal{A}, \phi_{\text{wp}}, \phi_{\text{crit}}$ obtained from the fit of Eq. 4 (see Fig. 6) we can compute the difference $\Delta\mathrm{E}(\phi_{\text{dry}})$. Together with the values of A_{dry} obtained in the simulations ~~with~~ the values of $\Delta\mathrm{E}(\phi_{\text{dry}})$ can be used to fit Eq. 9 ~~to obtain a value of the parameter $\mathcal{B} = 1.47$~~and compute a value for the parameter $\mathcal{B}$: in the DA cases $\mathcal{B} = 1.47$. This is smaller than the value that would be obtained by estimating instead $\langle u_{\text{front}} \rangle$ and L_{front} directly, as this latter approximation does not take into account moisture losses due to advection.~~Note that from Eq. 9 it immediately follows that $A_{\text{dry}}(\phi) = \mathcal{B}\Delta\mathrm{E}(\phi)$.~~

Figure 7 shows the values of A_{dry} and the fit performed using Eq. 9 for the basic set of experiments and for further cases, the latter used to test the ~~findings~~ finding that $\mathcal{B}$ does not depend on ϕ but solely on the atmospheric state. Overall the fit matches the variation of A_{dry} with ϕ_{dry} remarkably well given the various assumptions. Both the simulated decrease of A_{dry} with higher

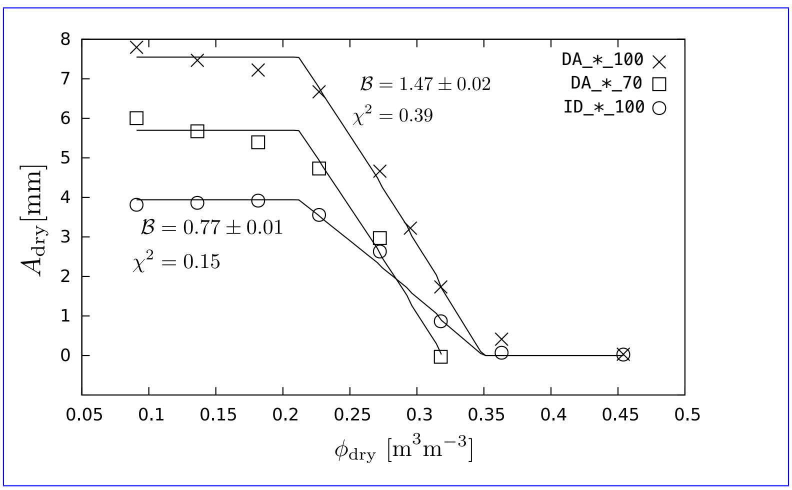

Figure 7. Fit of the advection in cases DA_*_100, DA_*_70 and ID_*_100. Symbols indicate the values obtained from the simulations while lines represent the fit performed using Eq. 9. The ~~different~~ obtained values of $\mathcal{B}$ ~~obtained~~ are reported in the insets, together with the absolute error and the χ^2 value (see Fig. 6 for the definition). Note that for the DA_*_100 and DA_*_70 cases the fits yielded similar results: for this reason the obtained value for $\mathcal{B}$ is reported only once.

values of soil moisture and the flattening of advection ~~with~~ by soil moisture lower than the wilting point are reproduced, although both effects seem to be overestimated by Eq. 9.

In the simulations where the initial value of ϕ_{wet} is reduced to just 70% of saturation the estimated value of $\mathcal{B}$ is almost the same as the one of the default configuration, confirming that $\mathcal{B}$ does not depend ~~both on ϕ_{wet} and ϕ_{dry}~~on soil moisture. Instead, in the `ID_` cases (Tab. 3), which ~~uses~~ use a different atmospheric profile and hence support distinct cold pool strength, the value of $\mathcal{B}$ is reduced by about half.

4.3 Computing the derivative of precipitation

Equations 2, 4 and 9 can be combined in order to compute P_{dry}. We are, however, interested in its variation with soil moisture, $\frac{\partial P_{\text{dry}}}{\partial \phi_{\text{dry}}}$, which can be computed as:

$$\begin{aligned}
\frac{\partial P_{\text{dry}}}{\partial \phi_{\text{dry}}} &= \eta_A \frac{\partial A_{\text{dry}}}{\partial \phi_{\text{dry}}} + \eta_E \frac{\partial E_{\text{dry}}}{\partial \phi_{\text{dry}}} \\
&= \eta_A \frac{\partial}{\partial \phi_{\text{dry}}} \left(\mathcal{B}\left(E_{\text{wet}} - E_{\text{dry}}\right)\right) + \eta_E \frac{\partial E_{\text{dry}}}{\partial \phi_{\text{dry}}} \\
&= -\eta_A \mathcal{B} \frac{\partial E_{\text{dry}}}{\partial \phi_{\text{dry}}} + \eta_E \frac{\partial E_{\text{dry}}}{\partial \phi_{\text{dry}}}
\end{aligned} \tag{10}$$

Note that the derivation of Eq. ~~9~~ 10 retains only one term of the difference given that E_{wet} does not depend on ϕ_{dry}. Using Eq. 4 it is straightforward to compute the derivative of E_{dry} as

$$\frac{\partial E_{\text{dry}}}{\partial \phi_{\text{dry}}} = \tau \mathcal{A} \langle Q_{\text{net}} \rangle \times \begin{cases} 0 & \text{for } \phi_{\text{dry}} < \phi_{\text{wp}} \\ \left(\phi_{\text{crit}} - \phi_{\text{wp}}\right)^{-1} & \text{for } \phi_{\text{wp}} \le \phi_{\text{dry}} \le \phi_{\text{crit}} \\ 0 & \text{for } \phi_{\text{dry}} > \phi_{\text{crit}} \end{cases} \tag{11}$$

which is a step-wise function constituted by constant values.

Equations 10 and 11 indicate that for ~~$\phi < \phi_{\text{wp}}$ and $\phi > \phi_{\text{crit}}$~~ $\phi_{\text{dry}} < \phi_{\text{wp}}$ and $\phi_{\text{dry}} > \phi_{\text{crit}}$ there is no change in precipitation with soil moisture independently of the value of the efficiencies. In contrast for ~~$\phi_{\text{wp}} < \phi < \phi_{\text{crit}}$~~ $\phi_{\text{wp}} \le \phi_{\text{dry}} \le \phi_{\text{crit}}$ then $\frac{\partial \mathrm{E}_{\text{dry}}}{\partial \phi_{\text{dry}}} \neq 0$ ~~and the equation can be rewritten to study the sign of $\frac{\partial P_{\text{dry}}}{\partial \phi_{\text{dry}}}$.~~

~~Putting the values of the efficiencies and of $\mathcal{B}$ obtained from the `DA_` simulations in the previous section in Eq. 12 confirms that, in that set-up~~but still the derivative of precipitation with respect to soil moisture does not depend upon the soil moisture content itself. These findings contrast with the ones of Lintner et al. (2013), who found a minimum of the derivative for intermediate values of soil moisture. This is a consequence of the formulation of E_{dry} as a linear function of ϕ_{dry} and the fact that A_{dry} also turned out to be a linear function of ϕ_{dry} as u_{front} is constant. This remains true as long as the convection is strongly organized by the front associated with the meso-scale circulation and produces strong cold pools that end up determining the propagation velocity. It should be noted that, $\frac{\partial P_{\text{dry}}}{\partial \phi_{\text{dry}}}$ ~~< 0, which agrees with the simulations results. For the `ID_` cases, Eq. 12~~

~~leads to a similar result,~~ although the ~~difference $\eta_E - \eta_A \mathcal{B}$ is just slightly positive. We will explain why hereinafter.Our simple theoretical derivation indicates that the variation of precipitation with soil moisture surprisingly does not~~ derivative $\frac{\partial P_{\mathrm{dry}}}{\partial \phi_{\mathrm{dry}}}$ does not depend on soil moisture, the absolute value of precipitation P_{dry} does indeed depend on soil moisture~~itself.Instead, the~~ , as we will shown later.

Coming back to Eqs. 10 and 11, we can now determine under which conditions P_{dry} will increase or decrease.

$$\frac{\partial P_{\mathrm{dry}}}{\partial \phi_{\mathrm{dry}}} \lesseqgtr 0 \Leftrightarrow -\eta_A \mathcal{B} + \eta_E \lesseqgtr 0 \Leftrightarrow \eta_E \lesseqgtr \eta_A \mathcal{B} \tag{12}$$

The atmospheric conditions, through the terms η_A, η_E and $\mathcal{B}$, determine whether increasing or decreasing the soil moisture of the dry patch is needed to increase the precipitation amount. ~~These findings contrast with the ones of Lintner et al. (2013), who found a minimum of the derivative for intermediate~~ Inserting the values of the efficiencies and of $\mathcal{B}$ obtained from the `DA_` simulations in Eq. 12 confirms that $\frac{\partial P_{\mathrm{dry}}}{\partial \phi_{\mathrm{dry}}} < 0$, which agrees with the simulated increase of precipitation with decreasing values of soil moisture. ~~This is most likely a consequence of our formulation of advection which is here tailored to the case of a soil-moisture induced (or sea-breeze like) circulation. The dependency of $\frac{\partial P_{\mathrm{dry}}}{\partial \phi_{\mathrm{dry}}}$ on η_A, η_E is illustrated for 3~~ These results are generalized with the help of Fig. 8 for three different values of $\mathcal{B}$~~in Fig.8.~~ .

~~Not surprisingly~~ In Fig. 8 positive values indicate an increase of precipitation over the dry patch with soil moisture, and

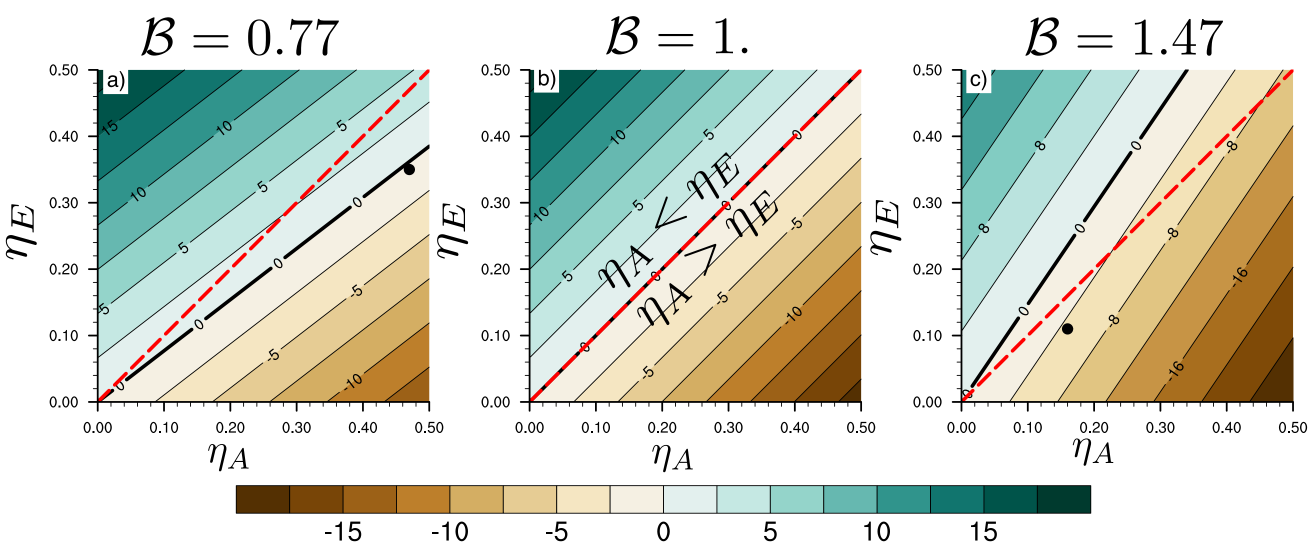

Figure 8. Contour plot of ~~$\frac{\partial P_{\mathrm{dry}}}{\phi_{\mathrm{dry}}}$~~ $\frac{\partial P_{\mathrm{dry}}}{\partial \phi_{\mathrm{dry}}}$ [mm m^3 m^{-3}] as a function of η_A, η_E for different values of the parameter $\mathcal{B}$. The black points in (a) and (c) are placed using the efficiencies obtained in the `ID_` and `DA_` cases, respectively. The dashed red line distinguishes the areas where $\eta_A > \eta_E$ and vice versa. Note the symmetric color scale and the thicker zero contour line.

vice-versa. Not surprisingly (see Eq. 12) using a value of $\mathcal{B} = 1$ ~~in Fig. 8 (b)~~ gives a symmetric picture ~~. Instead, different values~~ where an increase of precipitation with soil moisture is obtained for those cases when $\eta_E > \eta_A$. This relationship is modified by the value of $\mathcal{B}$~~slightly modify the conditions needed to obtain either a positive or negative derivative.~~.

Figure 8 overall shows that, as long as $\eta_A > \eta_E$ it is very unlikely to get a positive derivative~~, i. e. an increase of precipitation over the dry patch with increasing soil moisture.~~. Only with values of $\mathcal{B}$ small enough, which would mean weaker and slower cold pools, the derivative may change sign even with $\eta_A > \eta_E$. ~~Note how the values obtained in the ID_ simulations (black point in Fig. 8 a) lead to a smaller value of the derivative, which is close to the zero-line~~This situation almost happens in the ID simulation, where the theory predicts a derivative close to zero. This agrees with the weaker sensitivity of precipitation to soil moisture observed in ~~this~~ that case. Alternatively, to get a positive derivative, evaporation should become ~~more efficient of~~ much more efficient than advection, i.e. $\eta_E \gg \eta_A$. This, however, ~~does~~ did not happen in the performed simulations~~, which always lie in the rightmost lower quadrant (black points in Fig.8).~~.

These findings already answer the main question posed in the introduction and can be further generalized to the case when both ϕ_{wet} and ϕ_{dry} are changed at the same time. This allows one to investigate the dependency of precipitation on the soil moisture values of the two patches when the η_A, η_E and $\mathcal{B}$ parameters are fixed. First of all, $\frac{\partial P_{\text{dry}}}{\partial \phi_{\text{wet}}}$ can be computed with the same method ~~used~~ as before:

$$\frac{\partial P_{\text{dry}}}{\partial \phi_{\text{wet}}} = \eta_A \mathcal{B} \frac{\partial E_{\text{wet}}}{\partial \phi_{\text{wet}}} \tag{13}$$

given that the evaporation over the dry patch does not depend on the soil moisture of the wet patch. Second, the two derivatives $\frac{\partial P_{\text{dry}}}{\partial \phi_{\text{wet}}}, \frac{\partial P_{\text{dry}}}{\partial \phi_{\text{dry}}}$ can be combined to obtain the total precipitation change on the dry patch.

$$\begin{aligned}\Delta P_{\text{dry}} &= \frac{\partial P_{\text{dry}}}{\partial \phi_{\text{dry}}} \Delta \phi_{\text{dry}} + \frac{\partial P_{\text{dry}}}{\partial \phi_{\text{wet}}} \Delta \phi_{\text{wet}} \\ &= (\eta_E - \eta_A \mathcal{B}) \frac{\partial E_{\text{dry}}}{\partial \phi_{\text{dry}}} \Delta \phi_{\text{dry}} + \eta_A \mathcal{B} \frac{\partial E_{\text{wet}}}{\partial \phi_{\text{wet}}} \Delta \phi_{\text{wet}}\end{aligned} \tag{14}$$

Assuming that the soil type of both patches is the same~~from the various combinations between magnitude of ϕ_{wp}, $\phi_{\text{dry,wet}}$ and ϕ_{crit}~~, only the case $\phi_{\text{wp}} < \phi_{\text{dry,wet}} < \phi_{\text{crit}}$ is of interest. The other cases either revert to the previously discussed case (Eq. 12) or reduce to the trivial solution where only $\Delta\phi_{\text{wet}}$ is affecting ΔP_{dry}. For $\phi_{\text{wp}} < \phi_{\text{dry,wet}} < \phi_{\text{crit}}$ we obtain $\frac{\partial E_{\text{dry}}}{\partial \phi_{\text{dry}}} = \frac{\partial E_{\text{wet}}}{\partial \phi_{\text{wet}}}$. Thus, changes in precipitation in our idealized model can be formulated as

$$\Delta P_{\text{dry}} = \tau \cancel{\frac{\mathcal{A} Q_{\text{net}}}{\phi_{\text{crit}} - \phi_{\text{wp}}}} \frac{\mathcal{A} \langle Q_{\text{net}} \rangle}{\phi_{\text{crit}} - \phi_{\text{wp}}} \left((\eta_E - \eta_A \mathcal{B}) \Delta \phi_{\text{dry}} + \eta_A \mathcal{B} \Delta \phi_{\text{wet}} \right) \tag{15}$$

The behaviour of Eq. 15 as function of $\Delta\phi_{\text{dry}}$, $\Delta\phi_{\text{wet}}$ and $\mathcal{B}$ ~~can be investigated in~~ is investigated with the help of Fig. 9. In the default configuration described in section 3.1 the soil moisture of the wet patch was kept constant, i.e. $\Delta\phi_{\text{wet}} = 0$, while the soil moisture of the dry patch was increased, i.e. $\Delta\phi_{\text{dry}} > 0$. Figure 9 (a) shows that, in the aforementioned case, ΔP_{dry} is negative, as in our simulations. In this case decreasing ϕ_{dry} and increasing ϕ_{wet} is the most efficient way to increase precipitation.

Figure 9 (b) presents the case characteristic ~~for the ID_ simulations~~ of the simulations performed with the ID sounding. The flattening of the contour lines shows that there is little sensitivity on ϕ_{dry}, as ~~already discussed previously~~ previously discussed. Mainly increasing ϕ_{wet} would allow precipitation to increase. In the extreme case where $\mathcal{B}$ is further reduced (Fig. 9 c) the picture partly reverses. Both soil moisture of the wet and of the dry patch should be increased to sustain an increase of

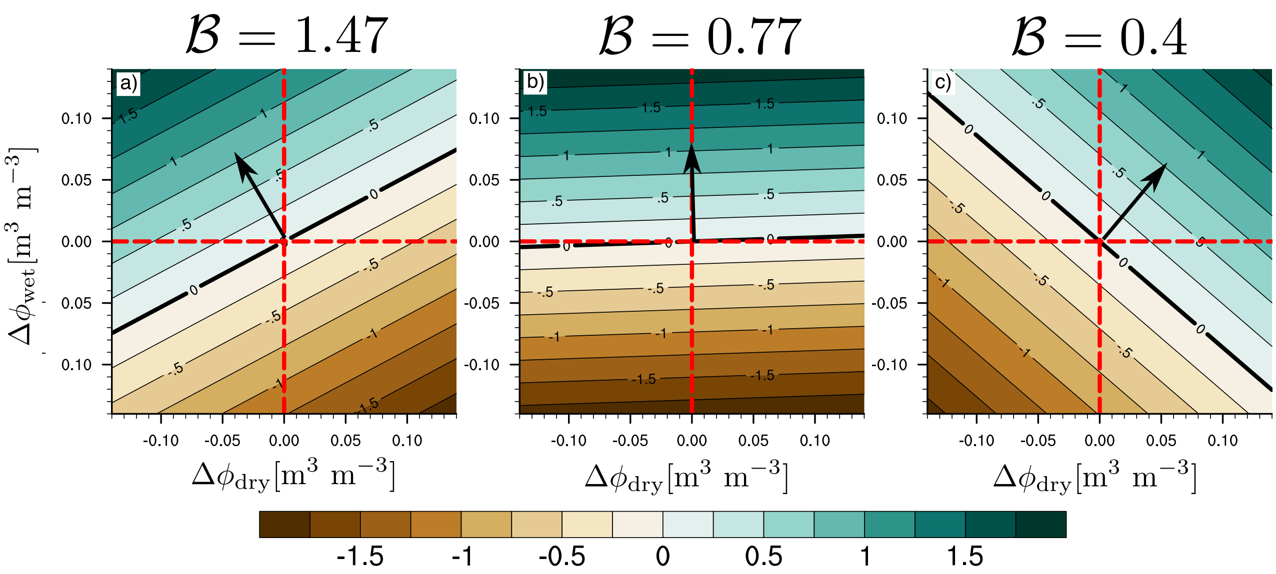

Figure 9. $\Delta P_{\rm dry}$ as function of $\Delta\phi_{\rm dry}$ and $\Delta\phi_{\rm wet}$ for different values of the parameter $\mathcal{B}$. The x and y axes represent the variation of $\phi_{\rm dry}$ and $\phi_{\rm wet}$, respectively. Note that the maximum variation is $\phi_{\rm crit} - \phi_{\rm wp}$, as $\Delta P_{\rm dry}$ is computed for the regime $\phi_{\rm wp} < \phi_{\rm dry,wet} < \phi_{\rm crit}$. The red dashed lines indicate no variation of the soil moisture of either one of the patches. The efficiencies are set to $(\eta_A, \eta_E) = (0.16, 0.11)$ in (a) and to $(\eta_A, \eta_E) = (0.47, 0.35)$ in (b) and (c) to match the simulation results. The black arrow indicates the direction of maximum growth, i.e. when an increase of precipitation is expected.

precipitation, as evaporation becomes now relevant and advection has a negligible contribution.

Figure 9 thus indicates that, in any case, the soil moisture of the wet patch should be increased to get more precipitation on the dry patch. The response to changes in soil moisture of the dry patch is more subtle, and the combination of the two responses can lead to positive or negative coupling depending on the atmosphere state. This may explain why in reality both signs of the coupling are observed with different atmospheric states.

5 Conclusions

Motivated by the ambiguous relationship between soil moisture, soil moisture heterogeneity and precipitation we designed idealized simulations of ~~the~~ a convective diurnal cycle that make use of a coupled configuration of an atmospheric Large Eddy Simulation (LES) model and a land-surface model. The heterogeneity in the land surface was prescribed by dividing the domain into two patches with different initial values of soil moisture. Inspired by the results of the simulations, we specifically wanted to derive a simple conceptual model that retains the minimum parameters that control precipitation over a spatially drier patch. Moreover, we wanted to use ~~such~~ this model to understand which is the most efficient way to increase precipitation by acting on soil moisture given the opposite control of soil moisture on advection and evaporation.

Since the main potential sources contributing to precipitation are constituted by remote moisture advection by the meso-scale circulation triggered by the soil moisture heterogeneity and local evaporation, we first aim at disentangling the effects of these two on precipitation. Results from the simulations show, as expected, that the moisture advection over the dry patch decreases with increasing local soil moisture, while evaporation increases. The interplay between these two effects produces a decrease of precipitation with increasing values of local soil moisture for the considered case.

More importantly the simulation results indicated that such a decrease can only be correctly reproduced by assuming that advection and evaporation processes contribute differently to precipitation. Hence we model precipitation as the sum of advection and evaporation each weighted by its own efficiency (see Eq. 2). By using two efficiencies they become independent of soil moisture and only ~~depend~~ dependent on the initial atmospheric state.

As a second step we conceptualize the variations of evaporation and advection with soil moisture. Evaporation can be approximated using the *bucket* model owing to Budyko (1961) (see Eq. 3). The advection is ~~estimate~~ estimated as the product of the breeze front ~~, triggered by the surface heterogeneity,~~ velocity and the gradient in near-surface specific humidity (see Eq. 5). ~~The latter depends on the gradient in latent heat fluxes, whereas the former is surprisingly independent of the degree of soil moisture heterogeneity. This is so because the breeze front velocity is, after a certain time, fully determined by cold pools whose strength is mainly controlled by the upper tropospheric profile~~A priori we would have expected a squared dependency of advection on soil moisture since both the velocity of the front and the gradient in specific humidity should be related to soil moisture. However, it turns out that the velocity of the front is independent of soil moisture as the development of convection at the breeze front and the generation of strong cold pools lead to a strong acceleration of the front that fully masks the effect of the initial surface heterogeneity.

Putting all the results together indicates that the ~~variations of precipitation over the dry patch do~~ derivative of precipitation with respect to the soil moisture of the dry patch does not depend on the actual soil moisture value. This is due to the fact that the ~~derivation of the~~ functional forms of advection and evaporation ends up to be ~~very similar and cancels out the dependency on soil moisture .~~ linear functions in soil moisture. The idealized model is valid as long as the evaporation keeps its linearity as function of soil moisture and the propagation speed of the front does not depend on the surface heterogeneity gradient, meaning strong enough cold pools.

The parameters that control the variations of precipitation with local soil moisture are the aforementioned efficiencies and a scale parameter that defines the magnitude of the advection. All these parameters depend solely on the atmospheric state. According to the values of these parameters, as estimated from the simulations, the most efficient way to increase precipitation over the dry patch is ~~always to decrease soil moisture over~~ to decrease the soil moisture of the dry patch. Thus, one can say that, in order to have more precipitation over spatially drier areas, more precipitation should first fall on spatially wetter ones. In other words, the most efficient way to obtain more precipitation over dry areas is to let them dry out for a long time so that a stronger gradient can build up and thus produce more explosive convective events due to a stronger meso-scale circulation. However, if either the efficiency of evaporation becomes much larger than the one of advection or the scale parameter that defines the importance of advection decreases under a certain threshold then the response of precipitation can be reversed. Although we did not find any evidence of this behaviour for the two atmospheric profiles tested in this work it would be interesting as a next step to derive ~~these~~ the three parameters predicted by the conceptual model from more realistic simulations to infer the frequency of occurrence of the various precipitation regimes.

Appendix A: Computation of the advection as residual term

The advection of every tracer ~~spatially averaged over a certain area A_{area}~~ is computed directly from the moisture balance equation as a residual. We use the following formulation~~:~~, which applies for a certain point (x,y) over a 2-dimensional domain:

$$\frac{1}{\rho_w}\int_0^\tau\int_0^H \cancel{\frac{\mathrm{D}}{\mathrm{D}t}}\frac{\mathrm{D}q_{\text{tot}}}{\mathrm{D}t}\rho_a \cancel{q_{\text{tot}\,\text{area}}}\,\mathrm{d}z\,\mathrm{d}t = \frac{1}{\rho_w}\int_0^\tau \cancel{\frac{\mathrm{D}\overline{q}_{\text{tot}}}{\mathrm{D}t}_{\text{area}}}\int_0^H\left[\frac{\partial q_{\text{tot}}}{\partial t} + \mathbf{v}\cdot\nabla q_{\text{tot}}\right]\rho_a\,\mathrm{d}z\,\mathrm{d}t = \cancel{E_{\text{area}} - P_{\text{area}}}E - P \tag{A1}$$

where D indicates the total derivative, ~~P_{area} is the area-averaged~~ P [m] is the accumulated precipitation, ~~E_{area} the area-averaged~~ E [m] the accumulated evaporation, ~~$\overline{q}_{\text{tot}}$ is the area-averaged vertically integrated~~ q_{tot} [kg kg^{-1}] represents the sum of all tracers (water vapour ~~, clouds, rain, snow, ice, graupel and hail~~ q_v, clouds q_c, rain q_r, snow q_s, ice q_i, graupel q_g and hail q_h) mixing ratios, ρ_w [kg m^{-3}] the density of water~~and~~ , ρ_a [kg m^{-3}] the air density ~~. As the quantities on the right-hand-side are accumulated over the length of the diurnal cycle τ, the left-hand-side is explicitly integrated over time~~ and $\mathbf{v}$ the velocity of air as a vector. H indicates the top of the simulation domain and τ the lenght of the accumulation period (18 hours in our experiments). The total derivative ~~term can be further~~ can be divided into its advective term:

$$\cancel{A_{\text{area}} =} \equiv -\frac{1}{\rho_w}\int_0^\tau\int_0^H \mathbf{v}\cdot\nabla\cancel{(}q_{\text{tot}}\cancel{\cdot\mathbf{v})}\rho_a\,\mathrm{d}z\,\mathrm{d}t \tag{A2}$$

and ~~a~~ the local derivative:

$$\frac{1}{\rho_w}\int_0^\tau \cancel{\frac{\partial\overline{q}_{\text{tot}}}{\partial t}_{\text{area}}}\int_0^H \frac{\partial q_{\text{tot}}}{\partial t}\rho_a\,\mathrm{d}z\,\mathrm{d}t = A_{\cancel{\text{area}}} + \cancel{E_{\text{area}} - P_{\text{area}}}E - P \tag{A3}$$

In both equations A2 and A3 the variables A, E, P are solely functions of (x,y) whereas q_{tot} depends also on time t and on the vertical coordinate z. We can eliminate the dependency on (x,y) by applying an average operator over a certain area, indicated with the subscript $_{\text{area}}$:

$$\frac{1}{\rho_w}\int_0^\tau\int_0^H \left.\frac{\partial q_{\text{tot}}}{\partial t}\right|_{\text{area}}\rho_a\,\mathrm{d}z\,\mathrm{d}t = A_{\text{area}} + E_{\text{area}} - P_{\text{area}} \tag{A4}$$

In the main text we use as area either the full domain, denoted with the suffix $_{\mathrm{dom}}$, or the dry patch only, denoted by the suffix $_{\mathrm{dry}}$. By indicating the weighted vertical integral of q_{tot} as $\overline{q}_{\mathrm{tot}} \equiv \int_0^H q_{\mathrm{tot}} \rho_a \, \mathrm{d}z$ we can further simplify the previous equation to:

$$\frac{1}{\rho_w} \int\limits_0^\tau \frac{\partial \overline{q}_{\mathrm{tot}}}{\partial t}\Big|_{\mathrm{area}} \mathrm{d}t = A_{\mathrm{area}} + E_{\mathrm{area}} - P_{\mathrm{area}} \tag{A5}$$

Although other studies only considered the advection of water vapour, i.e. of $\overline{q}_v$, in order to close the balance it is necessary to consider all species. In fact, although $\overline{q}_i, \overline{q}_g, \overline{q}_h, \overline{q}_s$ are order of magnitudes smaller than $\overline{q}_v, \overline{q}_c, \overline{q}_r$, their variations over time are not, so that neglecting these terms in Eq. ~~??~~ A5 would lead to an unbalance.
From the 5-min simulation output we use Eq. ~~??~~ A5 and estimate the advection as the residual ~~$R_{\mathrm{area}} = \int_0^\tau \frac{\partial \overline{q}_{\mathrm{tot}}}{\partial t}\Big|_{\mathrm{area}} \mathrm{d}t + P_{\mathrm{area}} - E_{\mathrm{area}} = A_{\mathrm{area}}$,~~
We verify that, when averaged over the entire domain, $R_{\mathrm{dom}} = 0$.

Competing interests. The authors declare that they have no competing interests.

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