

# 1 **Demonstrating the “Unit Hydrograph” and flow routing processes**

## 2 **involving active student participation – A university lecture experiment**

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### 11 **Abstract**

12 The unit hydrograph (UH) has been one of the most widely employed hydrological modelling techniques to predict  
13 rainfall-runoff behavior of hydrological catchments, and is still used to this day. Its concept is based on the idea  
14 that a unit of effective precipitation per time unit (e.g. mm h<sup>-1</sup>) will always lead to a specific catchment response  
15 in runoff. Given its relevance, the UH is an important topic that is addressed in most (engineering) hydrology  
16 courses at all academic levels. While the principles of the UH seem to be simple and easy to understand, teaching  
17 experiences in the past suggest strong difficulties in students’ perception of the UH theory and application.

18 In order to facilitate a deeper students’ understanding of the theory and application of the UH, we developed a  
19 simple and cheap lecture theatre experiment which involved active student participation. The seating of the  
20 students in the lecture theatre represented the “hydrological catchment” in its size and form. A set of plastic balls,  
21 prepared with a piece of magnetic strip to be tacked to any white/black board, each represented a unit amount of  
22 effective precipitation. The balls are evenly distributed over the lecture theatre and routed by some given rules  
23 down the catchment to the “catchment outlet”, where the resulting hydrograph is monitored and illustrated at the  
24 black/white board.

25 The experiment allowed an illustration of the underlying principles of the UH, including stationarity, linearity and  
26 superposition of the generated runoff and subsequent routing. In addition, some variations of the experimental  
27 setup extended the UH-concept to demonstrate the impact of elevation, different runoff regimes and non-uniform  
28 precipitation events on the resulting hydrograph.

29 In summary, our own experience in the classroom, a first set of student exams, as well as student feedback and  
30 formal evaluation suggest that the integration of such an experiment deepened the learning experience by active  
31 participation. The experiment also initialized a more experienced based discussion of the theory and assumptions  
32 behind the UH. Finally, the experiment was a welcome break within a 3-hour lecture setting, and great fun to  
33 prepare and run.

# 1 1 Introduction

## 2 1.1 Background

3 The prediction of catchment rainfall-runoff behavior is an important prerequisite of any effective flood risk and  
4 water resources management practice. Rainfall-runoff modelling has a long history with a starting point that can  
5 been dated back more than 150 years to the work of Mulvaney (1851) and even further (see e.g. Biswas et al.,  
6 1970). He first introduced a simple linear relationship between peak discharge and maximum catchment average  
7 rainfall intensity that is dependent on catchment size and an empirical coefficient that effectively represents all  
8 other catchment characteristics (Beven, 2012). It is known as the *rational method* in engineering hydrology and  
9 with modification is still used today (e.g. Hromadka, 1994; Plate, 1988).

10 The first attempt to develop a spatially distributed hydrological model can be attributed to Ross (1921). In his  
11 approach, the catchment was split up into zones of equal travel times to the catchment outlet, and runoff production  
12 is calculated for each area, dependent on the antecedent conditions and rainfall rates. The resulting time-area  
13 diagrams represent the delays for runoff from each part of the catchment.

14 While it has been long known that flow velocities change in a nonlinear way with flow rate or flow depth, the  
15 Ross-approach relies on the assumption of linearity in routing the runoff to the catchment outlet. This violation  
16 however, has been shown less critical compared to the problem of estimating the effective rainfall for each event  
17 (Beven, 2012). The effective precipitation is thereby the part of the total precipitation contributing to runoff during  
18 an event. The estimation of this proportion is generally a nonlinear process depending on the antecedent catchment  
19 conditions, including soil moisture and interception storage conditions, possible snow cover, as well as rainfall  
20 intensities. Common approaches to calculate effective precipitation are the *constant loss rate* ( $\phi$ -index) method,  
21 the *constant proportion* method, or infiltration based methods as suggested by Horton (1933).

22 Another major difficulty with the Ross-approach was to decide which areas of the catchment would contribute to  
23 the different time zones, since there were almost no information on flow velocities and pathways in the different  
24 soil compartments (surface runoff, interflow) and on different environmental conditions (soil moisture, climate)  
25 available. The unit hydrograph (UH) method developed by Sherman (1932) tried to avoid these difficulties by  
26 representing the various time delays for runoff generated within the catchment by a stationary time distribution  
27 that has not necessarily any direct link to a particular location. The principal idea of the method is that assuming a  
28 linear routing procedure, this distribution could be normalized to represent the response to a unit of runoff  
29 production, or effective rainfall, generated over the catchment in one time step. In other words, the UH represents  
30 a discrete transfer function for effective rainfall to reach the basin outlet, lumped to the scale of the catchment.

31 The UH has been one of the most widely employed hydrological modelling techniques to predict rainfall-runoff  
32 behavior of hydrological catchments, and is still used in current generations of hydrological forecasting systems  
33 (see e.g. Samaniego, 2010 who use a simple triangular UH to convolute runoff that is produced in each cell to  
34 represent spatially variable runoff production. The routing between cells in the catchment however is done  
35 differently).

1 Given the importance of the UH in the historical development of rainfall-runoff models, and the fact that its basic  
2 principles are still included in current spatially distributed hydrological models, the UH is an important subject in  
3 any of the academic hydrology courses at the undergraduate and graduate level. The UH theory and principles  
4 include general concepts of catchment hydrology that form the foundation for further, more advanced topics and  
5 therefore need to be introduced in an understandable way.

## 6 *1.2 Teaching Situation*

7 The principles of runoff generation processes and the basic concepts for rainfall-runoff modelling are essential  
8 parts of the 3<sup>rd</sup> semester “Hydrology and Water Resources Management 1” course that is mandatory within the  
9 “Civil Engineering and Water Management (H033231)” program at the University of Natural Resources and Life  
10 Sciences (BOKU), Vienna. All students in the program have previously attended introductory courses in mathe-  
11 matics, physics, statistics, mechanics, hydraulics, and went through an introduction into hydrography and  
12 hydrometry. While in theory the previous knowledge seems to be more than sufficient and adequate to understand  
13 and grasp the central ideas behind the UH, exam results over the last years consistently and repeatedly  
14 demonstrated significant gaps in students’ understanding. While the concept of “effective precipitation”  
15 contributing to catchment runoff did not seem to make any difficulties, it was in particular the interpretation of the  
16 UH within a hydrological system context and its use as a prediction tool that were identified as critical areas. This  
17 included the interpretation of the UH as a transfer function that describes the “generation” of a hydrograph given  
18 a unit of effective precipitation in a catchment. As a first attempt we included (for the case of a uniform distribution  
19 of effective precipitation) an interpretation of the UH as a distribution of travel times (similar to time-area  
20 histograms). It seemed not sufficient as still a large number of students were not able to formulate the functioning  
21 and interpretation of an UH properly.

22 Therefore, in addition to the standard slide presentation of the theory, we decided in the winter-semester 2016/17  
23 to additionally visualize the concept of the unit hydrograph, its underlying assumptions and the linear routing  
24 principles in a lecture theatre experiment. The main goal was to stimulate student interests and to help them in  
25 their scientific learning. The positive effect of experimental demonstrations on deeper understanding and learning  
26 has been found in a number of studies in a variety of natural sciences disciplines (e.g. Roberts et al., 2005, Savec  
27 et al., 2005).

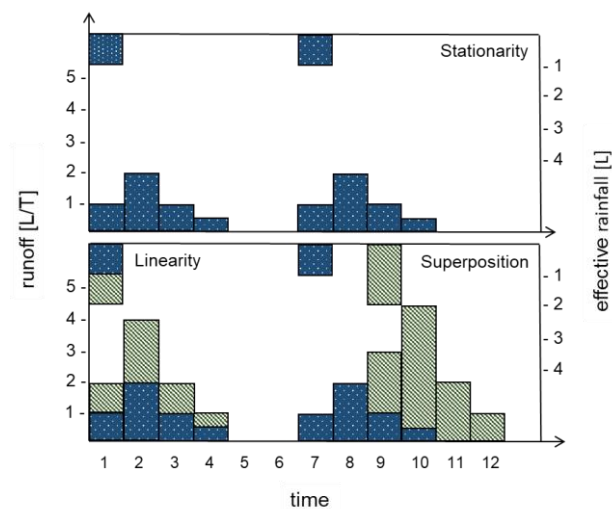
28 In our experiment, students and the location they were sitting represented a hydrological catchment in the lecture  
29 theatre, and they were actively involved in routing an effective precipitation event to a fictive runoff gauge in the  
30 room. This gauge was “monitored” over time and the resulting runoff data was visualized in an illustrative way.  
31 The setup of this experiment, the material required and its preparation as well as variants of the experiments and  
32 possible follow-up discussions to interpret these experiments will be described in some detail in the following.

## 1 2 Experimental Setup

### 2 2.1 The Unit Hydrograph (UH)

3 Summarizing the theory of the UH as briefly described in the previous section, three basic principles can be  
4 emphasized when transferring a unit of effective precipitation to the catchment outlet. These are: i) stationarity, ii)  
5 linearity, and iii) superposition. Stationarity of the UH concept assumes that a unit of effective precipitation will  
6 always generate the same hydrograph, independent of the time of the day/year and of the antecedent e.g. soil  
7 moisture conditions. Linearity in the routing process requires that any event with x units of effective precipitation  
8 will generate x-times as much predicted runoff in the hydrograph at the catchment outlet, having the same temporal  
9 distribution. The principle of superposition states that multiple consecutive effective precipitation events can be  
10 treated independently and predicted runoff in the hydrographs is superimposed by simply adding individual  
11 responses. An example unit hydrograph and these three fundamental principles are illustrated in Fig. 1.

12 We would like to point out here, that the UH concept per se does not explicitly link the runoff generation processes  
13 to any particular location. The UH rather represents a travel time distribution of a unit effective rainfall for a  
14 particular catchment. Being aware of this fact, we nevertheless chose a spatially explicit setting for illustrating the  
15 UH and its principles as well as runoff routing processes. While the UH theory does allow to consider spatially  
16 heterogeneous precipitation field that are stationary in time, the focus here is on uniformly distributed precipitation  
17 fields, as is often done in hydrology textbooks for simplicity (Maniak, 2016, p.350). The implementation into a  
18 lecture theatre experiment will be explained step-by-step in the next subsection.



19

20 Figure 1: An example of a UH (upper left) and an illustration of the principles of stationarity (upper right), linearity (lower  
21 left), and superposition (lower right). Units for runoff are provided as catchment area specific runoff in length [L] per time [T].

### 22 2.2 Lecture theatre experiment and teaching material

23 The following steps were implemented to realize an experimental illustration of the UH principle in a lecture  
24 theatre. The experiment took about 20-25 min including all instructions, explanations and some basic discussions.

25 It will be shown later (section 2.3) that different variants of the experiments can be performed to focus on further

1 aspects of catchment rainfall-runoff behaviour so that the experiment including discussions can be easily extended  
2 to 90 min. The experiment uses very simple and cheap materials that were easily available in any department store.  
3 Overall costs were in the range of €30,-.

4 Step 1 (ball preparation):

5 An effective precipitation of 1mm is realized using differently colored plastic balls. A short piece of magnetic  
6 stripe was glued to each ball so that they could be tacked to most of the lecture theatre and seminar room  
7 white/black boards. Figure 2 illustrates a prepared ball and its use at the white/black board.

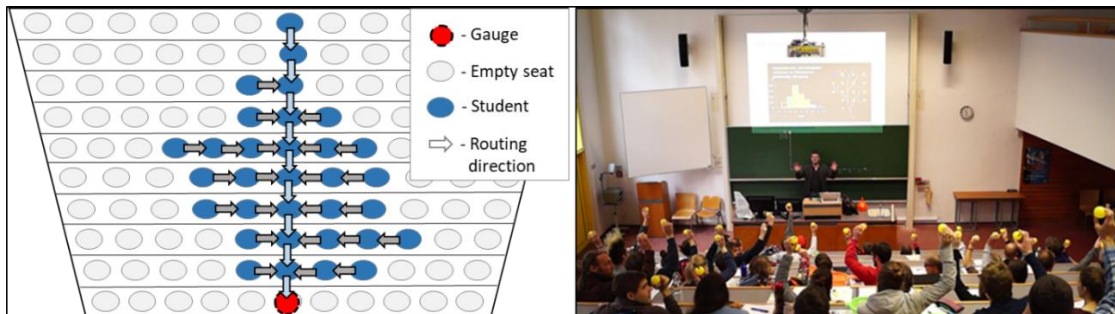
8 Step 2 (defining the catchment):

9 The “size” and the “form” of the catchment is defined by the positioning/seating of the individual students in the  
10 lecture theatre. By considering a different number of students within the experiment and changing the seating  
11 positions, the catchment size and form can be varied in order to examine and illustrate the effect of variations in  
12 both on the resulting hydrograph. Figure 3 shows the lecture theatre setup – each student holding a plastic ball  
13 representing 1mm of effective precipitation received by the catchment.

14 One person - approximately in the center of the first row or a teaching assistant (see Fig.3, left, gauge) - is defined  
15 as the catchment outlet, thus receives all the effective precipitation (balls) that have been routed “down” the  
16 catchment (lecture theatre).



17  
18 Figure 2: A piece of magnetic stripe glued to a plastic ball (left) that represents a unit amount (1mm) of effective rainfall, as  
19 well as generated specific runoff. It can be tacked to most of the classroom white/black boards (middle). The sieve is used to  
20 “collect” generated runoff during the routing procedure (right).



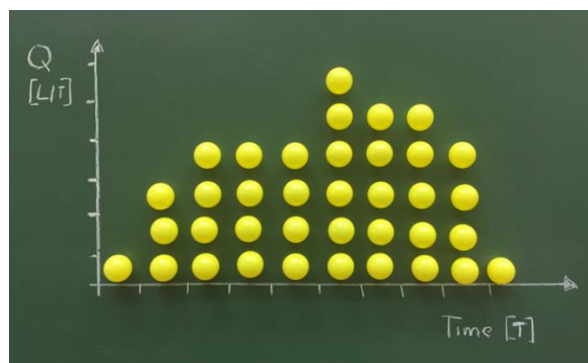
21  
22 Figure 3: Catchment setup by 32 students in the original lecture theatre (right). The catchment/each student received an effective  
23 precipitation of 1mm represented by a plastic ball. A sketch of student seating, flow routing and the gauge (left). Blue flow  
24 arrows indicate the main stream channel within the catchment.

1 Step 3 (routing rules):

2 The routing of the effective precipitation within the catchment has to follow some rules in order to generate runoff.  
3 To mimic some real catchment behavior, each student has to transport her/his 1mm of effective precipitation water  
4 package towards the outlet. A simple routing scheme that is easy to explain and to execute is sketched in Fig. 3  
5 (left) and proceeds as follows for each time step: i) vertical transport (blue arrows) of water packages of one  
6 position only along the main stream channel starting in the first row; the water packages in the gauge position are  
7 the measured runoff information for this time step; ii) horizontal transport (grey arrows) of one position towards  
8 the main stream channel in each row starting with the inside positions. The sampling of the water packages (balls)  
9 along the “main stream channel” (middle column of the lecture theatre) is carried out with plastic sieves as the  
10 number of balls might exceed the number of balls that can be handled by hand (see Fig. 2, right).

11 Step 4 (hydrograph representation):

12 With each time step, a different number of water packages (balls) are collected/sampled at the catchment gauge.  
13 They represent the catchment hydrograph resulting from 1mm of effective precipitation homogeneously spread  
14 over the entire catchment. Given the routing scheme that has been chosen for this particular experiment, the  
15 hydrograph in Fig. 4 represents the corresponding UH of our lecture theatre catchment. The units of the y-axis are  
16 in length per time and dependent on the units chosen for the time steps and the effective precipitation (here 1 mm  
17 per ball).



18

19 Figure 4: A lecture theatre catchment unit hydrograph resulting from 1mm effective precipitation (represented by a yellow  
20 plastic ball) homogeneously spread over the catchment.

### 21 3 Results and Discussion

22 The UH illustrated in Fig. 4 is the result of a specific lecture theatre setting including the form of the catchment  
23 defined by the positioning of the students in the lecture theatre (see Fig. 3) and the rules formulated to route the  
24 effective precipitation through the catchment. The basic experimental setup as described in section 2 can be  
25 extended in various ways to illustrate i) the basic principles behind the unit hydrograph, and ii) a number of  
26 additional factors that have a significant control on the generated hydrograph, such as the catchment terrain, the  
27 surface conditions and dominant runoff components, for example surface runoff or subsurface stormflow.

### 3.1 Illustration of unit hydrographs principles

The three principle underlying the UH are stationarity, linearity and superposition of runoff generated by different events (Fig. 1). The principle of stationarity is easily explained by simple repeating the experiment and thereby receiving the same hydrograph as result. Such a repetition might be not very exciting for the students, but experience shows us that a thought experiment is sufficient to foster understanding. The principle of linearity could be illustrated and discussed in two ways. First, given the initial experiment where each plastic ball represented a unit amount of effective precipitation (e.g. 1mm), this is simply redefined to twice the amount (e.g. 2mm) which exactly doubles the runoff generated in the hydrograph. Secondly, the experiment may be repeated with twice the number of balls per student, again resulting in a doubling of the runoff in the generated hydrograph. The second variant will obviously provide a more vivid illustration of the principle of linearity; however, it will require more resources in terms of number of balls and more experimental execution time, which might be more limiting given any lecture setting. The principle of superposition can be illustrated by considering two independent precipitation events, each producing e.g. 1mm of effective precipitation. The different precipitation events can be labeled by using differently colored balls (e.g. yellow and blue). While the first precipitation event will take place at the first time step, the second event will have a time delay of some steps. Figure 5 (right) shows the superposition of the two events (each 1mm of effective rainfall) and the resulting hydrograph.

As mentioned earlier, the unit hydrograph does not explicitly link any runoff generation to a specific location in the catchment. We here used an explicit setting to illustrate the principles of the UH – each explicit setting or form of a catchment and routing scheme will produce a specific hydrograph. The experiment in Fig. 5 provides a good opportunity to discuss this fact with the students. This point could be illustrated by designing a second but different experimental setting (form and/or routing scheme) that leads to the same hydrograph.

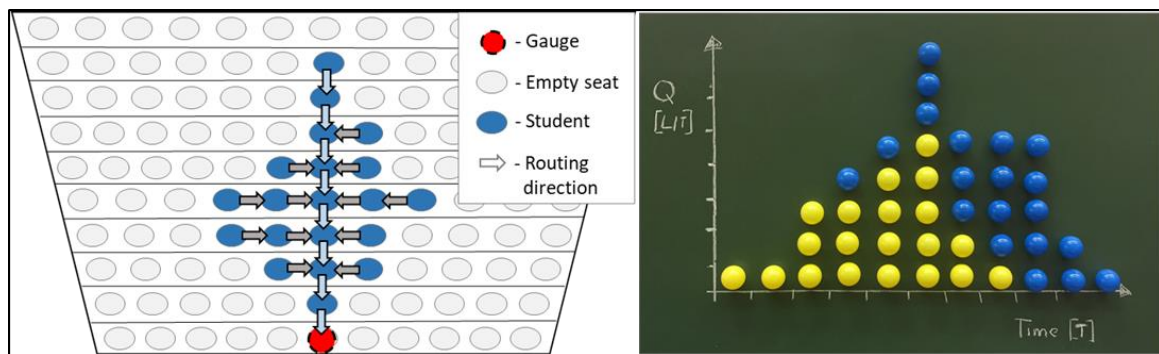


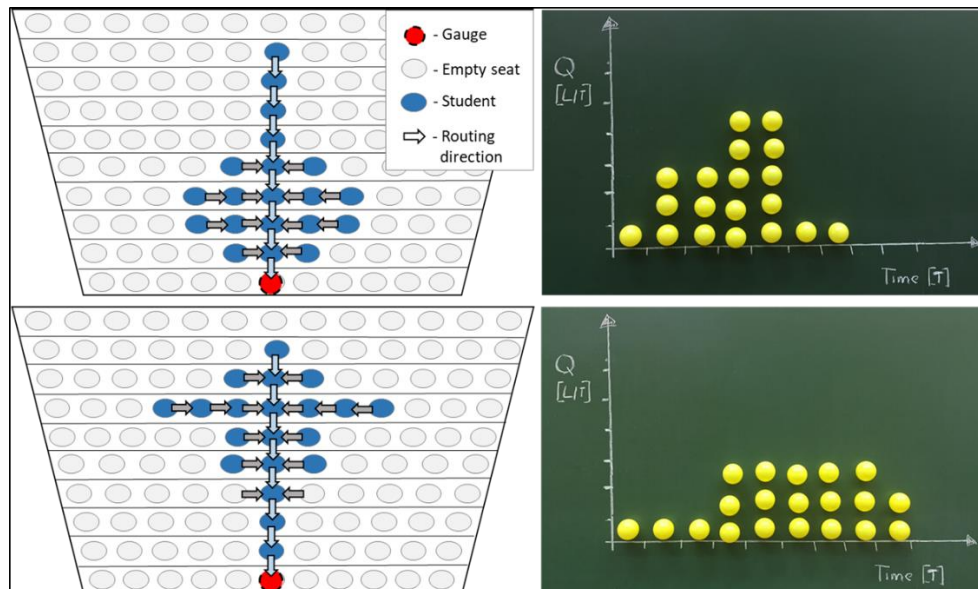
Figure 5: The superposition of two rainfall events, both producing an effective precipitation of 1mm. The first event is represented by yellow balls, the second event which is delayed by 4 time steps is represented by blue balls. The resulting hydrograph is shown on the right. The left figure illustrates the student seating and routing scheme applied in this experiment.

### 3.2 Analysis of factors controlling the unit hydrograph

One obvious factor to analyze is the form of the catchment and its impact on the dynamics of the hydrograph. While such an analysis is presented in recent hydrology textbooks (e.g. Baumgartner et al., 1996, p.525f; Ward and Trimble, 2003, p.126f), this effect is simply assessed by changing the seating positions of the students in the

1 lecture theatre and repeating the experiment. Figure 6 illustrates the representation of different catchment forms  
 2 by student seating in the lecturer theatre (top panel) and the resulting hydrographs (lower panel).

3 While the concept of the UH per-se does not require any linkage between the time delay of any runoff generated  
 4 in the catchment to a particular location (see section 1), the lecture theatre experiment is well suited to additionally  
 5 discuss any of the catchment properties that are related to the different forms of runoff processes in the catchment.  
 6 In the standard routing scheme as introduced in section 2, no effect of differences in topography is considered in  
 7 the routing scheme. In order to investigate the effects of topography, different areas with steep elevation gradients  
 8 (e.g. steep areas close to the catchment boundaries, where the transport of water is enhanced) are demonstrated by  
 9 allowing balls to “jump” over 2 positions, thus representing the increase in flow velocities due to increased  
 10 potential gradients. Differences in the soil texture and resulting infiltration capacities might be represented  
 11 similarly, indicating areas with more frequent surface runoff generation and quick flow conditions versus areas  
 12 with more dominant subsurface or interflow flow regimes that deliver water more slowly towards the catchment  
 13 outlet. In this way, different routing rules can be linked to different catchment properties and runoff/flow regimes,  
 14 which can be used for critical discussion of the assumptions and underlying principles of the UH, as well as the  
 15 possible effects in case those principles and assumptions are violated.



16  
 17 Figure 6: The effect of different catchment forms (same area/number of students, left) on the resulting hydrographs (right).

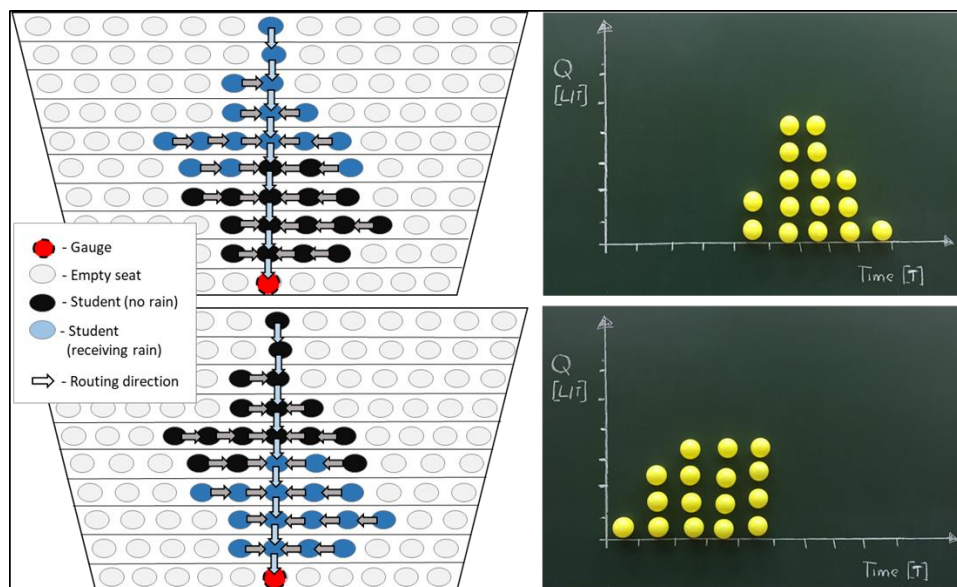
18 While beyond the underlying assumptions of the UH (spatially uniform effective precipitation), this lecture theatre  
 19 experiment is well suited to additionally demonstrate the effect of spatially non-uniform precipitation and runoff  
 20 generation processes, by limiting the distribution of plastic balls to only a subset of the students. Figure 7 illustrates  
 21 the resulting hydrographs when only the lower half (left) or the upper half (right) of the catchment receive a unit  
 22 amount of effective precipitation. The differences in the timing of the resulting hydrographs are clearly visible.



## 1 4 Experience and Outlook

2 Our personal experience with the unit-hydrograph and runoff routing lecture theatre experiment, as well as the  
3 direct student feedback and the student evaluation was extremely positive and revealed the following key aspects:

- 4 (1) The experiment was well suited to illustrate the concepts of the unit-hydrograph. The underlying  
5 principles such as stationarity, linearity and superposition, as well as the effect of different catchment  
6 form, catchment elevation, runoff mechanism and non-uniform precipitation events on the catchment  
7 hydrograph could be well demonstrated.
- 8 (2) The active participation of every student in the experiment allowed an intensive “active experience” of  
9 the UH and routing mechanism by being “one part of the catchment” and being “involved” in the transport  
10 of the water towards the catchment outlet. Student’s active participation in the experiment also highly  
11 supported an intensive discussion on the theoretical background of the unit-hydrograph. Assumptions  
12 such as the linearity of routing procedure could be questioned and discussed with regard to real catchment  
13 behaviour including the impact of antecedent soil moisture conditions on the runoff generation, and the  
14 non-linearity of flow depth – flow velocity relationship.
- 15 (3) For the basic experiment, a time slot of approximately 20-25min and the help of two student assistants  
16 was needed in the lecture. However, in case there is more time available the experiment and discussions  
17 can be easily extended to 90min and more by including some of the additional variants as illustrated in  
18 section 3.2. Given a 3-hour framework for our particular lecture, the experiment was a very welcome  
19 “active break” from the standard slide based lecturing. It refocussed students’ minds and attention.
- 20 (4) Actively participating in deriving the catchment rainfall-runoff relationship was fun, not only for the  
21 students, but also for everybody involved in the course teaching.
- 22



24 Figure 7: The effect of different spatial distributions of effective precipitation within the catchment on the resulting hydrograph.  
25 The student seating, the spatial distribution of effective precipitation and the routing scheme are indicated on the left, the  
26 hydrograph are illustrated on the right.

1 To summarize our experience with the integration of such a lecture theatre experiment into the course: We believe  
2 that overall a much deeper learning experience for the students could be achieved due to the visualization and  
3 active participation of the students. In order to quantify the impact of such experiments on the students learning  
4 process, a longer-term educational study using some kind of split group approach would be needed. While the  
5 education department of our university has expressed their interest in conducting such a study, we are currently  
6 not able to make a quantitative statement in that direction. Nevertheless, our subjective experience is well  
7 expressed by a statement of Confucius (551-479 BC) saying that “I hear and I forget, I see and I remember, I do  
8 and I understand” and will encourage us to further extend the integration of experiments into lecture-based  
9 teaching.

10  
11 *Data Availability.* All required information and appropriate references are given in the text.

12 *Author contributions.* KS (PI) designed the experiment and run the lecture. RB (M.Sc. student) developed the  
13 teaching material. JW (PhD student), DK (PhD student), MH (Co-PI) as well as RB contributed to the development  
14 of the teaching experiment. All authors were involved in the manuscript and figure preparation.

15 *Competing interest.* The authors declare that they have no conflict of interest.

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