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Interactive comment on "Technical note: Transit time distributions are not L-shaped" *by* Earl Bardsley

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This technical note proposes, not just that L-shaped transit time distributions cannot exist, because "the transit time of a particle must always be with reference to a store, the transit time being some finite duration of time between particle entry and exit." Elsewhere essentially the same argument is repeated in different forms, including "... the M tracer particles placed onto the recorder at t=0 never transited through any part of the catchment system and therefore have no connection to catchment transit times," and "... it is not possible to have transit times of exactly zero because any tracer particles initially present on the recorder have never entered the store concerned. That is, they did not transit to the recorder."

This is characterized as "a purely conceptual argument." A better characterization would be that it is a purely definitional argument. What has happened is that the author has chosen to DEFINE transit times as necessarily being non-zero, and thus transit times of exactly zero have been excluded purely by definition.

The author's responses to several previous comments, in which he says that transit times of zero are impossible because they imply infinite velocity, repeat the same definitional argument dressed up in different clothes. The author is simply asserting that by definition transit distances cannot be zero (otherwise no "transit" has occurred), and therefore this must require some finite time.

One can easily see that this is a question of definitions rather than physical reality. For example, rain falls everywhere in a catchment, including into the stream. Assuming the detector is located in the stream, then rain can fall directly on the detector and its transit time will be zero. There is no logical or physical reason why a tracer cannot enter at the same location as the detector, and thus have a travel time of zero. Why should we assume that rain can fall everywhere in the catchment, EXCEPT at the observation point? The author's argument is simply that we should not count this as part of the transit time distribution because no "transit" has occurred.

It is important to recognize that because this is a purely definitional matter, it has exactly zero implications for the physics of water movement in the environment. In the example outlined above, for example, whether we choose to include the particles with zero transit time as part of the transit-time distribution, or not, will have exactly zero implications for how each raindrop travels to the detector or the time it takes to do so.

The physical irrelevance of the definitional inclusion or exclusion of t=0 transit times is mirrored also in its mathematical irrelevance. Whether p(t) is greater than zero for t in the range (0...x] or in the range [0...x] – that is, whether the lower bound at zero is open or closed – makes precisely zero difference to any calculations performed over any continuous interval of time, for the reason that the total probability (not probability

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density, but probability) associated with the value of t=0 (or any other exact real-number value) is exactly zero. Therefore, even as a matter of rigorous mathematics, the definition that is adopted in this manuscript (that transit times can be any positive real number but not exactly zero) has exactly zero consequences.

The only way that this manuscript's argument can be consequential is if one excludes not only values of exactly t=0, but also values in some meaningfully large interval above zero. But the fundamental problem is that even if the author's argument (not really an argument, but just an arbitrary definition) were accepted, it does not establish any logical reason to reject any transit time greater than zero, no matter how small. Unless one can also rule out some finite range of non-zero transit times, the argument has no physical (or even mathematical) effect.

However, ruling out a finite range of non-zero transit times would require the manuscript to abandon its purely definitional approach to the problem, and to state that not only are transit times of zero impossible, but transit times of up to 1 minute (or 1 second, or 1 hour, or 1 something) are impossible. Doing that, however, entails a burden of proof that apparently cannot be met (because if lags of, say, 1 hour are possible, why not lags of one half hour, or one minute, or one second, or one nanosecond, or any other span of time)?

Much of the apparent force of the manuscript's definitional argument comes from a confusion between probabilities and probability densities. For example, the manuscript proposes a thought experiment where we distribute N particles over the catchment and place M particles exactly on the detector. The problem is that this creates a discontinuous and grossly nonphysical probability distribution, in which the density of particles at t=0 is infinitely higher than the density everywhere else, even infinitesimally close to t=0. As far as I know, nobody has proposed that transit time distributions might include Dirac delta functions at t=0. Thus, the context of the transit time literature, this is nothing more than a straw man argument.

In its treatment of my own work, the manuscript makes an analogous error: "An equivalent argument can be made by noting the two-parameter inverse Gaussian form of Eq. (8) of Kirchner at al. (2001). Therefore h(t) corresponds to an infinite mixture of inverse Gaussian transit time distributions. This infinite mixture distribution can be represented to any degree of accuracy as a finite mixture distribution – in this case a finite mixture of a sequence of inverse Gaussian distributions with progressively decreasing mean and variance as the tracer input point x* decreases by increments toward the observation point at x = 0."

But this is of course the central problem: saying that an infinite mixture of inverse Gaussians can be represented "to any degree of accuracy as a finite mixture distribution" is false for the argument that the author wants to make, because that argument concerns an interval that is not finite (but instead, the infinitesimal range around the exact value of zero). For this specific problem, any finite mixture distribution is an (infinitely) horrible approximation to the true distribution in the infinitesimal range around t=0. The rest of the manuscript's argument concerning my work also fails for analogous reasons.

One needs to be very careful in jumping between continuous and discrete distributions, or continuous and discrete mathematics more generally. The required degree of care has not been taken here.

The manuscript's abhorrence of transit times of zero leads to the very strange result that, for example, a gamma distribution with a shape factor of 1.00000000000001 would be declared to be realistic (because p=0 at t=0), while a gamma distribution with a shape factor of 1.000000000000000 (an exponential distribution) would be considered to be problematic because it has a finite probability density at t=0, and a gamma distribution with a shape factor of 0.999999999999999 would be regarded as anathema because its probability density climbs toward infinity as t approaches zero. This makes no sense, for two reasons. First, gamma distributions with shape factors of 1, 1+epsilon, and 1-epsilon are indistinguishable "to any degree of accuracy" (to borrow the manuscript's phrase). And second, in all three cases (and for all other gamma distributions), the probability of t=zero is, in each case, exactly zero (as it is with any continuous probability distribution).

That last statement re-casts the point made above, namely that the definitional distinction advanced in this manuscript has no physical or mathematical consequences. If the author wants to claim that the probability of t=0 must be zero, that's fine, since the probability of ANY exact value is ALWAYS zero for ANY continuous distribution over real numbers. But then the author's argument has no physical or mathematical consequences (and any claim of one distribution being more "realistic" than another is logically empty). For the author's argument to have any consequences, he would need to exclude some finite interval of time (corresponding to some finite probability), but then he would need to justify why a non-zero transit time is impossible. In the world of continuous space and time, that would be a difficult case to make.

The foregoing arguments are completely separate from the EMPIRICAL question of whether real-world transit time distributions have peaks at lag times greater than zero. This is certainly possible (and indeed in Kirchner et al. 2001, I described cases that would generate this result). But whether this is actually true in the real world is an empirical question, to be answered, within the limits of our ability, with data.

Several research groups (including mine) are doing the hard work of making tracer measurements over very small sampling intervals in real-world catchments, and they may (or may not) find that there is a measurable non-monotonic behavior in the transit-time distribution for very short lags. But unless and until they do, it would seem wise to refrain from claiming, on the basis of arbitrary definitional premises, and in contradiction to a large body of empirical evidence, that L-shaped distributions are fundamentally impossible.

Kirchner, J.W., X.H. Feng, and C. Neal, Catchment-scale advection and dispersion as a mechanism for fractal scaling in stream tracer concentrations, Journal of Hydrology, 254, 81-100, 2001.

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