

My thanks to James Kirchner (JK) for his comments – responses follow below.

By way of initial comment, we can consider the JK statement:

There is no logical or physical reason why a tracer cannot enter at the same location as the detector, and thus have a travel time of zero.

It is clear, therefore that JK has no issues concerning a transit time probability density function $f(t)$ to be such that $f(0) > 0$, thereby allowing the possibility of L-shaped distributions. Presumably this view is also held by some number of his associates well.

It is interesting to contrast this with the comments of Referee # 1 who, though feeling that the $f(0) > 0$ issue is of no great importance in a practical sense, makes the point that:

...we are all well aware that the property at $t=0$ is incorrect...

As it happens, the “all” in this case is obviously too encompassing because it evidently does not include the JK group. This is not to imply that the nature of $f(0)$ is a topic of current debate, but there is at least a need for some degree of comment to clarify matters.

Shifting now to the hydrological argument, which might be termed the raindrop-on-recorder situation, summarised by JK as:

Assuming the detector is located in the stream, then rain can fall directly on the detector and its transit time will be zero.

There is no question that the rain can fall directly onto the recorder. Indeed, a conceptual raindrop could already be present on the recorder even before $t=0$. However, all those tracer particles (one or more) present at the recorder at $t=0$ cannot be part of a transit time distribution with zero transit times because there is no transit involved. Assume for now that JK is in fact correct here. Transit time distributions would then be comprised of the transit times of two different populations of tracer particles: (i) particles which have transited, and (ii) particles which have not transited. The transit time distribution would then consist of (i) a probability density function which does not integrate to 1, and (ii) a fixed probability (not probability density) at $t=0$. With respect to such a distributional monstrosity, I can think of no better description than that given by JK:

The problem is that this creates a discontinuous and grossly nonphysical probability distribution, in which the density of particles at $t=0$ is infinitely higher than the density everywhere else, even infinitesimally close to $t=0$.

I should add, incidentally, that the thought experiment at the start of the submitted paper was purely in anticipation (correctly, as it turns out) of the raindrop-on-recorder argument being raised to make a case that L-shaped transit time distributions are conceptually possible. With that particular argument hopefully now disposed of, it would not be reproduced in any final paper.

At this point we can presumably agree that the discussion centres on definitions and probability density functions, and not on probabilities. In regard to definitions, I seem to stand accused by JK of inserting some sort of personal definition to exclude zero transit times, and then using that definition for my own ends to consequentially define the impossibility of L-shaped transit time distributions.

To demonstrate my lack of personal input here, we need to first recall just what is being transiting “to” in transit time distributions. As far as I am aware, hydrological transit time distributions always have transits terminating as a death process. For example, a tracer particle on reaching a recorder does not pop back out of the recorder, perhaps to be recorded a second time. That is, hydrological transits terminate when an absorbing barrier is reached.

Various forms of transits to absorbing barriers are much-researched in stochastic analysis, as HESS readers will be aware. However, for the purposes of the present discussion it is sufficient to refer briefly just to the discrete random walk between a reflecting and an absorbing barrier (Weesakul, 1961), as mentioned in the submitted paper.

Specifically, given an absorbing barrier at $x = 0$ and a reflecting barrier at $x = b$, a particle starts its transit at some point u , where $0 < u \leq b$. Once motion is initiated, the particle moves about by discrete random motion until the transit is terminated when reaching the absorbing barrier for the first time.

The important factor here is the definition of the range of starting points for a transit. Was Weesakul making an arbitrary decision for his own ends when he defined $0 < u \leq b$ and as opposed to $0 \leq u \leq b$? Of course not. Our mathematical colleagues are meticulous in their definitions and the specification of $0 < u \leq b$ arises from the basic logic that transits to an absorbing barrier cannot be initiated from within the absorbing barrier. This property is independent of any supposed “definition” of mine and will of course still remain regardless of whether the submitted paper is accepted or not.

It follows that because $u > 0$ when $t = 0$, transit time distributions to absorbing barriers must have $f(0) = 0$ because there are no zero-time transits. And it follows in turn that derivations in hydrology which supposedly obtain L-shaped transit time distributions must contain an error at some point in their argument, which will have influence on the form of the derived distributions near $t = 0$. This is not a mathematical error in algebra, but an error in the specification of the problem to be solved.

There is nothing to stop u from being located arbitrarily close to 0 but the requirement of $u > 0$ always remains. This relates as well to the inverse Gaussian mixture discussion for the L-shaped transit time distribution of Kirschner et al (2001). Regardless of whether we have a finite or infinite mixture of inverse Gaussian distributions, each inverse Gaussian distribution has the property of $f(0) = 0$. The summation of even an infinite number of zeroes at $t = 0$ still yields a mixture transit time probability density of zero at $t = 0$.

The conclusion (and title) of the paper remains exactly unchanged – transit time distributions are not L shaped. And they are not L-shaped because they are fundamentally impossible. As responsible editors of an international journal, the HESS editors will never consider publishing a paper whose very title they see to be nothing more than an unproven assertion. The publication decision is therefore of interest.

I would like to note that the paper was not written for the purposes of being critical of any previous publication. The paper stands on the basis of its own argument. Some critical reference to Kirchner et al (2001) was simply unavoidable because that paper is arguably the best-known published hydrological work deriving an L-shaped distribution. If no reference to it had been made then the referees would most probably have advocated rejection, pointing out that Kirchner et al (2001) had demonstrated via analytical argument that L-shaped transit time distributions can exist. If the HESS publication process advances then my personal preference would be for having a shorter paper avoiding listing specific criticisms of other works and focus on the positive aspects – while fully appreciating that JK sees zero of the latter.

Turning to one further specific point, with respect to the comment:

.. the definitional distinction advanced in this manuscript has no physical or mathematical consequences

I can only fully agree – at least to the extent that the paper makes no pretence to provide new insights into the physics of moving water in the environment and certainly does not offer anything of mathematical consequence. However, what it does demonstrate (as is well-known according to Referee #1) is that L-shaped transit time distributions cannot exist as mathematical entities, so L-shaped parametric transit time distributions would seem inappropriate for application to transit time data.

The remaining JK comments relate to aspects of observational limitations. These were discussed as part of the response to Referee #4 and are not repeated here.

It is fully appreciated that this response will be totally unsatisfying to JK but I would like to offer my thanks for his contribution in any case. The submitted manuscript has now (in effect) had six referee comments so there can certainly be no issue over it not having had wide-ranging review.