Reply to the comments of Referee #2

(Note that the page number and line number mentioned in the following responses are referred to those in the revised manuscript.)

- The paper provides an analytical solution for transient groundwater flow in an L-shaped aquifer, with strong connection to a stream. The so called analytical solution is not completely analytical, as numerical tools as the Stehfest algorithm are included to obtain the final result. When the results are compared with a MODFLOW solution, in fact two quite different numerical approaches are compared. Both of these approaches have their limitations and deliver approximate solutions only. The possible size of the errors is difficult to discuss and is not addressed in the manuscript.

Response:

1. Thanks for reviewer's reminder. The steady state solution derived in this study is analytical and the transient solution is semi-analytical because it needs a numerical tool to obtain the time-domain result. To avoid confusion, we therefore use the word "semi-analytical" in lieu of "transient" before the time-domain solution in the revised manuscript.
2. Figure A (Figure 3 in the revised manuscript) depicts the hydraulic head contours in L-shaped aquifer simulated by the present solution and MODFLOW. As shown in this figure, the head distribution simulated by the present solution agrees with that by MODFLOW except in the region near the no-flow boundary FG, which has the largest relative deviation 2.1% between these two models. Furthermore, field observations are available from Kihm et al. (2007) to compare the simulation results from the present solution and MODFLOW. Figure B (Figure 6 in the revised manuscript) plots the temporal hydraulic head distribution obtained from the present solution, MODFLOW, and FEM from Kihm et al. (2007) at piezometers O_1, O_2 and O_3, together with the field observations at these piezometers. Compare to the field data, the largest deviation is 0.03m and 0.08m for both MODFLOW and present solution at O_3 and O_2, respectively, and 0.04m for MODFLOW and 0.07m for present solution at O_1. The discussion of the comparison is addressed in the revised manuscript in lines 26-29, page 10 as "The hydraulic head distribution predicted by the present solution of Eqs. (26) and (27) and represented by the dotted line is shown in Figure 3. The figure indicates that the head distribution simulated by the present solution agrees with that by MODFLOW except in the region near the no-flow boundary FG, which has the largest relative

deviation 2.1% between these two models. The comparison of the head distributions predicted by the present solution and MODFLOW ensures that the simplification of aquifer layers in the present model is appropriate and gives a fairly good predicted results.” and lines 6-11, page 12 as “Compared with the field observation, the differences of predicted hydraulic head among FEM, present solution and MODFLOW are all less than 0.08m at these three piezometers during 0.1 to 10 day. In addition, the largest relative differences between measured heads and predicted heads by the present solution at O_1 to O_3 are respectively 1.64%, 1.74% and 0.62%, indicating that the present solution gives good predictions in the early pumping period. Moreover, the effects of unsaturated flow and land deformation on the groundwater flow in Yongpoong aquifer are small and may be negligible.”

- Usually analytical solutions are utilized for benchmarking numerical codes, because they are a more accurate representation of the exact solution. Obviously this property is not expected by the authors, when they present their approach. In contrary they use a numerical solution for benchmarking their method, not taking into account that the numerical solution is definitely only an approximation.

Response:

We agree that analytical solutions are the primary means for benchmarking numerical codes. Here we would like to mention that the use of MODFLOW is to examine the suitability of simplification made in our analytical solution using the approach of equivalent hydraulic conductivity. To avoid confusion, the title of section 3 “Solution validation and application” and subsection 3.1 “Solution validation by MODFLOW-2005” in page 9 in original manuscript is respectively replaced by the “Comparisons of present solution, numerical solutions and field observed data” and “Comparisons of present solution with MODFLOW solution”. The purpose of the MODFLOW simulation is further discussed in lines 3-31, page 10 in the revised manuscript with the following text: “The software MODFLOW is used to simulate the groundwater flow due to pumping in the L-shaped aquifer in Yongpoong 2 Agriculture District with different hydraulic conductivities for the two layers. The MODFLOW is a widely used finite-difference model developed by U.S. Geological Survey for the simulation of 3D groundwater flow problems under various hydrogeological conditions (USGS, 2005). As shown in Figure 1, region 1 has an area of $852m \times 222m$ (i.e., $l_1 \times d_1$) while the area of region 2 is $297m \times 183m$ (i.e., $(l_1 - l_2) \times (d_2 - d_1)$). Thus, the total area of these two regions is $243495\ m^2$ which is close to the area of the fluvial aquifer ($246500m^2$) reported in Kihm et al. (2007). In the simulation of MODFLOW, the plane of the L-shaped aquifer is discretized with a uniform cell size of $3m \times 3m$. The aquifer thickness is $6m$ and divided into two layers. The upper loam layer is $2.5m$ and lower sand layer $3.5m$. Within the aquifer domain, there is totally 54110 cells while the numbers of cell are 42032 and 12078 respectively for region 1 and region 2. The types of outer boundary specified for the L-shaped aquifer are the same as those defined in the mathematical model. The hydraulic heads along AG and DE are respectively $h_1 = 5.18m$ and $h_2 = 5.29m$ and the head at point B is $h_3 = 4.06m$. The fluvial aquifer reported in Kihm et al. (2007) is isotropic and homogeneous in horizontal direction. In other words, the hydraulic conductivities in *x* and *y* directions are identical in both regions 1 and 2 (i.e., $K_{x1} = K_{y1} = K_{x2} = K_{y2} = K$). However, the aquifer is heterogeneous in the vertical direction. It has two layers with hydraulic conductivity $K_1 = 3 \times 10^{-6} m/s$ for the upper layer and $K_2 = 2 \times 10^{-4} m/s$ for the lower layer. The specific storage of the aquifer in both regions 1 and 2 is $10^{-4} m^{-1}$. Consider that the pumping well P_w is located at $(609m, 9m)$ in region 1 shown in Figure 2 with a rate of $120m^3/day$ for one year pumping. The hydraulic head distribution predicted from the MODFLOW simulations is denoted as the dotted line shown in Figure 3. A multi-layered aquifer with heterogeneous hydraulic conductivity may be approximated as an equivalent homogeneous medium. The equivalent hydraulic conductivity K_h may be evaluated as (Schwartz and Zhang, 2003):

$$K_h = \sum_i^m b_i K_i / \sum_i^m b_i \tag{50}$$

where K_i is the hydraulic conductivity in the horizontal direction for layer i, b_i is the thickness of layer i, and m is the number of the layers. Accordingly, the equivalent horizontal hydraulic conductivity K_h for the two layered L-shaped aquifer is estimated as $1.2 \times 10^{-4} m/s$. The hydraulic head distribution predicted by the present solution of Eqs. (26) and (27) and represented by the dotted line is shown in Figure 3. The figure indicates that the head distribution simulated by the present solution agrees with that by MODFLOW except in the region near the no-flow boundary FG which has the largest relative deviation 2.1% between these two models. The comparison of the head distributions predicted by the present solution and MODFLOW ensures that the simplification of aquifer layers in the present model is suitable and gives a fairly good predicted results."

- Concerning the model region, the L-shaped domain is surely a big deviation from the real aquifer geometry, especially along boundary AG, but even more along boundaries FE and ED. Thus deviances, as shown in Fig. 3 could be expected. The problem with the manuscript is that it cannot trace back the differences to its causes: it could be the different numerical approach (MODFLOW, FEM, 'analytical') or the different model region. Were the results of the numerical models obtained with sufficient mesh refinement?

Response:

1. The aquifer geometry in real-world situation could be very complicated. In order to investigate the groundwater flow system in the real-world aquifer, the problem domain is simplified so that the analytical model or numerical model is easy to apply. This study conceptualizes an irregular aquifer in Kihm et al. (2007) as an L-shaped aquifer to simulate the flow due to groundwater pumping by MODFLOW and the present solution. The differences between the finite element solution presented by Kihm et al. (2007) and present solution (or MODFLOW) shown in Figure 3 are significant near the boundaries AG, FE and ED. However, their effects on groundwater head distribution and stream depletion rate near the pumping well are very small because those boundaries are far from the area near the pumping well that we focus on. The discussion on this issue is given in lines 25-36, page 11 as "The head distributions predicted by the FEM solution and present solution have obvious differences in the area far away from the pumping well. Those differences may be mainly caused by the difference in the physical domain considered in FEM solution and the simplified domain made in the present solution. In addition, the mathematical model in Kihm et al. (2007) considered the unsaturated flow and deformation of the unsaturated soil, which may also affect the head distribution after pumping. Notice that the pumping well is very close to the stream boundary AB, which is the main stream in that area and provides a large amount of filtration water to the well. Hence, it seems that the groundwater flows in the region 1 for $x \leq 300$m (near boundary AG) and in the region 2 for $y \geq 200$m (near boundaries FE and ED) are both far away from the well and almost not influenced by the pumping."
2. Figure C provides the spatial hydraulic head distributions with streamlines after one year pumping simulated by MODFLOW using two different cell sizes, 1m×1m (blue dashed line) and 3m×3m (pink solid line). The result shows no difference while using two different cell sizes, indicating that the cell size 3m×3m used in MODFLOW is good enough to predict the spatial head distribution.

- The production well is located quite near to the boundary AB. It can be expected that the strong head gradients that appear due to this constellation can only be reproduced numerically if strong mesh refinement is used in the direct vicinity of the well.

Response:

We agree that a finer mesh can give better results in the vicinity of the well. We think the mesh size ($3m \times 3$m) in MODFLOW simulation is relatively small compared to the length of boundary AB (852m) and may give fairly good results. The difference of hydraulic heads near the pumping well predicted by the MODFLOW using cell sizes ($3m \times 3$m) and ($1m \times 1$m) is negligibly small as mentioned in previous response. Accordingly, the use of $3m \times 3$m mesh in MODFLOW is capable of producing good prediction in head gradients in the area adjacent to the pumping well.

- Concerning the real world situation, it could be doubted that a numerical approach with a constant head boundary can address the physically relevant processes in that case. I would expect that strong or weak connection between aquifer and surface water body play a role in reality in addition.

Response:

We agree that the connection between aquifer and stream has an impact on the groundwater flow in the aquifer, but its impact in reality is strong only in the region near the stream. The Poonggye stream and its tributary are perennial stream and almost fully penetrate the fluvial aquifer system reported in Kihm et al. (2007). Unfortunately there is no information available regarding the streambed properties; thus, we consider that the stream has a prefect hydraulic connection with the aquifer. If the permeability of the streambed is significantly lower than that of the aquifer, then the Robin type condition should be employed as the stream boundary (see, e.g., Huang and Yeh, 2015, 2016). Such a treatment for the stream boundary however is beyond the scope of this study.

- If the paper could be re-written in a way to address the points made, I could deliver a more positive comment.

Response:

Thanks, we have largely revised the manuscript.

References

Huang, C. S., and Yeh, H. D.: Estimating stream filtration from a meandering stream under the Robin condition, Water Resources Research, 51, 4848-4857, doi:10.1002/2015WR016975, 2015.

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Kihm, J.-H., Kim, J.-M., Song, S.-H., and Lee, G.-S.: Three-dimensional numerical simulation of fully coupled groundwater flow and land deformation due to groundwater pumping in an unsaturated fluvial aquifer system, Journal of Hydrology, 335, 1-14, http://dx.doi.org/10.1016/j.jhydrol.2006.09.031, 2007.

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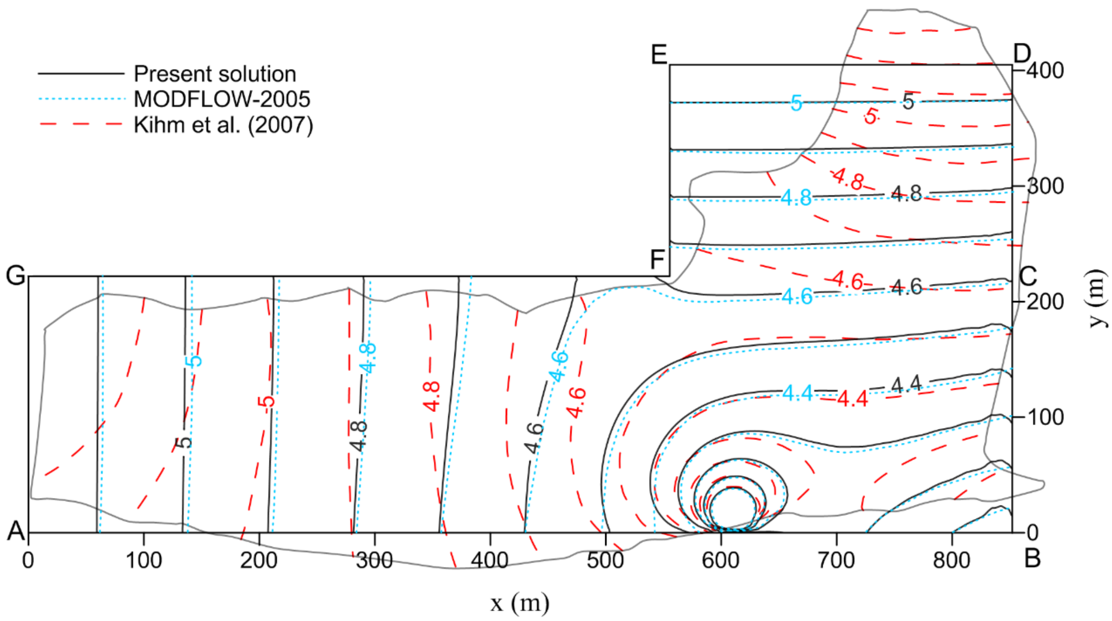

Figure A: Contours of hydraulic head in L-shaped aquifer predicted by the present solution, MODFLOW, and FEM simulations with irregular outer boundary reported in Kihm et al. (2007).

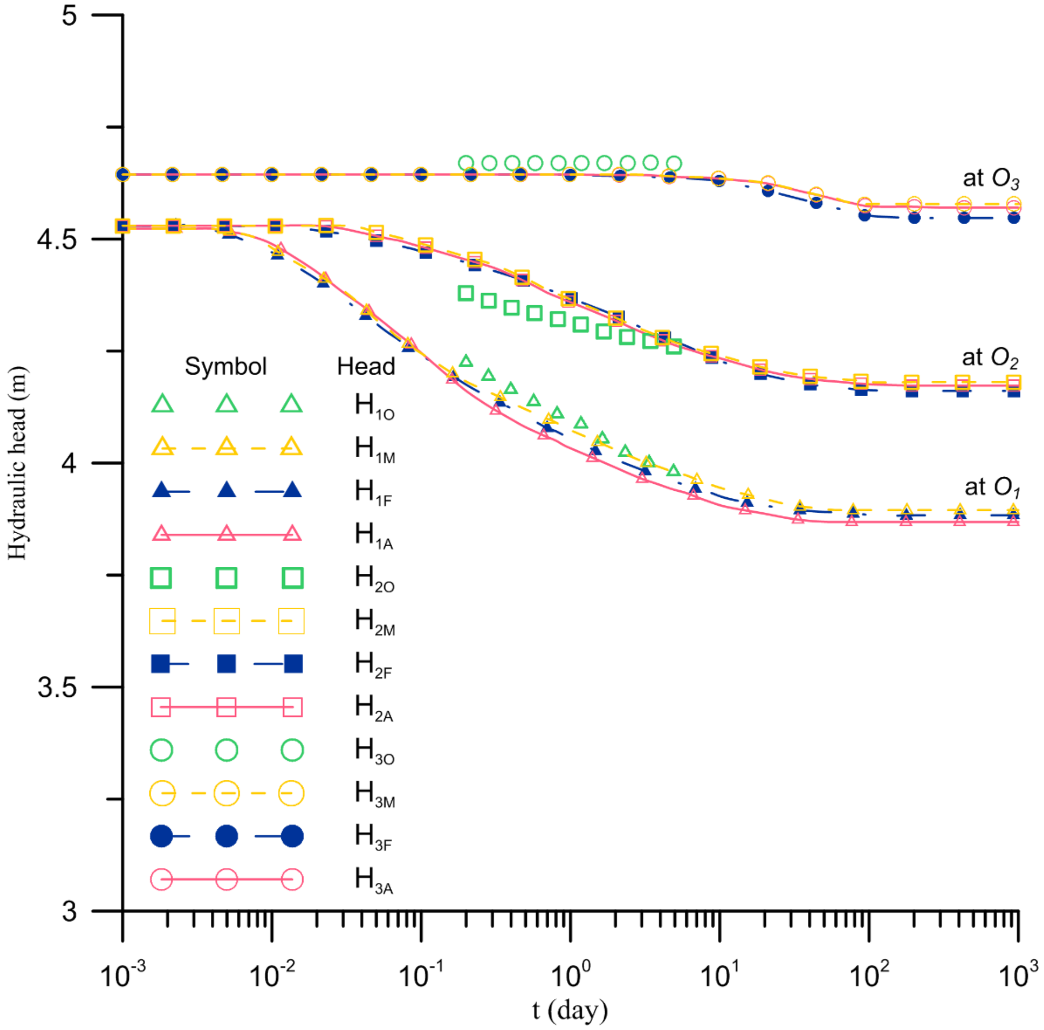

Figure B: **Temporal distributions of hydraulic head H_{io} observed at piezometer O_i and H_{iF} simulated by the FEM simulations both reported in Kihm et al. (2007) and H_{iA} and H_{iM} predicted by the present solution and MODFLOW, respectively, for i = 1 - 3.**

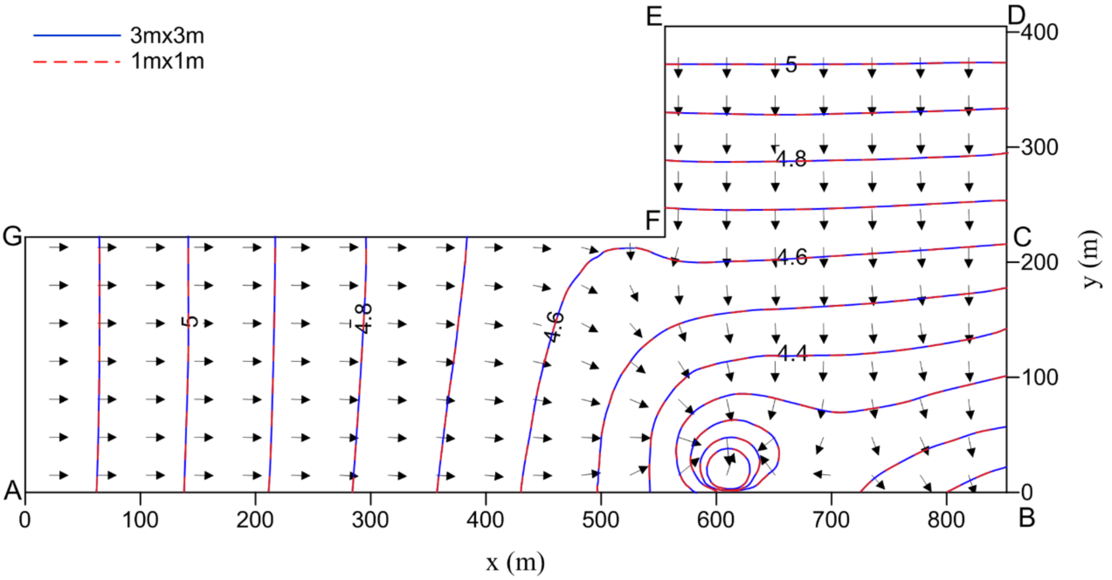

Figure C: Contours of hydraulic head with streamline in L-shaped aquifer simulated by MODFLOW with different cell size.