Reply to Anonymous Referee #1

General comments:

- There are few analytical studies addressing flow field in multiaquifer systems. This may be attributed to inadequacy of conventional solution techniques in dealing with such geometrically cornered entities. Aimed at reproducing a real-world scenario in a semi-analytical framework, the present manuscript also offers useful insights regarding the nature of multi-well hydraulics in L-shaped aquifers consisting of two anisotropic sub-regions with properly imposed interface conditions. Comparisons are also made with relevant numerical results and existing measurement data. The subject can further be clarified if the authors consider the comments listed below:

Response:

Thanks, we provide point-by-point response to each of your comment listed below. The page and line numbers given in our responses are referred to those in the revised manuscript.

Specific comments

- In addition to those reviewed in "Introduction", the following studies examine different ways of simplifying natural aquifer settings through non-rectangular domains: Variational method of Kantorovich for modeling rainfall induced mounds in trapezoidal-shaped aquifers (Mahdavi and Seyyedian, 2014); the method of Strack's discharge potential for groundwater hydraulics in coastal promontories (Kacimov et al., 2016); and more recently, holomorphic functions for flow fields defined in circular meniscus (Kacimov et al., 2017). Moreover, the case of L-shaped domains has been treated analytically in different fields of engineering such as torsion of elastic bars (Kantorovich and Krylov, 1958) as well as heat conduction in plates (Mackowski, 2011). It is suggested to include above-mentioned works in the literature review.

Response:

Thanks for the suggestion. These articles have been reviewed and listed in the revised manuscript for two parts. The first part is from lines 24-29, page 1 to lines 1-4, page 2 as: "Many studies have been devoted to the development of analytical models for describing flow in finite aquifers with a rectangular boundary …, a wedge-shaped boundary (Chan et al., 1978; Falade, 1982; Holzbecher, 2005; Yeh et al., 2008; Chen et al., 2009; Samani and Zarei-Doudeji, 2012; Samani and Sedghi, 2015; Kacimov et al. 2016), a triangle boundary (Asadi-Aghbolaghi et al., 2010) a trapezoidal-shaped boundary (Mahdavi and Seyyedian, 2014), or a meniscus-shaped domain (Kacimov et al. 2017). So far, the case of re-entrant angle (L-shaped) boundaries has been treated analytically in different fields such as torsion of elastic bars (Kantorovich and Krylov, 1958), head fluctuation problems for tidal aquifers (Sun, 1997; Li and Jiao, 2002), and heat conduction in plates (Mackowski, 2011). However, none of them are to deal with pumping or stream depletion problems."

Then, the second part of the new reviews is given after the sentence "Patel and Serrano (2011) solved nonlinear boundary value problems of multidimensional equations by Adomian's method of decomposition for groundwater flow in irregularly shaped aquifer domains." in lines 11-19, page 3 as "Mahdavi and Seyyedian (2014) developed a semi-analytical solution for hydraulic head distribution in trapezoidal-shaped aquifers in response to diffusive recharge of constant rate. The aquifer was surrounded by four fully penetrating and constant-head streams. Kacimov et al. (2016) used the Strack-Chernyshov model to investigate the unconfined groundwater flows in a wedge-shaped promontories with accretion along the water table and outflow from a groundwater mound into draining rays. Huang et al. (2016) presented 3D analytical solutions for hydraulic head distributions and *SDR*s induced by a radial collector well in a rectangular confined or unconfined aquifer bounded by two parallel streams and no-flow boundaries. Currently, the distribution of groundwater flow velocity in a circular meniscus aquifer was investigated analytically by theory of holomorphic functions and numerically by FEM (Kacimov et al., 2016)."

- Since (46) refers to water exchange along aquifer-stream interface AB (denoted by), it should take into account only contribution from hydraulic gradients in Region 1, i.e. the portion of aquifer which is directly in hydraulic connection with the stream. The second integral in this expression, which implies direct influence of Region 2 on SDR_A, thus seems irrelevant and should be removed. When evaluating SDR_B, the first and second integrals in (47) should be taken from 0 to b1 and from b1 to b2, respectively, for the same reasoning as described before.

Response:

Thanks for the comment. The stream depletion rates (in Laplace domain) from stream reaches AB and BD have been modified, respectively, as

$$\widetilde{SDR}_A = \frac{q_A}{Q} = -\frac{1}{Q}\int_0^{l_1} K_{y1} \left.\frac{\partial \tilde{\phi}_1(x,y,p)}{\partial y}\right|_{y=0} dx \qquad \text{(A1)}$$

and

$$\widetilde{SDR}_B = \frac{q_B}{Q} = \frac{1}{Q}\left(\int_0^{b_1} K_{x1} \left.\frac{\partial \tilde{\phi}_1(x,y,p)}{\partial x}\right|_{x=l_1} dy + \int_{b_1}^{b_2} K_{x2} \left.\frac{\partial \tilde{\phi}_2(x,y,p)}{\partial x}\right|_{x=l_1} dy\right) \qquad \text{(A2)}$$

in equations (46) and (47) in the revised manuscript.

- The extraction water comes from surrounding streams and compression of fully-saturated porous media, as clearly mentioned in the manuscript. Contribution from constant-head boundaries (AG and ED) is, however, ignored in the aquifer water-budget model and only the effects of AB and BD are addressed by (50). Obviously, Darcian flow (either inwardly or outwardly) is induced by non-zero head gradients perpendicular to AG and ED. Such water fluxes are also disregarded in Fig. 7.

Response:

Thanks for the comment. We replace the sentence “The hydraulic heads along AG and DE are fixed at their average water stages as did in Kihm et al. (2007).” with the following text “The hydraulic heads along AG and DE are assumed equal to their average head values as did in Kihm et al. (2007). In other words, the boundaries along AG and ED are assumed under the constant-head condition in our mathematical model. Physically, they are not streams and therefore not count for their contribution in the calculations of SDR in Sect. 2.5 Stream depletion rate.” (lines 17-20, page 4 in the revised manuscript). Note that we also evaluate the SDRs along the boundaries AG and ED and their estimated values are both less than 0.0008 over the entire pumping period, indicating that their effects are negligible.

Technical corrections

- The dimension of 1D Dirac’s delta function should be mentioned: [1/L]

Response:

Thanks, it has been added as: “The symbol δ represents one dimensional (1D) Dirac’s delta function $[1/T]$.” (line 12, page 5)

- The dimension of time should be changed to [T] in “Table 1”.

Response:

Thanks, it has been corrected.

- Unbalanced parenthesis is detected in (34).

Response:

Done as suggested.

- Equal sign is omitted in (24) and (25).

Response:

Thanks, it has been corrected.

References:

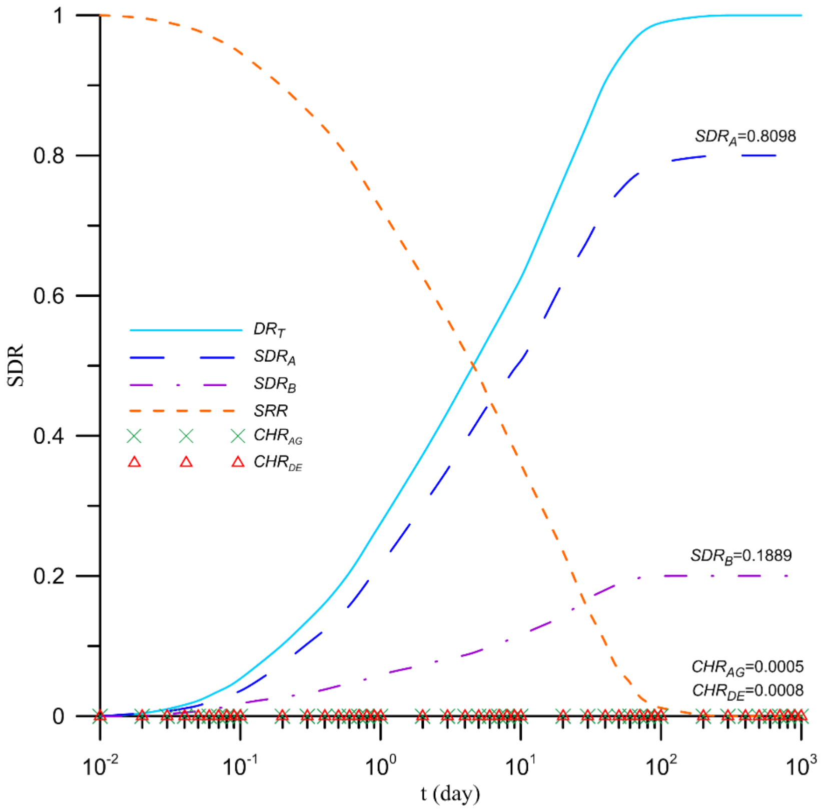
Kacimov, A. R., Kayumov, I. R., and Al-Maktoumi, A.: Rainfall induced groundwater mound in wedge-shaped promontories: The Strack–Chernyshov model revisited. Advances in Water Resources, 97, 110–119, 2016.

Kacimov, A. R., Maklakov, D. V., Kayumov, I. R., and Al-Futaisi, A.: Free Surface flow in a microfluidic corner and in an unconfined aquifer with accretion: The Signorini and Saint-Venant analytical techniques revisited. Transport in Porous Media, 116(1), 115–142, 2017

Kantorovich, L.V., and Krylov, V.I.: Approximate Methods of Higher Analysis. Interscience, New York, 1958.

Mackowski, D. W.: Conduction Heat Transfer: Notes for MECH 7210. Mechanical Engineering Department, Auburn University, 2011.

Mahdavi, A., and Seyyedian, H.: Steady-state groundwater recharge in trapezoidal-shaped aquifers: A semi-analytical approach based on variational calculus. Journal of Hydrology, 512, 457–462, 2014



Modified Figure 7. Temporal distributions of $SDRs$, $CHRs$ and SRR due to pumping at P_w.