Response to the comments of Dr. Skøien

1. This is a new version of a manuscript describing a coupling of the Budyko approach and hydro-stochastic interpolation. The manuscript has been improved, but there are still a few issues. I also still have some questions regarding the hydro-stochastic interpolation. I am suggesting minor revision here, as there might be good answers to my questions, in that case the editor can accept the manuscript after receiving these. If he is not satisfied with the answers, I'm happy to have another look at a new version.

Reply: Thanks for the reviewer's work and valuable comments that have helped improve every aspect of our manuscript. In this revision, we have explained your questions regarding the hydro-stochastic interpolation and revised the manuscript according to the reviewer's suggestions (P8L155- P10L193, P18L387- P19L406).

2. I am happy to see that the new version gives better and more sensible results for the hydro-stochastic interpolation. However, in the answer to the previous review, the authors added the weights for one of the stations. These weights puzzled me, and needs some explanation, by checking closer how the weights could occur.
The weights are for the HWH catchment, which is in the bottom part of the domain. The two closest catchments (BLY and a catchment where I cannot see the name) are not even mentioned in the list of weights. QL and HNZ are quite highly correlated, but still get large weights both of them. This is rather weird, even if hydro-stochastic interpolation is depending on the configuration of the observations and their spatial supports in addition to the distance between them.
Does the software have a correction for negative weights? That could be an explanation, but then it needs to be mentioned. Still it is more common to see the observations "in the shade" getting negative weights. By that I mean that I would expect a relatively large weight for BLY and the unnamed catchments, and negative weights for QL and/or HNZ. If negative weights is the cause, it is necessary to check how large they are before rescaling. Just deleting them and rescaling the rest might not be the best solution, it might be better to pick fewer neighbours for the interpolation in the first place.

Reply: To answer these questions, we added the calculated weights of HWH catchment and the two closer catchments (BLY and XHD – the unnamed before but we added the name of XHD now in the revision) that were not in the previous list of weights. We checked our calculations and found that:

(1) The two closest catchments are HPT and BLY, not BLY and XHD.

(2) Yes, there are corrections for negative weights in our software. As the reviewer pointed out, it is common to see the observations getting negative weights. According to the reviewer's suggestions, fewer neighbors of HWH (e.g., the eight stations in Table 1) were selected in our interpolation procedure. There were still negative weights for BLY, XHD, and MS catchments. The negative weight is small at BLY, and a little large at XHD and MS.

(3) In our previous interpolation, we deleted the catchments with the negative weights

and rescaled the remaining weights. We noticed that deleting the stations with negative weights may not obtain the best solution. In our study, in order to guarantee that the interpolation of $Z^{**}(u)$ *using the rescaled weights does not reduce the estimation accuracy of the runoff* $Z^{*}(u)$*, we rescaled the weights using the nonlinear programming (a posteriori correction): min* $\left(Z^{*}(u) - Z^{**}(u)\right)^2$*, s.t.* $\sum_k^r \lambda_j' = 1$ *and* $0 < \lambda_j' < 1, j = k, \ldots . r$.

In terms of the eight stations in Table 1, the weighted runoff using the rescaled positive weights for HPT, ZC, QL, HNZ, and JJJ catchments is much closer to the original weighted runoff with negative.

So, our rescaled weights don't affect the conclusions that our coupled Budyko and hydro-stochastic interpolation method is better than the Budyko and the hydro-stochastic analysis alone.

We didn't describe the rescaled method to avoid distraction or confusion and also to ease the reading for readers.

Table 1 Covariance and weights of HWH

No.	Sub-basins	Cov(A, B)	Runoff (mm)	weights	weighted R (mm)	Rescaled weights	weighted R using rescaled weights (mm)
1	HPT	218905	764	1.586	1212	0.789	603
2	BLY	210664	868	-0.013	-12	0	0
3	ZC	162552	838	0.064	53	0.035	30
4	XHD	114354	740	-0.760	-562	0	0
5	QL	71889	970	0.164	159	0.125	121
6	MS	32072	672	-0.202	-136	0	0
7	HNZ	24772	640	0.088	57	0.039	25
8	JJJ	14855	513	0.027	14	0.013	6
sum					784	1.00	784

Minor points

1. Section 2.2 is still too long – 3 pages is not necessary for methodologies that have been published before.

Reply: We shortened section 2.2 but kept the main points of that interpolation method.

2. In P16, it is mentioned that the area is divided into a 40x50 grid. I find this relatively course resolution for many of the smaller sub-basins, which are likely to get rather few grid points for the calculation of the Ghosh-distance (not geostatistical distance), if I understand correct.

Reply: Here the geostatistical distance between sub-basin A and B is also called Ghosh *distance. It was calculated by averaging the distance between pair of gridded points in two different sub-basins.*

We compared effects of the grid resolution on the geostatistical distance and the derived function of the covariogram. For example, one test was to double the resolution of the grid from 40×50 grid to 80×100 grid. The geostatistical distance changed from 94.54 km in the coarser grid to 89.13 km in the finer grid between the two sub-basins TJH and XX2 (using them as examples). Our additional tests have shown that this change (doubling the resolution) causes little change in the derived/fitted function for the covariogram in the HRB (the curve fit in Fig. 4). Thus, this resolution has a trivial effect on our interpolation results within the practical limit.

3. Section 5 is very much a repetition of the summary from 4.4. Some more discussion would instead be useful, such as how does the results in this paper compare to similar studies before.

Reply: We rewrote this section in the revision according to this suggestion.

Edits

There are still many grammatical errors – often related to articles and plural/singular. Already in the start of the abstract, it should either read "A Hydro-stochastic interpolation method…" or maybe better: "Hydro-stochastic interpolation methodS based on traditional block kriging HAVE often … A caveat in such methodS ARE that…" There are several more examples in the manuscript, such as P5L84 – such AN approach. P6 L114-119 "THE semi-empirical …" "into A deterministic trend" "calculated as THE difference …"

Reply: Yes, we noticed issues with use of articles, and have asked for help to correct them. We hope the revision reads better.

1. P4L63 Blöschl

Reply: We have changed it.

2. P4L65-66 I find it unclear what this sentence really says. It should anyway be constrains.

Reply: This sentence has been revised as "The river network constrains the water paths from upstream to downstream in a basin."

3. P4L73 What is an "integrated course"? Rephrase

Reply: It has been changed to "integrated process."

4. P5L80 I think there are more important reasons for the deviations than the influence FROM heterogeneous rainfall.

Reply: This sentence has been revised as "The observed patterns of runoff reveal systematic deviations from the homogeneity assumption, however, because of the influences from the heterogeneous climate and underlying surface factors."

5. P5L86 "method FOR describing complex runoff patterns suffers FROM AN inevitable …"
Reply: According to the reviewer's suggestion, it has been modified as "… the deterministic method for describing complex runoff patterns suffers from an inevitable loss of information."

6. P5L90-93 KED is not recent – maybe rather something like "A method that combines both deterministic patterns and stochastic variability is kriging … " And then "It takes deterministic patterns of spatial variables into account and incorporates these as a local trend, a smoothly varying secondary variable, instead of a function of spatial coordinates."
Reply: These sentences from P5L90-93 are revised to be "A method that combines both deterministic patterns and stochastic variability is the kriging with an external drift (KED) (Goovaerts, 1997; Li and Heap, 2008; Laaha et al., 2013). It takes the deterministic patterns of spatial variables into account and incorporates them as a local trend of a smoothly varying secondary variable, instead of a function of the spatial coordinates."

7. P6L117-118 comma after residuals, remove "both of"
Reply: It has been revised.

8. P13L266 "millions of tons of water" – this is not very precise, and actually not a particularly large number for a region of this size. As an example, 10 million m3/year equals 0.3 m^3/second.
Reply: This sentence has been changed to "About 18 billion m^3 of water was consumed in 1998 to meet the basin's domestic and agriculture needs."

The list of all relevant changes in the revised manuscript

1. Some grammatical errors in the manuscript, related to the use of articles and plural/singular, have been corrected;
2. P4L65: It has been changed to “Blöschl”;
3. P4L67: It has been changed to “The river network constrains the water paths from upstream to downstream in a basin”;
4. P4L75: It has been changed to “…integrated process…”;
5. P5L80-82: This sentence has been revised as “The observed patterns of runoff reveal systematic deviations from the homogeneity assumption, however, because of the influences from the heterogeneous climate and underlying surface factors”;
6. P5L88-89: Some words have been changed in this sentence;
7. P5L93-97: These sentences have been revised according to the reviewer’s suggestion;
8. P6L120-121: “both of” in the original sentence has been deleted and a comma has added;
9. P8L155- P10L193: This section has been shortened and the main points of the method has been kept;
10. P12L239-240: The sentence has been revised;
11. P18L387- P19L406: The section of discussions and conclusions has been rewritten;
12. P21L447-449: A reference has been added;
13. P26: Some articles have been added in the captions of the figures;
14. P27, P29: Some articles have been added in the captions of the tables;
15. The name of “XHD” has been added in Figure 1, 2, 3 and 7.

Hydro-Stochastic Interpolation Coupling with Budyko Approach for Prediction of Mean Annual Runoff

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Abstract

The hydro-stochastic interpolation method based on the traditional block-kriging has often been used to predict mean annual runoff in river basins. A caveat in such method is that the statistic technique provides little physical insight on relationships between the runoff and its external forcing, such as the climate and land-cover. In this study, the spatial runoff is decomposed into a deterministic trend and deviations from it caused by stochastic fluctuations. The former is described by the Budyko method (Fu's equation) and the latter by stochastic interpolation. This coupled method is applied to spatially interpolate runoff in the Huaihe River Basin of China. Results show that the coupled method significantly improves the prediction accuracy of the mean annual runoff. The error of the predicted runoff by the coupled method is much smaller than that from the Budyko method and the hydro-stochastic interpolation method alone. The determination coefficient for cross-validation, R_{cv}^2, from the coupled method is 0.87, larger than 0.81 from the Budyko method and 0.71 from the hydro-stochastic interpolation. Further comparisons indicate that the coupled method also has reduced the error in overestimating low runoff and underestimating high runoff suffered by the other two methods. These results support that the coupled method offers an effective and more accurate way to predict the mean annual runoff in river basins.

Keywords: Coupled Budyko and hydro-stochastic interpolation method; mean annual runoff; prediction accuracy; Huaihe River Basin

1. Introduction

The runoff observed at the outlet of a basin is a crucial element for investigating the hydrological cycle of the basin. Because runoff is influenced by both deterministic and stochastic processes, estimating the spatial patterns of runoff and associated distribution of water resources in ungauged basins has been one of the key problems in hydrology (Sivapalan et al., 2003), and a thorny issue in water management and planning (Imbach, 2010; Greenwood et al., 2011).

In estimating and predicting runoff and regional water resources availability, we have often used regional or global runoff mapping and geostatistical interpolation methods. In these methods, the value of a regional variable at a given location is often estimated as the weighted average of observed values at neighboring locations. This interpolation of runoff, which is assumed as an auto-correlated generalized stochastic field (Jones, 2009), uses secondary information from more than one variable (Li and Heap, 2008). Spatial autocorrelations of the runoff values are measured by the covariance or semi-variance between the runoffs at pairs of locations as a function of their Euclidian distance (such as in the ordinary kriging). The values obtained by the interpolation methods are the best linear unbiased estimate in the sense that the expected bias is zero and the mean squared error is minimized (Skøien et al., 2006). The ordinary kriging (OK) estimates the local mean as a constant; corresponding residuals are considered as random. Because the spatial mean could also be used as a trend or nonstationary variation in space, OK has been developed into various geostatistical interpolation methods, such as kriging with a trend by incorporating local trend within a confined neighborhood as a smoothly varying function of the coordinates. Block kriging (BK) is another extension of OK for estimating a block value instead of a point value by replacing the point-to-point covariance with point-to-block covariance (Wackernagel, 1995).

Unlike precipitation or evaporation which we often interpolate to find its values at specific locations, runoff is an integrated spatially continuous process in river basins (Lenton and RodriguezIturbe, 1977; Creutin and Obled, 1982; Tabios and Salas, 1985; Dingman et al., 1988; Barancourt et al., 1992; Blöschl, 2005). Streamflows are naturally organized in basins (Dooge, 1986; Sivapalan, 2005), e.g., rivers flow through sub-basins. The river network constrains the water paths from upstream to downstream in a basin. The hierarchically organized river network requires that the sum of the interpolated discharge from sub-basins equals to the observed runoff at the outlet of the entire basin. Previous studies have indicated that runoff interpolation may overestimate the actual runoff without adequate information of the spatial variation of the runoff (Arnell, 1995), e.g., neglecting the river network in connecting sub-basins or processing basin runoff at collective points in space (Villeneuve et al, 1979; Hisdal and Tveito, 1993). In nested basins, Gottschalk (1993a and b) developed a hydro-stochastic method to interpolate runoff. It uses the concept that runoff is an integrated process in the hierarchical structure of river network. Distance between a pair of basins is measured by geostatistical distance instead of the Euclidian distance. The covariogram among points in the conventional spatial interpolation is replaced by the covariogram between basins. In this concept, runoff is assumed spatially homogeneous in basins, i.e., the expected value of the runoff is constant in space (Sauquet, 2006). The observed patterns of runoff reveal systematic deviations from the homogeneity assumption, however, because of the influences from the heterogeneous climate and underlying surface factors.

An alternate method is to describe the hydrological variables of interest in deterministic forms of functions, curves or distributions, and construct conceptual and mathematical models to predict hydro-climate variability (Wagener et al, 2007). Qiao (1982), Arnell (1992), and Gao et al. (2017) have used such an approach and derived empirical relationships between runoff and its controlling factors of the climate, land-cover, and topography in various basins. However, the deterministic method for describing complex runoff patterns suffers from an inevitable loss of information (Wagener et al, 2007) because of existence of uncertainty in many hydrological processes and especially in observations. Thus, hydrological variables also contain the information of stochastic nature and should be treated as outcomes from deterministic and stochastic processes. A method that combines both deterministic patterns and stochastic variability is the kriging with an external drift (KED) (Goovaerts, 1997; Li and Heap, 2008; Laaha et al., 2013). It takes the deterministic patterns of spatial variables into account and incorporates them as a local trend of a smoothly varying secondary variable, instead of a function of the spatial coordinates.

The inclusion of deterministic terms in the geostatistical methods has been shown to increase the interpolation accuracy of basin variables, such as mean annual runoff (Sauquet, 2006), stream temperature (Laaha et al., 2013), and groundwater table (Holman et al., 2009). Those deterministic terms are often described by empirical formulae linking spatial features, e.g., variability of the mean annual runoff in elevation (Sauquet, 2006), and relationship between the mean annual stream temperature and the altitude of gauges (Laaha et al., 2013). As a semi-empirical approach to model the deterministic process of the runoff, the Budyko framework has been popularly used to analyze the relationship between mean annual runoff and the climatic factors, e.g., aridity index (Milly, 1994; Koster and Suarez, 1999; Zhang et al., 2001; Donohue et al., 2007; Li et al., 2013; Greve et al., 2014). Many efforts have been devoted to improving the Budyko method by, for example, including the effects of other external forcing factors, such as land-cover (Donohue et al., 2007; Li et al., 2013; Han et al., 2011; Yang et al., 2007), soil properties (Porporato et al., 2004; Donohue et al., 2012), topography (Shao et al., 2012; Xu et al., 2013; Gao et al., 2017), hydro-climatic variations of seasonality (Milly, 1994; Gentine et al., 2012; Berghuijs et al., 2014), and groundwater (Istanbulluoglu et al., 2012). However, it has been found that the use of the deterministic equation in the Budyko method alone still comes with large errors in the prediction of runoff in many basins (e.g., Potter and Zhang, 2009; Jiang et al., 2015).

The aim of this study is to combine the stochastic interpolation with the semi-empirical Budyko method to further improve the spatial interpolation/prediction of the mean annual runoff in the Huaihe River Basin (HRB), China. In this study, the spatial runoff from sub-basins in the HRB is separated into a deterministic trend and its residuals, which are estimated by the Budyko method and the interpolation method, respectively. The residuals are calculated as the difference between the observed and the estimated runoff from the Budyko method, and are used in the stochastic interpolation as described in Gottschalk (1993a, 1993b, and 2000). After that, the runoff of any sub-basin is predicted as the sum of the interpolated residuals and the Budyko estimated value. The improved method is tested in the HRB. In addition, the leave-one-out cross-validation approach is applied to evaluate and compare the performances of the three interpolation methods: the Budyko method, hydro-stochastic interpolation, and our coupled Budyko and stochastic interpolation method.

2. Methodologies

2.1 Spatial estimation of mean annual runoff by Budyko method

The Budyko method explains the variability of mean annual water balance on a regional or global scale. It describes the dependence of actual evapotranspiration (E) on precipitation (P) and potential evapotranspiration (E_0) (Williams et al., 2012). Their original relationship (E/P~E_0/P) derived by Budyko (1974) is deterministic and nonparametric. It was later developed into parametric forms (Fu, 1981; Choudhury, 1999; Yang et al., 2008; Gerrits et al., 2009; Wang and Tang, 2014). Among them, the one-parameter equation derived by Fu (Fu, 1981, Zhang et al. 2004) has been used frequently. This relationship is written

$$\frac{E}{P} = 1 + \frac{E_0}{P} - \left(1 + \left(\frac{E_0}{P}\right)^{\omega}\right)^{\frac{1}{\omega}} \quad (1)$$

or

$$R = P \cdot \left(1 + \left(\frac{E_0}{P}\right)^{\omega}\right)^{\frac{1}{\omega}} - E_0 \quad (2)$$

where, P, E, E_0, and R are mean annual precipitation, actual evapotranspiration, potential evapotranspiration, and runoff (units: mm), respectively, and ω is a dimensionless model parameter in the range of $(1, \infty)$. In these formulae, the larger the ω is, the smaller the partition of precipitation into the runoff.

The parameter ω in (1) is determined using observed P, E_0, and R in gauged sub-basins. The mean value of ω of a basin can be obtained by averaging ω of the sub-basins, or by minimizing the mean absolute error (*MAE*) in fitting the curve in Eq. (1) with E/P~E_0/P ($E = P - R$) (Legates and McCabe, 1999). Using the mean value of ω, Eq. (2) can be used to predict ungauged basin runoff or to interpolate the spatial variation of the runoff, using meteorological data in targeted sub-basins (Parajka and Szolgay, 1998).

2.2 Hydro-stochastic interpolation method

Gottschalk (1993a) described the hydro-stochastic interpolation method based on the kriging method to predict spatial runoff. Gottschalk's method redefines a relevant distance between basins, and identifies the river network and supplemental water balance constraints as follows.

As a spatially integrated continuous process, the predicted runoff of a specific unit of an area A_0 in a basin, $r^*(A_0)$, can be expressed as

$$r^*(A_0) = \sum_{i=1}^{n} \lambda_i r(A_i) \quad (3)$$

where, $r(A_i)$ is the observed runoff in a gauged basin i with area A_i ($i = 1, \ldots n$, n is the total number of gauged basins), and λ_i is the weight of basin i.

The weights are obtained by solving the following set of equations under the second order stationary assumption for hydrologic variables (Ripley, 1976),

$$\begin{cases} \sum_{j=1}^{n} \lambda_i Cov(u_i, u_j) + \mu = Cov(u_i, u_0), \quad i, j = 1,2,\ldots,n \\ \sum_{i=1}^{n} \lambda_i = 1\,. \end{cases} \quad (4)$$

In (4), $Cov(u_i, u_j)$ is the theoretical covariance function between each pair of gauged stations (*i*=1,…, *n*, j=1,2…, *n*), $Cov(u_i, u_0)$ is the theoretical covariance of runoff between the location of interest u_0 and each of the gauged stations u_i, and μ is the Lagrange multiplier.

The sum of the interpolated runoff for each non-overlapping sub-basin should be equal to the observed runoff at the river outlet. This constraint can be written as

$$R_T = \sum_{i=1}^{M} \Delta A_i \, r(\Delta A_i) \tag{5}$$

where, R_T is the streamflow observed at the outlet of the basin, ΔA_i is the non-overlapping area of sub-basin i, and $r(\Delta A_i)$ is the runoff depth for sub-basin i ($i = 1, \ldots, M$). The predicted runoff for each ΔA_i is a linear combination of the weights and the runoff observed in the *n* sub-basins, i.e., $r(\Delta A_i) = \sum_{j=1}^{n} \lambda_j^i r(A_j)$. Substituting it in (5) we get

$$R_T = \sum_{i=1}^{M} \Delta A_i \left(\sum_{j=1}^{n} \lambda_j^i r(A_j) \right). \tag{6}$$

In (6), $r(A_j)$ is the runoff depth for sub-basin *j* (*j* = 1, ..., *n*) with discharge observations, and λ_j^i is the weight (*i*= 1, ..., *M*; *j*= 1, ..., *n*). Further considering the basin area in the river network, Sauquet et al. (2000) derived the weight matrices and described a hydro-stochastic method to optimize the weights λ_j^i (*i*= 1, ..., *M*; *j*= 1, ..., *n*) in Eq. (6).

The theoretical covariogram, $Cov(A, B)$, is derived by averaging the point process covariance function Cov_p

$$Cov(A, B) = \frac{1}{AB} \int \int_{AB} Cov_p(||u_1 - u_2||) du_1 du_2 \tag{7}$$

where, $Cov_p(||u_1 - u_2||)$ is the theoretical covariance function value of pairs of points in basins A and B with distance d=$||u_1 - u_2||$.

The distance $d(A,B)$ is calculated based on grid division in each of the sub-basins (Sauquet et al., 2000). The trial-and-error fitting method is used to calibrate Cov_p(d) in Eq. (7) to best fit Cov_e(d). Only independent sub-basins are used to calculate the covariance function to avoid spatial correlation of nested sub-basins.

2.3 Coupling the stochastic interpolation with the Budyko method

The above stochastic interpolation procedure assumes a stationary stochastic variation of the runoff among sub-basins or spatial homogeneity in runoff (Sauquet, 2006), despite variations in river networks. For nonstationary variations in the runoff resulting from spatial heterogeneity in a river network, the spatial runoff can be decomposed into a nonstationary deterministic component and a stochastic component:

$$R(x) = R_d(x) + R_s(x). \quad (8)$$

In (8), $R(x)$ is the runoff at a location x, $R_d(x)$ is the deterministic component of the spatial trend or the external drift (Wackernagel, 1995) that results in nonstationary variability in space. $R_s(x)$ is the stochastic component considered to be stationary.

In this study, R in Eq. (2) is used as an external drift function in estimating the $R_d(x)$ in all sub-basins, i.e., $R_d(x)$ in Eq. (8) is substituted in Eq. (2) by setting $R_d(x) = R$. The residuals between $R_d(x)$ and the observed runoff are calculated for all gauged sub-basins. Furthermore, these residuals are interpolated for all ungauged sub-basins and set as the stochastic component $R_s(x)$ in Eq. (8) using the "residual kriging" method (Sauquet, 2006). In particular, $R_s(x)$ in Eq. (8) is replaced by $r^*(A_0)$ in Eq. (3) after setting $r^*(A_0) = R_s(x)$ for the stochastic interpolation scheme described in section 2.2. The superposition of these estimates of both components on the right-hand side in Eq. (8) yields the prediction of *R(x)*.

2.4 Cross validation

To validate this prediction procedure, we use the leave-one-out cross-validation method (Kearns, 1999). In addition to quantifying the performance of our coupled Budyko and the hydro-stochastic interpolation method, we compare and contrast its performance with the Budyko and the hydro-stochastic interpolation method alone. Their performances are evaluated by the following metrics (Laaha and Bloschl, 2006):

$$MAE = \frac{1}{n}\sum_{j=1}^{n}[R(x_i) - R^*(x_i)\,] \tag{9}$$

$$MSE = \frac{1}{n}\sum_{j=1}^{n}[R(x_i) - R^*(x_i)\,]^2 \tag{10}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n}[R(x_i) - R^*(x_i)\,]^2} \tag{11}$$

where, $R^*(x)$ and $R(x)$ are the predicted and the observed runoff, respectively, *MAE* is the mean absolute error, *MSE* is the mean square error, and *RMSE* is the root-mean-square error. The determination coefficient for cross-validation is

$$\mathrm{R}_{cv}^2 = 1 - \frac{V_{cv}}{V_{NK}} \tag{12}$$

where, V_{cv} is the mean square error (*MSE*), and V_{NK} is the spatial variance ($V_{NK} = \frac{\sum_{j=1}^{n}[R(x_i)-\bar{R}\,]^2}{n-1}$, in which $\bar{R}$ is the mean $R(x)$) of the runoff over all the tested sub-basins. In addition to these evaluation metrics, the prediction result is evaluated by regression analysis of the observation vs. the prediction.

3. Study catchment and data

The Huaihe River Basin (HRB) – the sixth largest river basin in China, is used in evaluation of our coupled model and in its comparison to the other two methods. The HRB has a strong precipitation gradient from the humid climate in the east and the semi-humid in the west (Hu, 2008). It is one of the major agricultural areas in China with the highest human population density in the country. About 18 billion m^3 of water was consumed in 1998 to meet the basin's domestic and agriculture needs. Water resources per capita and per unit area is less than one-fifth of the national average. Moreover, more than 50% of the water resources is exploited, much higher than the recommended 30% for inland river basins (Yan et al., 2011). Moreover, the concentrated annual precipitation in a few very rainy months makes the region highly vulnerable to severe floods or droughts (Zhang et al., 2015). Thus, having the knowledge of the spatial distribution of the runoff is vital for water resources planning and management in the region.

Our study area is in the upstream of the Bengbu Sluice in the HRB and is 121,000 km^2 (Fig. 1). The river network in the area is derived from data packages of the National Fundamental Geographic Information System, developed by the National Geomatics Center of China. The HRB is divided into 40 sub-basins, according to available hydrological stations with records from 1961-2000 (Fig. 2). The sub-basins vary in their size from the smallest of 17.9 km^2 to the largest of 30630 km^2. Among the 40 sub-basins, 27 are independent sub-basins and 13 are nested sub-basins.

Annual precipitation data used in this study are from 1961-2000 and are obtained from a monthly mean climatological dataset at 0.5-degree spatial resolution. The dataset was developed at China Meteorological Administration, and is accessible at: http://data.cma.cn/data/detail/dataCode/SURF_CLI_CHN_PRE_MON_GRID_0.5.html. The dataset was derived from the observations at 2472 stations in China, using the Thin Plate Spline (TPS) interpolation method and the ANUSPLIN software. Pan evaporation data at 21 meteorological stations in the HRB are used to interpolate E_0 by the ordinary kriging method and the ArcGIS. The interpolated E_0 are used to derive the annual potential evapotranspiration in the sub-basins. The statistical features of the mean annual precipitation (P), E_0, and the runoff depth (R) from 1961-2000 are summarized in Table 1. They show that P varied between 638-1629 mm, annual temperature was between 11°-16°C, and the mean annual E_0 between 900-1200 mm. The sub-basins in the north, e.g., ZM, ZQ, XY, and ZK in Fig. 2, are relatively dry with the dryness index (E_0/P) above 1.3. The sub-basins in the south, e.g., MS, HBT, and HC, are wetter with dryness index below 0.8. The average mean annual R is about 400 mm, fluctuating from 90 mm in the north to 1000 mm in the south. The temporal and spatial variations in the runoff are relatively small in the south and large in the north.

4 Results

4.1 Prediction of runoff by the Budyko method

Actual evapotranspiration E is estimated using long-term mean annual water balance (E=P-R) from 1961–2000 at the 40 sub-basins, and the results are shown in Table 1. Also shown in Table 1 are the calculated ω values for the sub-basins. They vary from 1.43 in the sub-basin HWH to 3.16 in JJJ. The average ω is 2.32 for the 40 sub-basins. The comparison E/P vs. E_0/P is shown in Fig. 3. The best fit (curve) for E/P vs. E_0/P, or

R vs. E_0/P, is also shown in Fig, 3; it gives an alternative for average ω of the sub-basins. The fitted value of ω for the 40 sub-basins determined from this process is 2.213, very close to that calculated directly from the 40 individual sub-basins.

Using ω=*2.213* in the HRB, Fu's equation in Eq. (2) can be written as

$$R = P \cdot \left(1 + \left(\frac{E_0}{P}\right)^{2.213}\right)^{\frac{1}{2.213}} - E_0. \tag{13}$$

Eq. (13) and Fig. 3 clearly show the deterministic trend of the runoff in the HRB. According to the water limit criterion, $E = P$, and the energy limit criterion, $E = E_0$, in Fig. 3a, the smaller the index $\frac{E_0}{P}$ is the smaller the $\frac{E}{P}$ will be (Fig. 3a) or the larger the runoff will be (Fig. 3b) from the sub-basins in the HRB. In Figs. 3b and 3c, the lower R in the northern sub-basins indicates drier conditions (E_0/P>1.4), whereas the higher R in the southern sub-basins assures wetter conditions (E_0/P<0.8).

Using P and E_0 given in Table 1 for the 40 sub-basins, we predict the runoff R by Eq. (13), the Budyko method, and the deviations of their predictions from the observation. The results are summarized in Tables 1 and 2. The *MAE* of predicted R is 94 mm, and *RMSE* is 112 mm. The largest absolute error is in the sub-basin HWH (328 mm), and the smallest in XX (24 mm). The largest relative error is 81.6% of the observed runoff in the sub-basin XZ, and the smallest is 5.0% of the observed runoff in XHD. They represent absolute errors of 91 and 37 mm in those two sub-basins, respectively.

4.2 Runoff by the hydro-stochastic interpolation method

For comparison, the observed runoff is used in the hydro-stochastic interpolation following the procedure detailed in section 2.2. In order to obtain the distance d

between pairs of the sub-basins, the study area is divided into 40 row × 50 column. The geostatistical distance between any two sub-basins, A and B, is calculated by averaging the distances between all pairs of grid points in A and B (all the possible pairs of the sub-basins are 40×41/2 for the 40 sub-basins in this study). According to the estimated distance for the pairs of sub-basins and the observed runoff at the 40 sub-basins (Table 1), the empirical covariance $Cov_e(d)$ is estimated for each pair of the sub-basins. From the plots of the mean $Cov_e(d)$ of all the independent sub-basin pairs vs. the corresponding distance d with an interval of 20 km, we fit the function of empirical covariogram shown in Fig. 4. The fitting theoretical covariance function $Cov_p(d)$ to the empirical covariogram is

$$Cov_p(d) = 6 \times 10^5 \exp(-d/28.62). \tag{14}$$

This function is used to calculate the average theoretical covariance $Cov(A,B)$ in Eq. (7). Finally, the weight matrices are determined using our programs in MatLab.

The interpolated runoff depth (R) over the 40 sub-basins along with the deviations from the observation are shown in Table 1. The *MAE* and *RMSE* of R are 103 and 140 mm, respectively. The largest absolute and relative error is in the sub-basin JZ (401 mm and 68.8%), and the smallest is in DPL (1 mm and 0.3%) (Table 2). These results indicate that the errors from this interpolation method are in general larger than those from the Budyko method, suggesting that the observed runoff is more influenced by the deterministic trend in the basin.

4.3 Hydro-stochastic interpolation with Fu's equation (our coupled method)

We use Fu's equation, Eq. (2), to evaluate the deterministic trend or the external drift function, $R_d{}^*(x)$, and deviation of the trend from the observation, $R_s{}^*(x)$, assuming a spatially auto-correlated process. The $R_s{}^*(x)$ is then used in the stochastic interpolation.

The empirical residual covariogram of $R_s{}^*(x)$ for each pair of sub-basins vs. sub-basin distance is shown in Fig. 5. From the result in Fig. 5a, we obtain the exponential function for $Cov_p(d)$

$$Cov_p(d) = 13030\exp(-d/23.9). \tag{15}$$

From (15), the weight matrices of runoff deviation are determined by Eq. (4) using our program in MatLab. They are then used to predict the runoff deviation. The scatterplot of the predicted residuals vs. the observed residuals shown in Fig. 5b delineates a positive correlation between the predicted and the observed residuals. However, the large scatter indicates limited performance by the residual model alone. Because this interpolation scheme represents the spatial runoff deviation, the sum of the interpolated runoff deviation and the simulated runoff by Fu's equation is the total interpolated runoff in the sub-basins.

The predicted runoff using this procedure is given in Table 1, with the *MAE* at 71 mm and *RMSE* at 93 mm over the 40 sub-basins. The largest absolute error is in the sub-basin QL (220 mm), and the smallest in ZM (4 mm) (Table 2). The largest relative error is 47.2% of the observed runoff in XZ, and the smallest is 1% of the observed runoff in BLY. They represent the absolute error of 52 and 8 mm, respectively.

4.4 Comparisons of the predicted runoff by the three methods

Comparing the results in Table 2, we find that our coupled method of the deterministic and stochastic processes substantially reduces the runoff prediction error in the HRB. The *MAE* and *RMSE* of the runoff from our coupled method are much smaller than those from the Budyko or the hydro-stochastic interpolation method. In cross-validation (Table 2), our coupled method has R_{cv}^2=0.87, which is larger than 0.81 and 0.71 from the Budyko method and the hydro-stochastic interpolation, respectively. The errors in runoff at the sub-basins are significantly reduced as well. The error in the sub-basin HWH is 216 mm from the coupled method, compared to 328 mm from the Budyko method and 300 mm from the hydro-stochastic interpolation. The error in JZ is 120 mm from the coupled method, smaller than 179 mm from the Budyko method and 401 mm from the hydro-stochastic interpolation.

Our correlation analysis between the predicted and the observed *R* is shown in Fig. 6. The predicted runoff from our coupled method shows higher correlation with the observed (R^2=0.87), in comparison to the Budyko method (R^2=0.82) and the hydro-stochastic interpolation (R^2=0.79). Our analysis indicates that the latter two methods overestimate low runoff and underestimate high runoff, as indicated by large departures from the 1:1 line in Fig. 6. Similarly, large deviations of the runoff predicted by the hydro-stochastic interpolation have also been reported by Sauquet et al. (2000), Laaha and Bloschl (2006), and Yan et al. (2011).

The spatial distributions of the runoff in the HRB calculated from the three methods are shown in Fig. 7. They again show significant differences. Compared to the result from our coupled method (Fig. 7c), the Budyko method overestimates the runoff in most of the northern sub-basins (Fig. 7a), where the climate is relatively dry and runoff is small (ranging from 140-280 mm). The hydro-stochastic interpolation method underestimates the runoff in some southern sub-basins (Fig. 7b), where the wet climate has fostered extremely high runoff (800~1100mm), such as in the sub-basins HWH, BLY, and ZC (Table 1). The results from our coupled method are closest to the observed distribution of the runoff among the three methods (Fig. 7d). Compared to the errors in the predicted runoff by the Budyko method and the hydro-stochastic interpolation (Fig. 7 and Table 1), our coupled method reduces the error in 70% of all the sub-basins (28 of the 40 sub-basins).

5. Discussions and conclusions

In this study, we use the Budyko's deterministic method to describe the mean annual runoff, which is an integrated spatially continuous process and determined by both the hydro-climatic elements and the hierarchical river network. A deviation from the Budyko estimated runoff is used by the stochastic interpolation that assumes spatially auto-correlated error. The deterministic aspects of the runoff described in Budyko method are reflected in the trends at locations (sub-basins), and deviations from the trends caused by the stochastic processes are described by the weights as a function of the autocorrelation and distance. Information from both the Budyko method and the stochastic interpolation are integrated in our coupled method to predict the runoff.

Different from the universal kriging method, in which the trend is represented as a linear function of coordinate variables and determined solely through spatial data calibration (i.e., semi-variogram analysis), the Budyko method couples water and energy balance and could directly predict streamflow in ungauged basins. This physically based method relies on using the spatial trend of runoff and, in our study, it yields the deterministic coefficient of cross-validation, R_{cv}^2, to be 0.81, better than that from the hydro-stochastic interpolation method.

Incorporating secondary information into the geostatistical methods improves the estimate of a predictive variable, e.g., the estimate of groundwater level by incorporating topography into the collocated co-kriging (Boezio et al., 2006), or the estimate of mean annual stream temperature by incorporating a nonlinear relationship between the mean annual stream temperature and altitude of the stream gauge into the Top-Kriging (Laaha et al., 2013). By incorporating such secondary information and the relationship between the mean runoff and the climate conditions (the aridity index) in the Budyko method through coupling with the hydro-stochastic interpolation, we develop our new coupled Budyko-hydro-stochastic interpolation method. It can substantially improve the prediction of streamflow in ungauged basins. This improvement is shown by the higher R_{cv}^2 of 0.87 in the HRB, compared to 0.81 and 0.71 by the Budyko and the hydro-stochastic interpolation method, respectively. Moreover, for high and low runoffs in the sub-basins of the HRB our coupled method gives more accurate predictions.

While substantial progress has been made by our coupled method, its results show rooms for improvement to further increase the accuracy of runoff prediction. For example, runoff prediction errors remain large from our coupled method in some sub-basins in the HRB. In the sub-basins MS, QL, HWH, and HNZ, the absolute error of predicted runoff is larger than 150mm and the relative error of predicted runoff is larger than 20% of the observed runoff. In the sub-basins BGS and XZ, the relative error of the predicted runoff is larger than 40% of the observed runoff. These errors are largely attributable to large prediction errors intrinsic to the Budyko method (e.g., MS, QL, HWH, and XZ in Table 1). Possible causes to the errors could be from additional external factors influencing the runoff, such as land-cover, soil properties, hydro-climatic variations, and the groundwater. Including some or all these effects to improve the Budyko method or incorporating these effects as secondary information (e.g., multi-collocated co-kriging) in our coupled model would help aid our understanding of the deterministic processes and increase the runoff prediction accuracy.

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Captions of figures:

Figure 1: The topography and river network of the study area.

Figure 2: The sub-basins and hydrological stations in the study area.

Figure 3: (a) $E/P \sim E_0/P$ and (b) $R \sim E_0/P$ for the 40 sub-basins (the solid line is the best fit function). (c) The sub-basins in the north and south of the study basin. Note: in (b) and (c), blue color indicates wetter climate in the south and yellow color indicates drier climate in the north.

Figure 4: Empirical covariogram ($Cov_e(d)$) from the sub-basin runoff data and theoretical covariogram by fitted covariance function $Cov_p(d)$ of the study area.

Figure 5: (a) Empirical covariogram ($Cov_e(d)$) from the residual $R_s(x)$ and theoretical covariogram by fitted covariance function $Cov_p(d)$ of the study area. (b) The scatterplot of the predicted vs. the observed residuals.

Figure 6: Cross validation of the predicted runoff vs. the observation by (a) Budyko method, (b) hydro-stochastic interpolation, (c) our coupled method, and (d) the scatterplot of the predicted vs. the observed residuals for (c). The dashed-line is 1:1.

Figure 7: Spatial distribution of the mean annul runoff estimated from (a) Budyko method, (b) hydro-stochastic interpolation, (c) our coupled method, and (d) the observation.

Table 1: Summary of hydro-meteorological data and predicted runoff of the sub-basins in the HRB.

No.	Stations	Basin area (km^2)	P (mm)	R (mm)	E_0 (mm)	E_0/P	E (mm)	Budyko method			Hydro-stochastic interpolation		Coupled method	
								ω	Predicted R (mm)	Error (mm)	Predicted R (mm)	Error (mm)	Predicted R (mm)	Error (mm)
1	CTG	3090	1012	366	932	0.92	646	2.41	399	32.85	357	8.29	442	75.89
2	XHD	1431	1517	740	974	0.64	776	2.41	777	36.94	819	78.85	785	44.21
3	SQ	3094	822	168	1024	1.25	653	2.83	248	79.29	154	14.34	189	20.40
4	MS	1970	1517	672	957	0.63	845	3.06	786	114.28	705	33.18	833	161.55
5	BGS	2730	877	225	1029	1.17	651	2.57	279	53.93	331	105.51	321	95.80
6	XC	4110	945	225	997	1.06	720	3.02	332	106.82	197	27.83	261	35.87
7	BT	11280	910	223	993	1.09	687	2.85	310	86.94	205	18.10	220	3.73
8	ZK	25800	678	123	1061	1.56	555	2.54	163	39.96	101	21.54	101	21.60
9	JJJ	5930	1347	513	969	0.72	834	3.16	640	127.27	369	143.29	555	42.76
10	HB	16005	1092	335	937	0.86	757	3.15	455	120.48	197	137.61	383	48.20
11	ZQ	3410	739	118	1083	1.47	621	2.83	190	71.71	101	17.02	125	7.56
12	HPT	4370	1629	764	984	0.60	865	2.92	868	103.53	729	34.69	896	131.58
13	XX	10190	987	367	1053	1.07	620	2.10	343	23.77	297	70.54	325	41.95
14	BB	121330	850	215	1024	1.20	635	2.54	264	49.48	71	143.43	175	39.74
15	WJB	30630	1003	294	957	0.95	709	2.85	384	90.29	225	68.43	280	14.17
16	LZ	390	963	345	1078	1.12	618	2.09	320	24.96	335	10.87	337	8.57
17	NLD	1500	1019	439	1101	1.08	581	1.86	351	88.30	350	88.75	388	50.60
18	ZMD	109	690	212	1093	1.58	478	1.94	163	48.65	265	52.90	157	54.73
19	BLY	737	1504	868	1126	0.75	635	1.69	695	173.27	783	85.32	861	7.54
20	HWH	292	1560	1068	1127	0.72	492	1.43	740	328.03	768	299.97	852	216.14
21	ZC	493	1512	838	1112	0.74	674	1.79	708	130.23	700	137.94	790	48.34
22	BQY	284	1268	693	1094	0.86	575	1.68	527	166.21	543	150.04	568	125.47
23	QL	178	1559	970	1090	0.70	589	1.60	756	214.17	749	221.28	749	220.34
24	HNZ	805	1480	640	1114	0.75	840	2.41	681	41.37	576	63.94	816	175.57
25	TJH	152	1305	699	1090	0.84	605	1.74	556	143.66	309	390.52	556	143.05
26	LX	77.8	1025	484	1079	1.05	540	1.75	361	123.77	302	182.46	368	116.82
27	ZLS	1880	755	253	1104	1.46	502	1.91	194	58.45	197	55.37	223	29.21
28	ZT	501	1021	437	1101	1.08	583	1.87	351	85.87	212	225.14	452	14.74

29	XGS	375	830	302	1088	1.31	528	1.91	238	63.74	99	202.58	317	15.33
30	JZ	46	1103	583	1107	1.00	520	1.63	404	178.81	182	401.32	463	120.48
31	GC	620	638	111	1055	1.65	528	2.51	145	34.18	53	57.92	125	14.85
32	ZM	2106	645	97	1039	1.61	548	2.72	150	53.48	72	24.71	100	3.62
33	YZ	814	979	235	1083	1.11	743	2.85	329	94.07	271	35.66	321	85.76
34	XZ	1120	746	111	1040	1.39	636	3.06	202	90.66	84	27.12	163	52.32
35	GZ	1030	855	342	1098	1.28	513	1.81	250	92.10	230	111.80	260	81.82
36	DPL	1770	1067	331	1066	1.00	736	2.57	393	61.62	330	1.02	437	105.29
37	XX2	256	1301	606	1092	0.84	695	2.00	552	53.68	708	101.78	732	126.63
38	PH	17.9	1248	708	1094	0.88	540	1.61	512	196.04	605	102.78	564	144.41
39	HC	2050	1255	454	1095	0.87	802	2.54	517	63.36	328	125.79	537	83.61
40	HK	2141	871	227	1077	1.24	644	2.44	264	37.28	273	46.15	243	16.02

Table 2: Interpolation cross-validation errors between the predicted and the observed runoff in the 40 sub-basins in the HRB from the three methods.

Evaluation indicators	Budyko method	Hydro-stochastic interpolation	Coupling method
MAE (mm)	94	103	71
MSE (mm^2)	12561	19828	8557
RMSE (mm)	112	140	93
Max absolute error (mm)	328	401	220
Min absolute error (mm)	24	1	4
Max relative error (%)	82	69	47
Min relative error (%)	5	0.3	1
R_{cv}^2	0.81	0.71	0.87

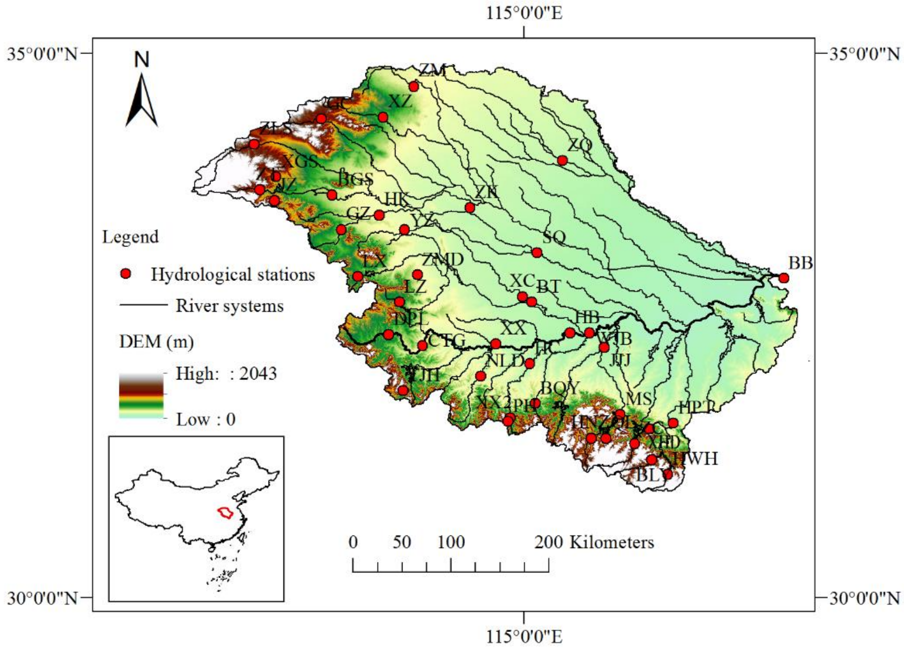


Figure 1: The topography and river network of the study area.

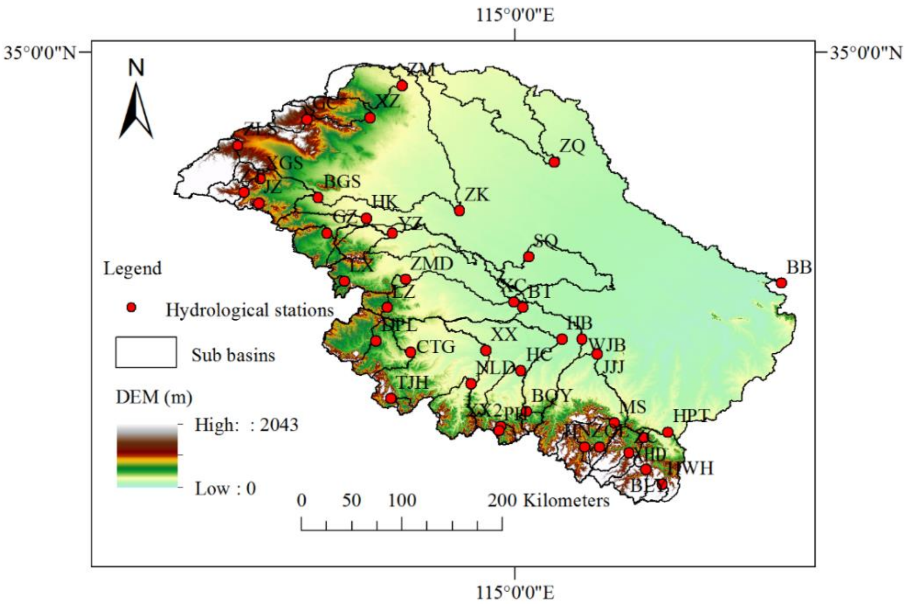


Figure 2: The sub-basins and hydrological stations in the study area.

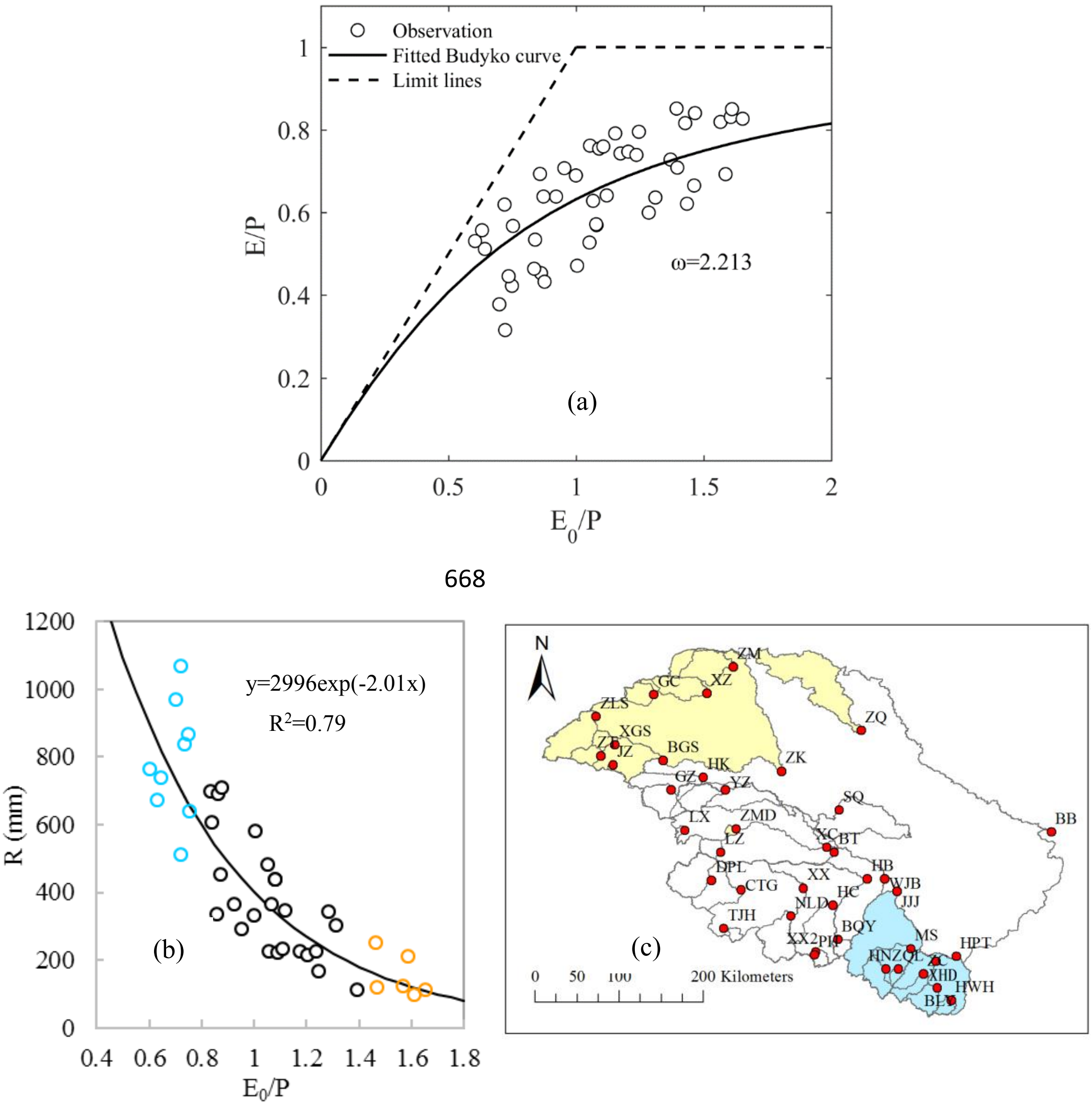


Figure 3: (a) $E/P{\sim}E_0/P$ and (b) $R{\sim}E_0/P$ for the 40 sub-basins (the solid line is the best fit function). (c) The sub-basins in the north and south of the study basin. Note: in (b) and (c), blue color indicates wetter climate in the south and yellow color indicates drier climate in the north.

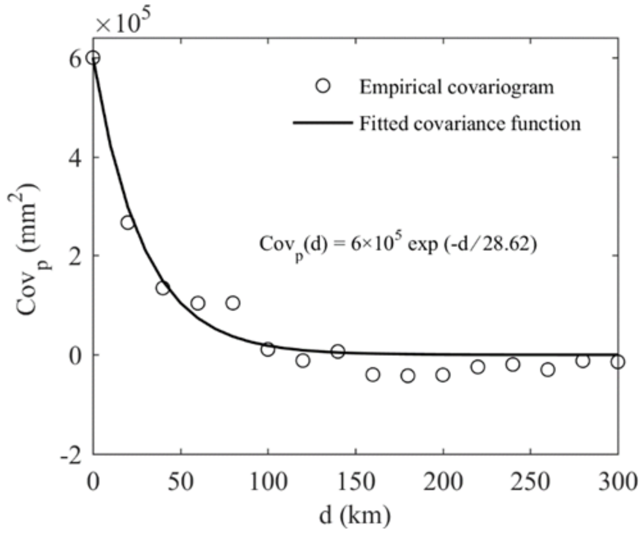
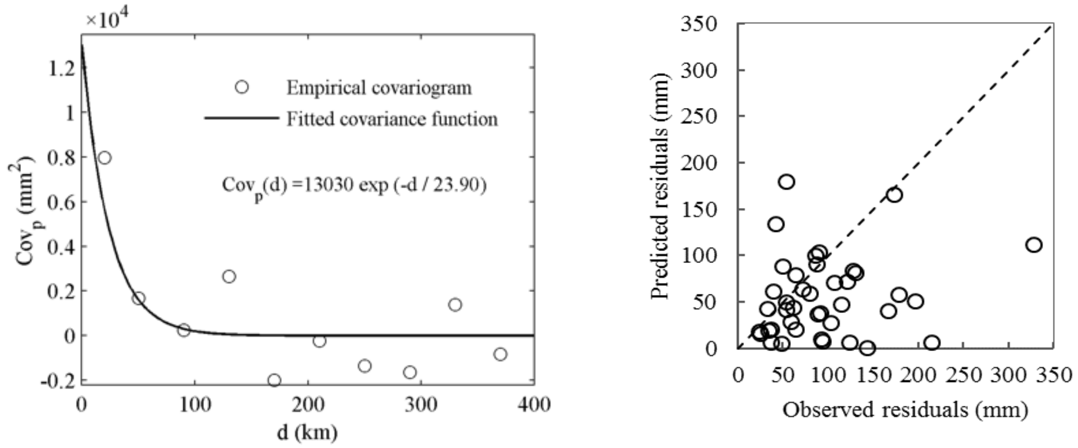


Figure 4: Empirical covariogram ($Cov_e\,(d)$) from the sub-basin runoff data and theoretical covariogram by fitted covariance function $Cov_p\,(d)$ of the study area.



(a) (b)

Figure 5: (a) Empirical covariogram ($Cov_e(d)$) from the residual $R_s(x)$ and theoretical covariogram by fitted covariance function $Cov_p(d)$ of the study area. (b) The scatterplot of the predicted vs. the observed residuals.

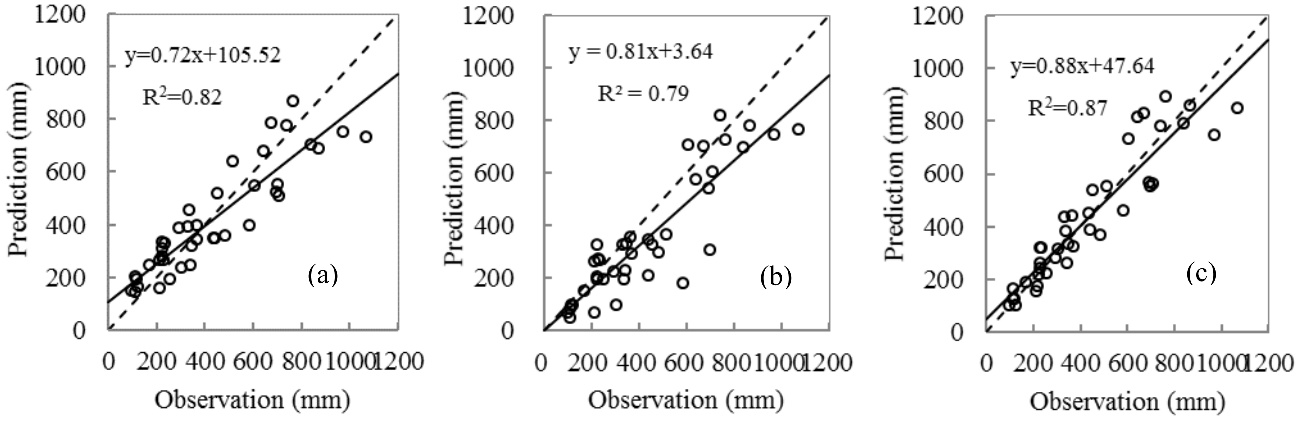


Figure 6: Cross validation of the predicted runoff vs. the observation by (a) Budyko method, (b) hydro-stochastic interpolation, (c) our coupled method, and (d) the scatterplot of the predicted vs. the observed residuals for (c). The dashed-line is 1:1.

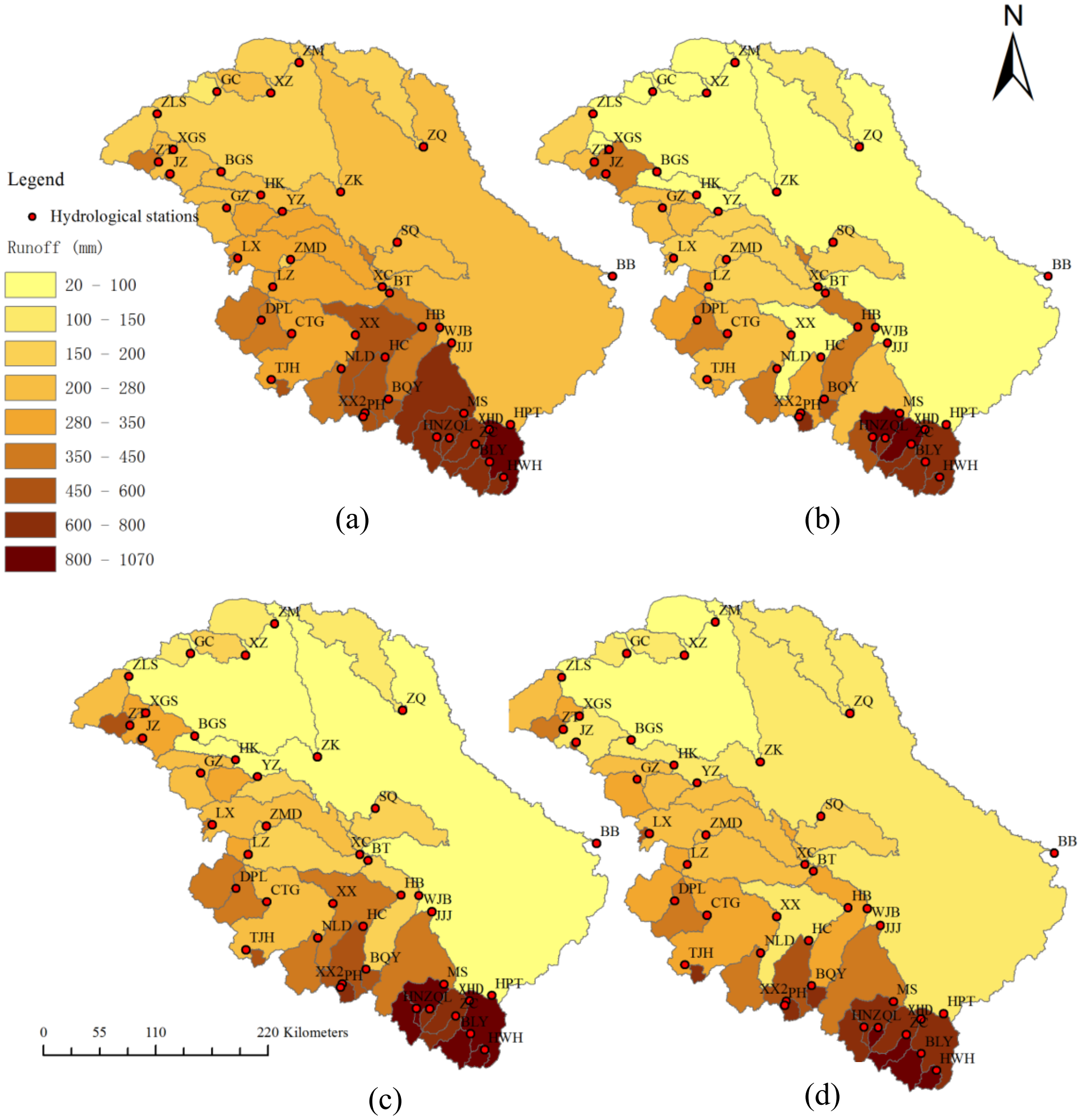


Figure 7: Spatial distribution of the mean annul runoff estimated from (a) Budyko method, (b) hydro-stochastic interpolation, (c) our coupled method, and (d) the observation.