Response to the comments of Dr. Alberto Bellin from the Short Comment published on September 12, 2017 concerning the manuscript with reference number: hess-2017-394.

We would like to thank Dr. Bellin for his insightful comments. Our responses to the specific points raised by him are provided below. Please note that his comments will be presented in italics, preceded by a "**C**", while our responses will be presented in normal typeface with a blue font, preceded by an "**R**". For some responses, text which was changed or added to the manuscript (based on suggestions from the comments) is quoted and placed under "Specific author changes". Please note that the pages and line numbers provided in this document are from the original version of the manuscript.

C1: *In this manuscript the authors present an illustrative example of the Fokker Planck equation associated to the De Saint Venant equation for a spatially uniform but random roughness coefficient.*

R1: We thank Dr. Bellin for his review of this companion paper.

C2: *As I already commented in my review of the previous manuscript I think that the material is not enough to justify a two-paper series. Therefore, my suggestion is for merging the two manuscripts into one. The first part (up to page 5) of the second manuscript summaries what already presented in the first one and is not longer needed in the merged manuscript, while the second part can be easily merged with the first manuscript.*

R2: We again thank Dr. Bellin for his comment and concern regarding this matter. However, as we have mentioned in our reply to comment **C2** from his review of our companion paper (Referee #2 for hess-2017-393), we still believe in the ability for both manuscripts to remain as two standalone, companion papers. We would also like to note that neither Referee #1 of the companion paper, nor Referees #1 and #2 of this manuscript have presented any concerns regarding having two standalone manuscripts, nor have they presented the desire to join the two manuscripts into one. Below is our original reply to Dr. Bellin's comment **C2** from our companion paper regarding the same matter:

> *"We thank the referee for the comment. The ensemble-averaging technique used in this study was indeed developed by Kavvas (2003) and this technique has been applied to other processes with different governing equations where Fokker–Planck Equations (FPEs) specific to those processes were obtained and applied successfully. However, this technique had never been applied to the Saint-Venant equations to tackle the stochastic unsteady open-channel flow problem. As such, the novelty of the proposed FPE methodology that was developed in this manuscript was, firstly, to figure out how to apply the Kavvas (2003) technique to the Saint-Venant equations especially through the transformations that provided us with the state variables α and β and that allowed us to write the Saint-Venant equations as four Ordinary Difference Equations (ODEs) in the specific forms of Eqs. (15) to (18) (for this was not a straightforward process), and secondly, to go forth with developing the FPE that is specifically for the stochastic unsteady open-channel flow process, an equation which has not been developed before. Hence, this study clearly derives and presents an entirely new FPE that can be used to solve for the probability density of the state variables of a stochastic open-channel system, which is not found elsewhere in the literature. And while the numerical discretization was made following the Chang and Cooper (1970) scheme, the scheme was generalized from its original one-dimensional form and adapted to the four-dimensional FPE that was being solved in this study. Therefore, joining this manuscript and the companion manuscript*

into one manuscript, while placing a large portion of this first manuscript in the appendix, would mostly take away from the importance of these equations and from the work that was done in arriving to those equations. As such, we believe in the novelty of the equations derived in this manuscript, and thus we believe in its ability to stand as its own manuscript. We would also like to note that Referee #1, who has read and reviewed both manuscripts, has not mentioned any desire for the joining of the two papers into one, in which case we would assume that Referee #1 may not have seen any concern with them being two standalone, companion papers."

C3: *The boundary condition used at the lower end of the channel in the solution of the de Saint Venant Equation in a Monte Carlo framework are unclear and should be better specified and justified. At page 7 line 12 I read "... while at the downstream end, the channel was assumed to be hydraulically long so that the flow can be taken as normal flow, thus satisfying the Manning's equation". This sentence is unclear: what is normal flow? Mathematically the downstream condition can be of imposed water depth y, with the velocity V obtained through the Manning's equation, or imposed velocity (or water discharge) and again the Manning's equation provides the water depth. The imposed condition can be either stationary, i.e. constant, or transient. It seems from the results that the authors choose the second option (Figures 2, 3 4 and 5 shows that the ensemble mean of both y and Q changes with time at the lower end of the channel), but no details on the specific boundary conditions are provided in the text. Assuming that one of the above two boundary conditions have been selected at the lower end of the channel, this choice should reflect to the boundary condition for the stochastic variables in the FPE equation. For instance, if y is imposed its PDF is a Dirac delta, while the pdf of the velocity is related to the pdf of the roughness coefficient through the Manning's equation. In turn, the PDFs of* α *and* β *depends from the PDFs of y and V through equations (13) and (14) of the first manuscript. Similar arguments can be applied if the BC is of imposed water discharge (i.e. velocity) and the water depth is computed through the Manning's equation. What puzzles me is that the authors impose a reflection boundary at the end of the channel (line 15, page 8), which is apparently not compatible with the previous conditions and with those imposed in the MC simulations. Since Figures from 2 to 5 show clearly that the water wave interacted with the downstream boundary, I am expecting that the boundary condition here should have an impact on the solution. This reinforce the need to select compatible boundary conditions in both models.*

R3: We thank Dr. Bellin for this comprehensive and crucial comment regarding the downstream boundary condition (BC). We certainly agree with his final sentiment regarding the importance of selecting a compatible downstream BC for both the MC and FPE models. Regarding the first point Dr. Bellin raises, it is true that the downstream BC may be imposed as water depth (y) or as velocity (V). However, to add to this, we provide a quote from the book *Open Channel Hydraulics* by Sturm (2001) specifying that "the downstream boundary condition in a flood routing problem also might be a stage or discharge hydrograph, but in some cases, it could be a depth-discharge relationship." Hence, concerning the last part of Page 7: Line 12 which Dr. Bellin indicated, what we meant by normal flow was that the Manning's equation is assumed valid at this downstream boundary, and that our downstream BC is not a specified/imposed depth or velocity, but instead it is the depth-discharge relationship represented by the Manning's equation. The way we computed the depth and velocity downstream will be explained in the following paragraph.

Recall that the manner with which we solve our MC simulation is through the method of characteristics, which is explained in detail in Viessman et al. (1977) and Sturm (2001). We describe here, in short, the steps used in order to compute the values of the flow variables at the downstream boundary. When using the method of characteristics, the solution is usually found at the intersection of the forward (C_1)

and backward (C_2) characteristic curves. The point of intersection is denoted by *P*. However, when calculating the values for the downstream boundary condition, only C_1 is used (Eq. (1)), along with its compatibility equation (Eq. (2)). Now, for the downstream BC, the unknown value is at the intersection of C_1 with the downstream boundary (again this point is denoted by *P*). C_1 originates from a point from the previous time step, and this point (denoted by *L*) is upstream of point *P*. We use the known flow variables at point *L* (V_L and y_L) from the previous known time step (t_L) located at a known location (x_L) in order to compute t_P from the equation of C_1 (Eq. (1)). Then, from the compatibility equation along C_1 (Eq. (2)), we get a nonlinear expression in depth (y_P) which results from the substitution of V_P with its expression from Manning's equation as a function of y_P. As such, this nonlinear expression is solved to find y_P which is then used to compute V_P. Therefore, while no imposed values for y_P and V_P are set at the downstream boundary, the expression of the Manning's equation in combination with the equations corresponding to the forward characteristic curve C_1 are used to determine these variables.

Since none of the flow variables (*y* or *V*) are imposed at the downstream boundary conditions, we do not need to specify any Dirac delta function for these variables at the downstream boundary. Moreover, we would like to mention that the reflecting boundary conditions for the FPE method were used to describe the boundaries of the α and β dimensions, as mentioned in the original manuscript on Page 8: Lines 17-19. However, for the upstream boundary condition, the probability densities were known. Finally, the downstream boundary condition for FPE was formulated in a way that would replicate the downstream BC of the MC model with its depth-discharge relationship (Manning's). Moreover, in the manuscript we note that we also extend the downstream boundary condition much further than the required reach length in the downstream direction in order to eliminate any of its effects on the numerical solution.

<u>Specific author changes</u>

The manuscript text describing the downstream boundary conditions for the MC and FPE simulations have been adjusted, and the new versions are shown below.

The text of Page 7: Lines 11-13 (for MC) now reads as follows:

> *"As for the boundary conditions, the flow hydrograph at the channel entrance was given, while at the downstream end, the channel was assumed to be hydraulically long so that the flow can be taken as normal flow, thus satisfying Manning's equation. As such, the downstream boundary condition was chosen as the depth-discharge relationship represented by Manning's equation. This equation, along with the equations corresponding to the positive characteristic (i.e., Eqs. (1) and (2)), was used to compute the flow variables at the downstream boundary."*

The text of Page 8: Lines 20-22 (for FPE) now reads as follows:

> *"As for the upstream boundary, recall that the discharge hydrograph was assumed to be known upstream, in which case the probability densities at the upstream boundary of the x–dimension were known for all t > 0. Finally, the downstream boundary in the x–dimension was formulated to replicate that of the MC model, and it was extended downstream much further than the required length of the reach so as to eliminate any of its effect on the numerical solution."*

C4: *In addition, given the boundary condition of the FPE at the initial section I was expecting here the standard deviation of the water discharge equal to zero, as in the MC simulations. However, this is not the case, as shown in Figures 8 and 9. This unexpected result needs justification.*

R4: We kindly refer Dr. Bellin to our reply to **C10** of Referee #1 of this manuscript, which includes our full response regarding the same matter.

References

Sturm, T. W.: Open channel hydraulics, McGraw-Hill series in water resources and environmental engineering, McGraw-Hill, Boston, 493 pp., 2001.

Viessman, W., Knapp, J. W., Lewis, G. L., and Harbaugh, T. E.: Introduction to hydrology, 2nd ed., Series in civil engineering, IEP-Dun-Donnelley, Harper & Row, New York, 704 pp., 1977.