## **Supporting Information**

## Evaluation of statistical methods for quantifying fractal scaling in water quality time series with irregular sampling

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**Figure S1**. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 0.01, \mu = 1$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



Figure S2. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 0.1, \mu = 1$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



**Figure S3**. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 1, \mu = 1$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



**Figure S4**. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 10, \mu = 1$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



Figure S5. Comparison of standard deviation in estimated spectral slope in irregular data that are simulated with varying prescribed  $\beta$  values (30 replicates), series length of 9125, and mean gap interval of 2 (*i.e.*,  $\mu = 1$ ).



Figure S6. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 0.01$ ,  $\mu = 14$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



Figure S7. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 0.1, \mu = 14$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



**Figure S8**. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 1, \mu = 14$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



**Figure S9**. Comparison of methods for estimating spectral slope in irregular data (30 replicates) that are simulated with varying prescribed  $\beta$  values, series length of 9125, and NB ( $\lambda = 10, \mu = 14$ ) distributed gap intervals. The blue dashed lines indicate the true  $\beta$  values.



**Figure S10**. Comparison of standard deviation in estimated spectral slope in irregular data that are simulated with varying prescribed  $\beta$  values (30 replicates), series length of 9125, and mean gap interval of 15 (*i.e.*,  $\mu = 14$ ).



**Figure S11**. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for total nitrogen (TN) at the nine Chesapeake Bay monitoring sites. See **Table 1** for site and data details.



**Figure S12**. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for total phosphorus (TP) at the nine Chesapeake Bay monitoring sites. See **Table 1** for site and data details.



Figure S13. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for nitrateplus-nitrite ( $NO_x$ ) at the six Lake Erie and Ohio River monitoring sites. See **Table 1** for site and data details.



**Figure S14**. Histogram of concentration residuals from the WRTDS method, expressed in natural log concentration units, for total phosphorus (TP) at the six Lake Erie and Ohio River monitoring sites. See **Table 1** for site and data details.