Authors' response to Referee #2

For clarity, authors' responses are inserted as blue text.

I find the manuscript is well-written and technically rigorous, with results that can be generalized beyond hydrologic time series. This manuscript tackles a challenging and highly relevant topic - the quantification of fractal scaling behavior for irregularly sampled data - and provides needed synthesis on the most promising methods to estimate this behavior. For these reasons, I recommend the manuscript be accepted subject to minor revisions.

Response: Thank you for these comments.

I do, however, have a number of comments that would help improve clarity of the manuscript and emphasize the more practical aspects of this work.

Major comments:

1) Lines 127-129: It would be interesting to the reader and for understanding the important contribution of this work to detail the effects of non-normal data and persistence, seasonality, and the presence of long-term trends on the estimation of Beta.

Response: We agree this would be potentially useful to do, but it is simply beyond the scope of the paper -- new modeling experiments for each of these effects would multiply the length and complexity of the paper by large factors. We recognize this would be an important area for future research, which we explicitly put in the Section 3.3 (lines 459-464): "*such real data are typically much more complex than our simulated time series, because of (1) strong deviations from normal distributions and (2) effects of flow-dependence, seasonality, and temporal trend (Hirsch et al., 1991; Helsel and Hirsch, 2002). In this regard, future research may simulate time series with these important characteristics and evaluate the performance of various estimation approaches, perhaps following the modeling framework described herein.*"

2) Lines 264-265: It is noted that the results which demonstrate that the approach used in this manuscript to mimic the sampling irregularity performs well as compared to other simulation methods are not shown. I think these results are important to show, as this approach is what underlies the remainder of the analysis of the methods. This can be added to the supplementary material.

Response: Thanks for this suggestion. We will provide these results in the supplementary material.

3) There are a large number of interpolation methods (n=11) presented here. I would argue that some of these methods are not very realistic in the context of what one would experience in terms interpolation for irregular samples. Unless the authors provide sound technical justification for each scenario, I would consider removing scenarios that would not generally be considered in standard practices (examples are scenarios B3, B4, and select a smaller subset of LOESS smoothing parameter values). This would also streamline the results and text.

Response: The interpolation methods were selected not only on the basis of their frequency of use, but to ensure a certain degree of completeness -- we felt it would ultimately be more useful to include obvious variations of common methods than to exclude one that someone might have been considering and looked to our paper for guidance. Although methods B3-B5 seem not plausible, they have been discussed and used in the scientific literature (e.g., Blankers et al. 2010; Graham 2009). Furthermore, some R packages have been developed (e.g., "na.locf"), making these methods readily available to general users without any prior knowledge on the methods' performance. Therefore, we think it is worthwhile to keep the results and discussion of these methods' performance, which is a useful contribution to the topic of "missing data analysis." Furthermore, removing one or two methods would do little to shorten or simplify the paper at this stage, so we have chosen not to.

4) Line 412: For Monte Carlo analysis, average values of the simulated parameter of interest are computed from sample sizes of 100 or more - not 30. Was this tested in your experiments?

Response: We used 30 samples because it was quite sufficient to constrain estimates of the average in most cases. Standard error of the mean beta for most methods is much smaller than the variation between methods. Nonetheless, to follow common practice, we will adopt this suggestion and modify the figures and text based on 100 runs.

<u>Minor clarification comments:</u>

1) Lines 3-4: Consider adding a phrase or sentence to explain why spectral slope is important to trend detection.

Response: We will add the phrase "to avoid false inference on the statistical significance of trends" at the end of this sentence.

2) Line 15: Is the "modified form" being newly introduced here? Or does it already exist. Clarify.

Response: It is a published method, see method C1b in Section 3.1 where it is introduced. For clarity, we will revise "a modified form" to "a previously-published modified form".

3) line 38-39: The fact that ACF is summable seems a non-sequitor here. It is later that the connection is made to summability and the presence of fractal behavior. Perhaps it is not necessary to comment on the summability of the ACF?

Response: We will delete the comment on "summability".

4) lines 90-103: Moving this paragraph to the end of Section 1.1 would provide more immediate clarity as to the scope and value of this work.

Response: Thanks. It will be moved to the suggested location.

5) Line 158: Consider the of the work "interpolating" instead of "modeling"

Response: We will keep the word "modeling".

6) Section 2.1: Highly clever way to define sampling irregularity.

Response: Thank you for this comment.

7) line 216 (and throughout): I do not think "gappy" is a word and chuckled at its appearance. Please replace with "irregularly-spaced".

Response: We will replace the word "gappy" with "irregularly-spaced" throughout the manuscript.

8) line 237: Please be more specific in how you arrived at this equation for the shape parameter.

Response: As the line states, $\lambda = \mu^2/[\mathrm{var}(\Delta t^*) - \mu] = (\mathrm{mean}(\Delta t^*) - 1)^2/[\mathrm{var}(\Delta t^*) - \mathrm{mean}(\Delta t^*) + 1]$. The first equality represents a well-established property for the negative binomial distribution. The second equality is achieved through the substitution of μ by "mean(Δt^*) – 1".We think it is already clear and hence further modification is not necessary.

9) line 247 (as an example): Please add units to values provided in this section and throughout. This will help the reader follow the results and methods.

Response: Units will be added throughout for $\Delta t_{average}$ (days). Note that λ and μ are fitted (negative binomial distribution) parameters for the *non-dimensionalized* time series (Δt^*) -- see Section 2.1 -- therefore they do not have units.

10) line 473: Hirsch and DeCicco (2015) is the reference to the user manual for WRTDS. The method itself is explained in Hirsch et al. (2010). I would cite the original paper.
Hirsch, R. M., Moyer, D. L. and Archfield, S. A. (2010), Weighted Regressions on Time, Discharge, and Season (WRTDS), with an Application to Chesapeake Bay River Inputs. JAWRA Journal of the American Water Resources Association, 46: 857–880. doi: 10.1111/j.1752-1688.2010.00482.x

Response: Reference will be corrected to Hirsch et al. (2010).

References Cited

Blankers, M., Koeter, M. W., & Schippers, G. M. (2010). Missing data approaches in eHealth research: simulation study and a tutorial for nonmathematically inclined researchers. Journal of medical Internet research, 12(5).

Graham, J. W. (2009). Missing data analysis: Making it work in the real world. Annual review of psychology, 60, 549-576.