The authors would like to thank the reviewer for their comments and questions. The remarks made by the Reviewer are written in italics, and the replies in normal font.

Reviewer #1

Thank you for the chance to review this manuscript. The manuscript is generally well written. However, there are a number of issues that need to be resolved before this manuscript can be accepted for publication. 1. Innovation and contribution of the paper needs to be better defined.

Thank you for your review of our manuscript. A statement addressing why the study is important is included in the abstract on page 1 Lines 1-4. The introduction has also been reworded to clarify this. To be brief, this study is a novel step towards estimating errors associated with input uncertainty and model structure.

The authors compared two methods commonly used in signal processing (i.e. DCT and DWT) for reconstructing rainfall information. But why is this study important?
The introduction has been reworded and restructured to clarify why the study is important. In summary, the use of model input data reduction allows modern parameter estimation algorithms to more efficiently estimate errors associated with input uncertainty and model structure.
And why these two methods were selected? Are these two methods better than currently used methods?
A brief reasoning for choosing these two of the many possible transforms is that they are the two most commonly used transforms for model input data reduction techniques in other fields. A more comprehensive reasoning for selecting these two methods is addressed on page 3 lines 4-16. Model Input Data Reduction is currently not performed in hydrology, however the techniques are the most frequently used in other fields.
What about other methods used in signal processing, such as short-time Fourier transform (STFT)?

The Discrete Cosine Transform (DCT) is a version of the STFT. In the manuscript on lines 32-33 page 2 we have stated that the Windowed Fourier Transform (WFT) is also sometimes referred to as the STFT. We also mention on line 10 page 5 that the DCT is a version of the WFT.

2. Description of experiment design is not very clear. [1] Can you please use a flow chart to illustrate the steps taken during the experiment? If space is of concern, this reviewer recommends to remove current Figure 1, which is not described in detail and does not have much value.

To avoid duplication when presenting the experiment design the authors would prefer not to include a flow chart. However, if it would be deemed necessary, we are certainly willing to provide this. To clarify the steps undertaken, more detail has been added to the Experiment Design section, more specifically on pages 7 lines 16-25. If further clarification is needed is it possible to outline in detail outline which areas of the experiment design are not clear? The caption of Figure 1 has been expanded to provide more detail. This is discussed further in a later comment.

[2] What is the role of stream flow in this study? In the last paragraph on page 7, PE (peak error) is defined as "the peak streamflow error over the 10 year period". Can the authors explain how this error is calculated? How is this error linked to reconstructed rainfall and the performance of the two methods? Are rainfall-runoff models used? If so, these rainfall-runoff models need to be described in the experiment design section. All these information can be included in the flow chart mentioned above, it will help the readers to understand the experiment process.

The mention of streamflow is a typographical error. Thank you for pointing out this error. In the manuscript Peak error in fact refers to peak rainfall error.

[3] The two methods were not validated – please refer to comment 3.3. 3. Results analysis [1] It is obvious that DWT performs better than DCT from the results obtained. But why is this the case? Is this because the nature of cosine functions oscillating at different frequencies makes DCT unsuitable for rainfall signals that is not cosine in nature? If this is the case, it comes back to my comment 1 above, why is DCT selected for this study at the first place?

It was clear prior to the study that the DCT would not perform as well as the DWT in reconstructing hydrologic data which are generated by transient mechanisms. This was discussed on page 3 line 13-15. Yet the use of Fourier transforms remains prevalent in hydrologic studies. Consequently, some of the reasons for the selection of the DCT are as follows:

1. To demonstrate that the DCT and Fourier based transforms are not the best transform to use for transformations involving hydrologic rain gauge data,
2. As a baseline from which to compare the DWT to,
3. In the literature it is a commonly used transform for model input data reduction.

[2] Figure 4 is a scatter plot of RSR generated using the two methods. It is obvious that they are linearly correlated and the RSR from DWT is always lower than that from DCT. However, what evidence included in this figure show that "DWT is able to reconstruct the input rainfall signal" (line 23 page 8)?

It was never intended for Figure 4 to demonstrate that the DWT is able to reconstruct the input rainfall signal. This information is outlined in Section 2.3. The RSR that is mentioned throughout the text is the RMSE/standard deviation of the reconstructed signal when compared to the observed signal.

[3] The authors claimed that in this study the two methods were "validated" "using several simulation performance summary metrics". Line 24, page 3. This reviewer disagrees. In this study, the performance of the two methods was evaluated using a number of different metrics; however, no independent validation was conducted.

Thank you for pointing this out, the wording has been corrected to evaluate, page 3 line 23, page 10 line 614

Minor comments: 1) There a few typos throughout the manuscript. For example, Line 7, page 2: "prediction uncertainty" should be "prediction of uncertainty"; Line 22, page 8 "is always able reconstruct" should be "is always able to construct".

The first instance is not a typo, it is both parameter and prediction uncertainty that the authors refer to. The second typo has been fixed.

2) Line 12 page 1: The sentence does not really make sense here. "Unfortunately, errors in rainfall time series data may lead to hydrological model parameter estimates that produce adequate streamflow simulations during calibration".

To provide more clarity this sentence has been rephrased to "Unfortunately, errors in rainfall time series data may lead to hydrological model parameter estimates that produce adequate streamflow simulations only during the calibration period ". Line 12 page 1.

3) Figure 1 has only symbols, which is rather confusing. Please add descriptions in both the figure and caption so the figure stands alone and makes sense.

The Figure 1 caption has been updated to provide a stand-alone explanation:

"A schematic showing the pyramid algorithm used to decompose and down sample ($\downarrow 2$) an input signal ($\hat{\mathbf{R}}$) into high and low frequency components. The input signal is filtered using the high and low pass filters described in Equations 7 & 8 before being down sampled to produce the level one high and low pass parameters. The low pass parameters are now used as input for the high and low pass filters. This process of filtering and down sampling is repeated until the desired level of decomposition is met."

Reviewer #2

The paper presents a comparison of two methods of model order reduction, cosine transform and wavelet transform. It is general a smooth good read. However, the contribution of the author is not clear as the similar comparison has been investigated before in fields other than hydrology (please refer to the papers on the model order reduction methods for the modulation schemes on communication channels for example). I think the authors need to clearly state their contribution to this paper. Other than that, the authors have to compare the results with other methods of model reduction, i.e. projection-based methods.

The manuscript presents a comparison of two methods of model input data reduction, the discrete cosine transform and the discrete wavelet transform, in a hydrological context. The novelty of the paper is that these transforms are discussed in a hydrological context. This is clarified on page 3 line 19-23. Reasons for selecting these two transform are addressed on page 2, lines 29-35 and page 3 lines 1-16. There are a large number of transforms that could be used, the authors chose the two most common transforms that are used for model input data reduction techniques in other fields.

We would also like to clarify that methods of model order reduction are a related field, yet are outside the scope of this paper.

Reviewer #3

OVERVIEW The study investigates the use of Discrete Cosine and Wavelet transforms for the reduction of input data dimensionality in hydrological modelling. GENERAL COMMENTS I am reviewing the paper after reading the comments raised by previous reviewers on the interactive discussion. As specific comments were already given by previous reviewers, I included here only my general comments for the paper. The paper topic seems to be relevant for the HESS readerships. However, I found some important issues that need to be addressed before the publication.

1) It is not clear to me how the DCT and DWT methods are applied. If I well understood, for each basin the authors used streamflow and precipitation data, together with a hydrological model, for applying DCT (and DWT) and thus reducing the dimensionality of precipitation data. However, no hydrological model is mentioned in the paper. What are the input and the output data? Is a hydrological model used? Can the procedure be applied in a testing period? What is the targeted application of the proposed approach (something is mentioned in the introduction, but needs clarifications)? All these questions need to be addressed. Otherwise, I have not clear why the study is relevant for the hydrological community.

We would like to clarify that the authors have only used rainfall data, there is no mention of streamflow data in the Data Set section as streamflow has not been used. The input data is rainfall and the output data is rainfall represented by a smaller number of parameters than the number of rainfall observations. The authors now state that no streamflow data are used on page 7 line 9.

No hydrological model is used, this is now clarified on page 7 line 11.

It is now stated in the Experiment design that there are no calibration of evaluation periods.

To clarify the targeted application, sections of the Abstract, Introduction Model Input Data Reduction Theory sections have been reworded and restructured. In summary, the reduction of model input data allows input data such as rainfall to be reduced to a small number of parameters. Using modern parameter estimation algorithms, the representation of rainfall as parameters allows for the uncertainty in input data to be explored.

If there are specific lines that are unclear, we would like to ask the reviewer to make these known such that modifications can be made to the manuscript. The authors acknowledge that some ambiguity may have arisen due to a typo in which peak streamflow error was mentioned. We apologize for this and would like to clarify that it is peak rainfall error.

2) An important issue in the analysis of rainfall time series is related to the zeros, i.e., days with no rainfall. By looking at the results, good performances are obtained for POP values larger than 30-40

The analysis of no rainfall days is indeed important when analysing rainfall time series. As a larger Percentage of Original Parameters (POP) is used, it is expected that the reduced rainfall product correctly represents more days in which no rainfall was observed. Obtaining the best possible representation of rainfall using a minimal number of parameters is a primary concern for model input data reduction. Consequently, it is the reduced rainfall products that have a POP of 40% or less that are of interest to this study.

3) Besides the performance metrics related to precipitation, also the peak discharge error is mentioned. However, it is not clear how it is computed (see also comment 1). If a hydrological model is used, it should be mentioned. I expect that results depend also on the quality and reliability of discharge time series. If yes, it should be investigated and discussed. All these information are totally missing in the current version of the paper and should be added.

As was mentioned earlier, the authors made a typo when referring to peak streamflow error. We apologise for any confusion caused. The results shown do not depend on the discharge time series.

4) Some parts of the paper seem to be written quickly without much attention. Therefore, typos and grammatical errors are present. I suggest a detailed review of the whole Discussion paper text, and of the figures (e.g., y-axis labels in Figure 2 are wrong).

We have conducted a detailed review of the entire paper and Figure 2 is now amended.

RECCOMMENDATION On this basis, I found the topic of the paper relevant, but as I mentioned above, the analysis and the text need major revisions before the possible publication on Hydrology and Earth System Sciences.

A Comparison of the Discrete Cosine and Wavelet Transforms for Hydrologic Model Input Data Reduction

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Abstract. The treatment of input data uncertainty in hydrologic models is of crucial importance in the analysis, diagnosis and detection of model structural errors. ~~Model input data~~ Data reduction techniques decrease the dimensionality of input data, thus allowing modern parameter estimation algorithms to more efficiently estimate errors associated with input uncertainty and model structure. The Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT) are used to reduce the dimensionality of observed rainfall time series ~~observations from~~ for the 438 catchments in the MOdel Parameter Estimation eXperiment (MOPEX) data set. The rainfall time signals are then reconstructed and compared to the ~~measured~~ observed hyetographs using standard simulation performance summary metrics and descriptive statistics~~as well as peak discharge errors~~. The results convincingly demonstrate that the DWT is superior to the DCT ~~and best preserves and characterizes~~ at preserving and characterizing the observed rainfall data records. It is recommended that the DWT be used for model input data reduction in hydrology in preference over the DCT.

1 Introduction

Rainfall uncertainty is the biggest obstacle hydrologists face in their pursuit of accurate, precise and timely streamflow forecasts (McMillan et al., 2011). Unfortunately, errors in rainfall time series data may lead to hydrological model parameter estimates that produce adequate streamflow simulations ~~during calibration~~ only during the calibration period (Beven, 2006). This can lead to poor quality streamflow predictions for independent periods, and low confidence in the ability of streamflow forecasts~~for short and long term forecasts~~. Consequently, a precise and accurate representation of rainfall uncertainty is paramount for robust ~~parameter estimation~~ hydrological model parameter estimation and streamflow forecasting and Quantitative Precipitation Forecasts (QPFs). ~~Furthermore,~~ Robertson et al. (2013) and Shrestha et al. (2015) have demonstrated that skill can be added to QPFs by postprocessing with past observations. As such, skill can be added to QPFs and consequently flood forecasts, through developing a greater understanding of rainfall uncertainty.

The propagation of input errors in rainfall runoff modelling impedes the hydrologic community's ability to validate model structural error. Despite the vast amount of literature on rainfall measurement, estimation, statistical analysis (Testik and Gebremichael, 2010) and quality control procedures (World Meteorological Organization, 2008), a shroud of uncertainty still surrounds how rainfall and its associated uncertainty should be addressed in rainfall runoff modeling. The implementation of uncertainty analysis in many hydrological applications is also often limited by computational power.

Recent advancements in computational power as well as remote sensing have led to considerable improvements in data availability and quality of hydrological observations (Cloke and Pappenberger, 2009). These improvements can be leveraged to increase the hydrological and flood forecasting knowledge base and consequently provide water policy decision makers and emergency management services with higher quality information.

The advancement of computational power has also aided the search for optimal model parameters hydrological model parameters that optimally simulate hydrological observations. These approaches intially focussed on finding only the global optimum values of the parameters for a given objective function (Duan et al., 1994; Gan and Biftu, 1996; Thyer et al., 1999). In However, in the past two decades the interest has switched to the assessment of it has been recognized that the uncertainties in model parameters and predictions need to be estimated. Methods that seek to estimate parameter and prediction uncertainty . Examples of such methods includeinclude: Bayesian recursive parameter estimation (Thiemann et al., 2001), the limits of acceptability approach (Beven, 2006; Blazkova and Beven, 2009), the BAyesian Total Error Analyis (BATEA) framework (Kavetski et al., 2006a, b; Kuczera et al., 2006; Thyer et al., 2009; Renard et al., 2011), the Simultaneous optimization and data assimilation (SODA) (Vrugt et al., 2005), the DREAM algorithm and its variations (Vrugt et al., 2005, 2008, 2009a, b; Vrugt and Ter Braak, 2011; Laloy and Vrugt, 2012; Sadegh and Vrugt, 2014), Bayesian model averaging (Butts et al., 2004; Ajami et al., 2007; Vrugt and Robinson, 2007), the hypothetico-inductive data based mechanistic modeling framework of Young (2013) and Bayesian data assimilation (Bulygina and Gupta, 2011). It is through the development of these parameter estimation algorithms that hydrologists are able to explore input uncertainty.

Kavetski et al. (2006b) and Vrugt et al. (2008) identified the need represent true catchment rainfall and its associated uncertainty using parameters, both applied a rainfall parameteric approach to estimating true catchment rainfall and its associated uncertainty using a rainfall multiplier to storm events. The use of a parametric representation of rainfall with an effective sampler has sampling algorithm provides the ability to jointly estimate hydrologic model parameter distributions as well as input uncertainty. As in most hydrological problems there is a lack of sufficient data to obtain a unique solution. However, Kavetski et al. (2006b) and Vrugt et al. (2008) found there was insufficient data to uniquely identify both the uncertainty in hydrological model parameters and rainfall input, because estimating a rainfall multiplier at each time step made the inference underdetermined. Thus, estimating a rainfall multiplier for each time step further complicates the task at hand by making the problem more underdetermined. Data reduction transformations offer the potential to reduce the dimensionality of the parameter estimation problem to enable and thus enabling a more robust inference. The combination of the use of model input data reduction techniques with parameter estimation algorithms allows links to be explored between rainfall input error, QPF postprocessing algorithms and errors associated with model structure, parameter estimation, and systematic and random errors associated with observations. Signal transformssuch as the Discrete Cosine Transform (DCT) and the Discrete Wavelet Transform (DWT) can be used as tools for model input data reduction.

Kumar and Foufoula-Georgiou (1997) introduced wavelet analysis to the geophysical sciences. Schaefli and Zehe (2009) analyzed hydrological model performance and conducted parameter estimation in the wavelet domain and Nalley et al. (2012) used

DWTs to analyze rapidly and slowly changing events in both streamflow and precipitation time series. Montanari and Toth (2007) discussed the opportunities calibration of hydrologic models in the spectral domain offers to ungauged basins and Pauwels and De Lannoy (2011) used a discrete Fourier transform to calibrate water and energy balance models in the spectral domain, whilst De Vleeschouwer and Pauwels (201 the rainfall runoff model known as the Probability Distributed Model (PDM) in the spectral domain. To date there have been no instances in which the suitability of different transformshas been assessed for their use as a tool in hydrologic model input data reduction. Further, model input data reduction has not been used to infer hydrologic input uncertainty. Consequently, other fields need to be looked at for examples of model input data reduction . Bruce et al. (2002) have highlighted the efficiency of the DWT to reduce hyperspectral data and automatically classify ground vegetation from hyperspectral data. Studies in geophysics and hydrogeology have used the DCT (Jafarpour et al., 2009, 2010; Linde and Vrugt, 2013; Lochbühler et al., 2014, 2015), Discrete Wavelet Transform (Davis and Li, 2011; Jafarpour, 2011) and Karhunen-Loève expansion (Dostert et al., 2006, 2009; Marzouk ar transforms, such as Fourier and wavelet transforms, are examples of data reduction transformations that have been applied in hydrology, however they have not previously been used to reduce the 2-D and 3-D parameterization of subsurface structures before model inversion takes places. Amongst many other applications, DCTs and DWTs have been used extensively in image and signal processing. dimensionality of input data.

The motivation for the development of wavelets grew out of limitations of the Fourier transform and it's respective derivations. Fourier Fourier transforms use sinusoidal functions to represent the spectral component of an input signal, thus a periodic signal could be represented using a smaller number of Fourier coefficients than the number of input data points. A pitfall of the Fourier transform is that it represents the spectral components of a signal, without any indication of the time localization of those specific spectral components. In order to account for this, the Windowed Fourier Transform (WFT), sometimes referred to as the short time Fourier transform, segments the signal into discrete time windows before performing the Fourier analysis. A major drawback to this approach is that the uncertainty principle of signal processing imposes a limitation on the time and frequency resolutions that can be obtained for a given signal. As a response to this Daubechies (1990) produced discrete basis functions with good time and frequency localization. In conjunction with the pyramid algorithm, as described by Mallat (1989), This this work formed the basis for multi-resolution analysis with the Discrete Wavelet Transform DWT (Polikar, 1999). The pyramid algorithm DWT decomposes an input signal into high and low frequency components.

Wavelet analysis was first introduced to the geophysical sciences by Kumar and Foufoula-Georgiou (1997) and has been adopted for several different applications. Wavelet analysis has been used to assess the performance of hydrological models for parameter estimation (Schaefli and Zehe, 2009) to analyze changes at over different time periods for both streamflow and preciptiation data (Nalley et al., 2012). Various spectral methods have also been applied in hydrology, including the application of discrete Fourier transforms to calibrate water and energy balance models (Pauwels and De Lannoy, 2011), and for the calibration of the conceptual rainfall runoff model known as the Probability Distributed Model (PDM) (De Vleeschouwer and Pauwels, 201 While wavelet and spectral methods have been applied in the hydrological sciences, to date there have been no instances in which the suitability of different transforms has been compared for hydrological data reduction applications. Labat (2005) has pointed out that Fourier transforms and their derivatives are not well suited to reconstruct hydrologic data, which are generated by transient mechanisms. This is due to the Fourier transforms poor capability to represent sporadic high frequency events when dimensionally reduced. If model input data reduction techniques are to be accepted by the hydrologic community it is of critical important that the transform used is able to reconstruct transient events. Through a comparative study it will be shown that DWTs are a good multi-resolution alternative to the Discrete Cosine Transform DCT.

Traditionally, transform coefficients are the result of a convolution operation on an input signal. However, the aim of model input data reduction is to estimate these transform coefficients. Hence, they shall be referred to as transform parameters from herein. This paper provides ~~a comparison~~ novel theoretical and numerical comparisons of the DCT and DWT ~~and~~ in a hydrological context. The ability of both transforms to reproduce key components of hydrological data sets are investigated. The extent to which each transform can reproduce hydrologic data using a decreasing number of parameters will serve as a metric upon which their ability to be used as a tool for model input data reduction for hydrological data will be evaluated. To address the requirements for hydrologic model input data reduction, this paper ~~addresses~~ details (i) theoretical differences between the DCT and DWT, (ii) methodologies to reduce input rainfall to parameters, and (iii) ~~validation~~ an evaluation of the proposed methodologies using several simulation performance summary metrics.

2 Model Input Data Reduction Theory

For this study, model input data reduction theory is introduced using a lumped conceptual watershed model. Consider a non-linear model $\mathcal{F}(\cdot)$, which simulates n discharge values, $\widehat{\boldsymbol{Y}} = \{\widehat{y}_1, \ldots, \widehat{y}_n\}$ in mm/day according to

$$\widehat{\mathbf{Y}} = \mathcal{F}(\boldsymbol{\theta}, \widetilde{\mathbf{x}_0}, \widehat{\mathbf{E}}, \widehat{\mathbf{R}}), \tag{1}$$

where the model input arguments are the $1 \times d$ vector $\boldsymbol{\theta}$, with arbitrary model parameter values, the $1 \times m$ vector $\widetilde{\boldsymbol{x}_0}$ with values of the initial states in mm, and the $1 \times n$ vectors $\widehat{\mathbf{E}} = \{\widehat{e}_1, \ldots, \widehat{e}_n\}$ and $\widehat{\mathbf{R}} = \{\widehat{r}_1, \ldots, \widehat{r}_n\}$ store the observed values of the potential evapotranspiration (PET) and rainfall in mm/day, respectively. Note that $\widehat{\mathbf{R}}$ is used to represent rainfall and not precipitation, as snow, hail and other forms of precipitation are not considered. The $\widehat{\ }$(hat) symbol is used to denote measured quantities and the $\widetilde{\ }$(tilde) symbol reflects variables that are either reconstructed or could, in theory be observed in the field but due to their conceptual nature are difficult to determine accurately. If the traditional hydrological perspective in which the inputs $\boldsymbol{E}$ and $\boldsymbol{R}$ are considered to be fixed and known quantities is relaxed, and rainfall is now considered unknown, then a new inference problem arises in which the input rainfall is estimated via treating the input rainfall as a series of parameters. Inference problems in which the input is considered unknown can be dealt with using a Bayesian framework. Such inference problems have been considered by Kavetski et al. (2006a) and Vrugt et al. (2008) but are outside the scope of this paper. Consequently, for rainfall to be inferred a suitable parametric representation of rainfall must be determined. Given a daily ~~precipitation~~ rainfall data record with n observations in mm, n rainfall parameters could be used to represent the input hyetograph. This approach would be particularly elegant and parsimonious, yet for a 10 year record of daily discharge data, the inference problem would grow from d model parameters to roughly $10 \times 365 + d = 3650 + d$ parameters. These values would need to be estimated from the observed rainfall and discharge data record, respectively. As many hydrological models are already underdetermined the introduction of additional parameters would make the model even less determinable. Additionally an excessive amount of

CPU-time is required to solve for a 3600+ dimensional posterior parameter distribution. An alternative approach is therefore necessary. ~~The reduction of model input data is investigated by representing rainfall data records of the MOdel Parameter Estimation eXperiment (MOPEX) data with a much lower dimensional sparse transform.~~

Sparse transforms convey large amounts of data using fewer parameters than data points in the ~~initial~~ observed signal. An input rainfall signal can be reduced to sparse transform parameters~~. The modification of even one parameter~~ , which will allow multiple rainfall observations to be ~~altered~~modified using a single parameter. Some or all of these transform parameters can be altered before the transform is inverted to produce a new input signal for streamflow simulation and posterior analysis. The use of sparse transforms to represent input time series enables input uncertainty to be explored. ~~To adequately compare the~~ The ability of discrete wavelet and Fourier ~~based transforms, to reduce hydrologic~~ transformations to reduce hydrological input data to a set of parameters for uncertainty estimation ~~, both~~ are compared using theoretical and analytical ~~comparisons are made~~methods.

2.1 Overview of the DCT and DWT

Wavelet and Fourier transforms are invertible transforms in which a forward convolution operation can be used to decompose a signal into various components. Similarly, a backwards deconvolution operation can be applied to retrieve the original signal. Fourier based transforms decompose signals into frequency components and are best used for regular time-invariant signals that do not exhibit time specific information. Alternatively, wavelet based transforms decompose signals into frequency and time components. The advantage of using wavelet functions to transform data is that time specific information about when higher frequency components occur can be preserved.To obtain time specific information, Fourier based transforms can be applied over pre-specified temporal windows. Yet, this approach is limited by the uncertainty principle of signal processing. The uncertainty principle of signal processing imposes a lower limit on obtainable resolutions in the time-frequency domain such that

$$\sigma_{\mathrm{t}}\sigma_{\omega} \geq \frac{1}{2}, \tag{2}$$

where σ_{t} $[\mathrm{s}]$ and σ_{ω} $[\mathrm{s}^{-1}]$ are the respective temporal and frequency widths used in the sparse transform.

Applying the uncertainty principle of signal processing (Equation 2) it is clear that any attempt to narrow the temporal period analyzed to gain increased resolution in the time domain would be met by a widening of the frequency spectrum, and consequently a loss of resolution in the frequency domain.

Considering, there is no time-frequency window that is able to obtain limitless resolution in both the time and frequency domains, it is clear that an alternative solution must be found. Wavelet transforms can be used to decompose a signal into different levels that consist of different time and frequency resolution windows. Thus the wavelet transform is able to be configured to simultaneously obtain high levels of resolution in both the time and frequency domains. For a more detailed discussion on wavelets and sparse transforms the reader is referred to Mallat (2009).

2.2 Discrete Cosine Transform

The DCT (Ahmed et al., 1974) is a version of the WFT that has advantageous properties for the field of data compression. Due to the boundary conditions of the cosine function, the DCT is well suited to represent an observed input signal with a minimal number of parameters, in this case rainfall $\widehat{\mathbf{R}}(t)$. The DCT parameters $\mathbf{p}(i)$ are calculated as

$$\mathbf{p}(i) = \mathbf{w}(i) \sum_{t=1}^{\mathrm{n}} \widehat{\mathbf{R}}(t) \cos\left[\frac{\pi}{2\mathrm{n}}(2t-1)(i-1)\right], \tag{3}$$

where $i = 1, 2, \ldots, \mathrm{n}$ and

$$\mathbf{w}(i) = \begin{cases} \frac{1}{\sqrt{\mathrm{n}}}, & i = 1 \\ \sqrt{\frac{2}{\mathrm{n}}}, & 2 \leqslant i \leqslant \mathrm{n}. \end{cases} \tag{4}$$

~~This~~ The convolution process can be reversed to reconstruct the observed signal using the inverse transform

$$\widetilde{\mathbf{R}}(t) = \sum_{i=1}^{\mathrm{n}} \mathbf{w}(i)\mathbf{p}(i) \cos\left[\frac{\pi(2t-1)(i-1)}{2\mathrm{n}}\right], \tag{5}$$

where $t = 1, 2, \ldots, \mathrm{n}$.

2.3 Discrete Wavelet Transform

Using the pyramid algorithm, depicted in Fig 1, Mallat (1989) first described the decomposition of an input signal into multi-resolution components using high and low ~~bandpass~~ pass filters. Each stage of decomposition is referred to as a level. ~~The wavelet~~ An advantage of using wavelets is that decomposition can be performed using a variety of different wavelet families. This allows for signals with differing properties to be analyzed using the same methodology. The most commonly used wavelet family is the Daubechies wavelets (Daubechies, 1990). Each wavelet, within each family, consists of a scaling $\mathrm{h}(m)$ and wavelet $\mathrm{w}(m)$ function, where m is the length along the scaling and wavelet function. The scaling and wavelet functions are used in the low and high pass filtering sequences, respectively. Whilst there are numerous wavelet families that can be chosen for analysis, this study applies the most commonly used Daubechies wavelets. ~~Each~~ Depending on the choice of wavelet ~~will perform~~ stepwise convolutions of the input signal are performed over the filter length L. $j_{\max}$ imposes an upper limit on the level of decomposition j that a signal can be decomposed into, where

$$j_{\max} = \left\lfloor \log_2\left(\frac{\mathrm{n}+\mathrm{L}-1}{2}\right)\right\rfloor, \tag{6}$$

$\lfloor . \rfloor$ is the floor operator~~and L is the length of the filter being used~~. The input signal is then convoluted by being passed through high and low pass filters, where

$$\mathrm{p}_j(i) = \begin{cases} \sum_{m=1}^{\mathrm{L}} \widehat{\mathbf{R}}(2i-m-1)\mathrm{w}(m), & j = 1 \\ \sum_{m=1}^{\mathrm{L}} \mathbf{p}_{j-1}(2i-m-1)\mathbf{w}(m), & j > 1. \end{cases} \tag{7}$$

is the low pass and

$$\boldsymbol{p_j^{\mathbf{H}}(i)} = \begin{cases} \sum_{m=1}^{\mathrm{L}} \widehat{\mathbf{R}}(2i-m-1)\mathrm{h}(m), & j=1 \\ \sum_{m=1}^{\mathbf{L}} \mathbf{p}_{j-1}(2i-m-1)\mathrm{h}(m), & j>1. \end{cases} \tag{8}$$

is the high pass, $i = 1,\ldots,\mathrm{n}_{j-1}+\mathrm{L}-1$ and refers to the ith parameter, $j = 1,\ldots,j_{\max}$ and refers to the jth level, m refers to the mth filter coefficient~~and n_{j-1} is the length of the input time series~~. The resultant low pass $\mathbf{p}_j(i)$ and high pass $\mathbf{p}_j^{\mathrm{H}}(i)$ parameters are commonly referred to as approximation and detail parameters, respectively. After the input signal is passed through the high and low pass filters there is an issue of redundancy that needs to be dealt with. The filters split the input signal into high and low frequency components that each contain roughly half the information of the input signal. As the length of each of the resultant approximation and detail parameter series is equivalent to the length of the input signal, each of the parameter series must be down sampled. The process of down sampling removes every other parameter. It is the process of high and low pass filtering followed by down sampling that enables the DWT to analyze multi-resolution components of a signal. After down sampling the length of the resultant approximation and detail parameter series is

$$\mathrm{n}_j = \begin{cases} \left\lfloor \frac{\mathrm{n}+\mathrm{L}-1}{2} \right\rfloor, & j=1 \\ \left\lfloor \frac{\mathrm{n}_{j-1}+\mathbf{L}-1}{2} \right\rfloor, & j>1. \end{cases} \tag{9}$$

where n_j refers to the length of the series at the jth level. If further decomposition is required the downsampled low pass may be fed back into the filters until the resultant parameters can no longer be split any further. An iteration of this process is shown in Fig 1. To reverse the decomposition process and reconstruct a signal, up sampling is performed on the parameter series before the lower level parameters are obtained through

$$\mathbf{p}_{j-1}(i) = \sum_{m=\lceil i/2 \rceil}^{\lfloor (\mathrm{L}-1+i)/2 \rfloor} \left(\mathbf{p}_j^{\mathrm{H}}(i)\mathrm{h}(2m-i)\right)\left(\mathbf{p}_j(i)\mathrm{w}(2m-i)\right), \quad j>1, \tag{10}$$

where $\lceil . \rceil$ is the ceiling operator and the input signal is reconstructed using

$$\widetilde{\mathbf{R}}(i) = \sum_{m=\lceil i/2 \rceil}^{\lfloor (\mathrm{L}-1+i)/2 \rfloor} \left(\mathbf{p}_j^{\mathrm{H}}(i)\mathbf{h}(2m-i)\right)\left(\mathbf{p}_j(i)w(2m-i)\right), \quad j=1, \tag{11}$$

3 Data~~Set~~

This study utilizes data from the ~~MOPEX~~ MOdel Parameter Estimation eXperiment (MOPEX) data set. Ten years of rainfall data spanning the 1990's for 438 catchments in the United States of America (USA) are used to compare the suitability of the DWT and DCT to represent rainfall time series. The catchments used in this study were chose to ensure they had sufficient rain gauge density and represented a range of catchment sizes and climates. Rainfall for the Leaf River catchment (Collins, Mississipi), a catchment that is frequently used for hydrological studies (Sivakumar, 2001; Tang et al., 2006; Bulygina and

Gupta, 2011), is used to compare the DWT and DCTs ability to reconstruct high magnitude rainfall events. A single rainfall product for each catchment is used for analysis at a daily time step. A complete description of the selection process and MOPEX data set is given by Schaake et al. (2006). To summarize, only catchments with sufficient rainfall gauge density were selected. Also, the catchments were selected to represent a range of intermediate scale (500 - 10 000 km^2) river basins for a range of climatesNo streamflow data are used in the experiment.

4 Experiment Design

This experiment does not involve the use of any hydrological models. Owing to this and the nature of the transforms there are no calibration and evaluation periods. A major use of both the DWT and DCTs has been in image compression, consequently the observed input signals were compressed and decompressed using a methodology similar to that used in image compression. In order to determine which transform's parameters are able to effectively store the most hydrological input data, both DWT and DCT parameters will be compressed to varying extents for the MOPEX rainfall time series.Before

The process undertaken involves a number of steps. Firstly, before any compression is applied, the original rainfall signal for a given catchment is transformed into DCT and DWT parameters using the processes described in sections 2.2 and 2.3. Each equations 3 and 4 and equations 7 to 9 for the DCT and DWT respectively. Secondly, each transform is compressed by iteratively zeroing out parameters that provide a low degree of information, these parameters are those closest to zero. A threshold value T $[\mathrm{mm}]$ applies a lower limit, for which transform parameters above the threshold are retained. This threshold is iteratively increased until the compressed transform is composed of the desired number of remaining parameters k and percent of Original Parameters POP is met.

$$\mathrm{POP}(T) = 100 \cdot \left(\frac{k}{\mathrm{n}}\right), \tag{12}$$

where k becomes smaller as the threshold T increases and $\lim_{\mathrm{T}\to\infty} \mathrm{POP} = 0$. The next step is to reconstruct the observed signal from the compressed transform parameters using Equations 5 and 11 for the DCT and DWT respectively. After the reconstruction has been performed, a comparison between the reconstructed and observed rainfall can be made. Lastly, this process is iterated for different POPs as well as for each catchment within the data set.

To provide a meaningful comparison between the DCT and the DWTs ability to reproduce different rainfall time series with an increasing POP, a number of simulation performance summary metrics are used. Moriasi et al. (2007) recommend that Following Moriasi et al. (2007), a combination of graphical techniques and dimensionless and error index statistics be used that are widely accepted by the hydrological community were adopted for model evaluation. For reasons mainly due to their widespread acceptance by the hydrologic community, Moriasi et al. (2007) recommend the use of the The Nash-Sutcliffe Efficiency (NSE) and the Root Mean Square Error (RMSE) to Standard deviation of the observed input signal $(\mathrm{RSR} = \mathrm{RMSE}/\sigma_{obs})$ are used to compare the performance of the reconstructed rainfall signal with the observed rainfall signal. Once the reconstructed signals are obtained, further comparison with the observed rainfall will be made using the bias summary metric. The variance, kurtosis and skewness of the reconstructed signals will be compared with that of the observed signal. The bias is calculated as $\sum_{t=1}^{n}[\widehat{\mathbf{R}}(t)-\widetilde{\mathbf{R}}(t)]/n$, where $\widehat{\mathbf{R}}(t)$ and $\widetilde{\mathbf{R}}(t)$ are the observed and reconstructed rainfall signals, respectively. The reconstructed variance, kurtosis and skewness are all normalized by the observed input signals variance, kurtosis and skewness respectively. The Peak Error (PE) is the peak ~~streamflow~~ rainfall error over the 10 year period. It is used to compare the reconstructed and observed signals for seasonal and flood forecasting situations. The PE is normalized by the peak height of the observed input signal. Further, the number of rain events missed is computed for each reconstruction by; flagging original or reconstructed observations that exhibit no rainfall~~and observations where~~, and either the absolute difference between the reconstructed and original observation is less than 0.01 or the ratio of the reconstructed and original observation is equal to zero or larger than 10. Lastly, reconstructed rainfall using the DCT and DWT will be presented for the Leaf River catchment to compare each transforms ability to reconstruct high magnitude rainfall events.

5 Results

Fig 2 shows the relationships between RSR and the number of transform parameters using the DCT and DWT for three different catchments, Arroyo Chico, Skykomish River and Ohio Brush Creek. These catchments represent the smallest, largest and mean rainfall volumes for the MOPEX data set, resepectively. It is clear that for all but the highest POP the DWT is able to reconstruct the observed signal with lower RSR than the DCT and that as the rainfall volume increases the RSR decreases. For intermediate POPs the DWT is able to reconstruct the observed signal with significantly better RSR than the DCT. As the POP approaches both 100% and 0% there is little discernible difference between the DCT and DWT reconstructions.

~~Using~~ By comparing the reconstructed DWT and DCT signals~~with~~, using 20 POP~~and using~~, with the observed rainfall signal as a reference, a histogram for the Nash-Sutcliffe Efficiency (NSE) is shown for all catchments in Fig 3. Each frequency count in the histogram represents a catchment from the MOPEX data set. The reconstructed DWT signals are clearly able to better simulate the observed rainfall signal~~with all~~. All DWT reconstructed rainfall signals ~~scoring~~ obtained a higher NSE than the DCT reconstructred rainfall signals. Table 1 shows that as the transforms are compressed and fewer parameters are used in the reconstruction, the mean NSE for the DWT stays much closer to the ideal value of one than the DCT. Further the standard deviation of NSE becomes much larger for the DCT.

Fig 4 compares the RSR for the DCT and DWT using four different POPs. A one to one line is included in all subplots and each point represents a catchment from the data set. If the data points fall above the one to one line, then for that catchment and POP the DWT is able to reconstruct the input rainfall signal with lower RSR. Again it is found that DWT is always able to reconstruct the original signal with lower RSR for all POPs. In a similar fashion to that discussed regarding Fig 2 as the POP approaches 0% the difference between the DWT and DCT reconstructions becomes smaller.

The bias, variance and skewness observed in the reconstructed signals for each catchment are shown in Fig 5 for different POPs. The DWT is able to maintain a smaller bias at different POPs for all of the catchments. As the POP decreases the bias becomes increasingly positive and negative for the DWT and DCT, respectively. The distribution of the bias becomes more dispersed for both the DCT and DWT as the POP decreases. The bias can be seen to be dependent on the transform and POP used as well as the catchment being analyzed. Both the DWT and DCT never reconstruct the observed signal with greater variance than that of the observed rainfall signal. As the POP decreases the normalized variance for the DCT moves further away from unity than the normalized variance for the DWT. The reduction in normalized variance means that, as the POP decreases, both the DWT and especially the DCT reconstructions will have fewer extreme values, when compared to the observed rainfall. The normalized skewness is a measure of symmetry that describes whether or not the reconstructed signal is more positively skewed (more than one) or less positively skewed (less than one) than the observed input signal. All of the reconstructed and observed signals had a positive normalized skewness. When compared to the observed signal the DWT becomes increasingly skewed as the POP is reduced. The opposite of this is observed for the DCT. This indicates that, when compressed ~~both transforms~~ the DWT and DCT will reconstruct the observed rainfall signal with a greater and lower number of values ~~closer~~ close to zero when compared to the observed signal, respectively. This does not mean that the total volume will be any lower than the total volume of rainfall observed. This is made evident by the low bias observed in Fig 5.

The normalized kurtosis and PE for all catchments using different POPs are shown in Fig 6. The measure of kurtosis describes how much the fraction of the distributions variance is explained by extreme deviations. Consequently, a normalized kurtosis value larger than one indicates that the reconstructed signals variance is explained more by extreme deviations than the observed input signal. This is likely to be the result of more rainfall values being reconstructed at the extremities than that of the observed rainfall series. A value smaller than one indicates that the variance is described less by extreme deviations than the observed input signal. Similarly, this is likely to be the result of fewer rainfall values being reconstructed at the extremities than that of the observed rainfall series. It is worth noting that a reconstructed time series can have the same variance yet different kurtosis than the observed rainfall time series. As the POP decreases, the dispersion of normalized kurtosis and skewness increases, and the normalized kurtosis and skewness for the DWT and DCT reconstructions becomes larger and smaller than unity, respectively. With decreasing POP the normalized PE for the reconstructed DWT signal remains small and relatively consistent when compared to the normalized PE for the reconstructed DCT signal.

6 Discussion

Fig 3 shows that the DWT and DCT are able to reconstruct the observed input signals with good efficiency using 20 POP. However, the DWT consistently outperforms the DCT. ~~It is observed in Fig 2~~ Fig 2 shows that as the POP decreases from 100% the DWT is able to reconstruct the input signal with increasingly lower RSR than the DCT, the gap in performance is largest ~~when an approximate POP of~~ for 40 ~~% is used. From this point onwards, as~~ POP. As the POP continues to decrease towards 0%, the gap in RSR ~~fades~~ reduces to zero. It is interesting to note that the DWT perfectly reconstructs the observed rainfall signal with as many parameters as there are rainy days whereas the DCT does not.

As the bias for the DWT is consistently close to zero, the use of the DWT for rainfall input data reduction is likely to be beneficial for hydrologic studies that have short time steps and involve rainfall as an input. Whilst modification of the DWT parameters may slightly overestimate input rainfall, it is not as significant as the consistent underestimation of input rainfall by the DCT. The diminishing ability of both the DWT and DCT to match the input rainfall signal variance indicates that both transforms smooth out input data towards the mean. This ~~trend~~ behaviour is more significant ~~in~~ for the DCT than the DWT.

~~The impact of this is that~~Consequently, when used as a technique for input data reduction, the DWT will reconstruct temporal variances better than the DCT. The increased skewness for the reconstructed DWT signals compared to the observed input signals indicates that there is an ~~increase in~~ increased reconstruction of low magnitude rainfall events. On the contrary, the decreased normalized skewness for the reconstructed DCT signals indicates that a number of the low magnitude rainfall events are tending to ~~increase~~ be reconstructed towards the mean. The kurtosis results shown in Figure 6 demonstrate that, when compared to the observed input signal, events of extreme deviation explain more of the variance for the reconstructed DWT and less of the variance for the reconstructed DCT. Consequently, as the nature of the extreme deviations is a critical piece of information, the use of the DCT for model input data reduction for hydrologic studies that have short time steps and involving rainfall as an input is not recommended. It is also seen in Figure 6 that the DCT is more likely to miss peak ~~river~~ rainfall height information. Consequently, care needs to be taken when choosing a transform when ~~total volume~~ peak height is critical. Further, the DCT should not be used for studies involving flood forecasting situations where the accuracy of peak ~~river flow~~ height is critical.

Whilst it is important that rain gauges measure high ~~volume~~ magnitude rainfall events with accuracy and precision, it is also important that low magnitude rainfall events are recorded. Consequently, when evaluating the merits of the DCT and DWT to reconstruct rainfall it would be prudent to analyze the frequency in which each transform is either unable to reconstruct a rainfall event or erroneously constructs a rainfall event. Table 2 illustrates that, at times, both transforms will either fail to reconstruct a low magnitude rainfall event or will erroneously construct a rainfall event when there was none observed in the original rainfall time series. In general the DWT outperforms the DCT. The exception to this is at 10 POP. This is a ~~results~~ result of the discrete nature of the DWT analysis function as opposed to the continuous analysis function used in the DCT. As the POP decreases towards zero both transforms miss more rainfall events.

Due to rapid increases in rainfall intensity, high magnitude rainfall events tend to have high frequency components. In Fig 7 the ~~gradient of the DCT least squares fit to the observed rainfall is lower than the DWT gradient. This indicates that when compressed, the DCT will smooth out high frequency input detail. The~~ smoothing of high frequency high magnitude rainfall events by the DCT is made evident by the lower slope of the linear least squares fit for the DCT reconstruction of Leaf River ~~rainfall data~~ observed rainfall data when compared of the DWT. This shows that the compressed DWT is able to retain more detail for high magnitude rainfall events than the DCT. Using 20 POP, 730 DWT parameters are able to reconstruct observed rainfall with $\mathrm{RSR} = 0.315$, whereas 730 DCT parameters are able to reconstruct observed rainfall with $\mathrm{RSR} = 0.540$. Fig 7 and Fig 8 shows that the DWT often misses and sometimes smooths out low magnitude rainfall events, the DCT however does reconstruct inaccurate rainfall ~~events~~ at these times. Figure 8 ~~validates prior conclusions that ,~~ also demonstrates that at lower POPs, the DCT will smooth out and underestimate high magnitudes events whilst the DWT will maintain accuracy and precision.

7 Conclusions

Succint descriptions of the DCT and DWT were provided to determine the suitability of each transform to be used as a tool for hydrologic model input data reduction. Due to their different construction, each transform provides different possibilities for use in model input data reduction. Since it is infeasible to estimate all transform parameters, the modeller could choose to estimate high or low frequency parameters of the DCT. This would result in minimal control of the temporal component being modified. Due to the multilevel decomposition of an input signal into high and low frequency parameters by the DWT, the modeller is able to specify the estimation of both time and frequency components. Hence, portions of the input data record can be targeted for estimation. The use of the DWT as a hydrologic model input data reduction technique allows the modeller more flexible options. A comparison of the DWT and DCTs ability to reconstruct MOPEX rainfall data using standard simulation performance summary metrics, descriptive statistics, and peak errors was then made and it was found that the DWT is most efficient at preserving high magnitude and transient rainfall events. Thus, it is recommended that the DWT be used as a model input data reduction technique for hydrologic studies that have short time steps and involve rainfall as an input. Considering that the bias for the reconstructed DWT rainfall signal is consistently lower than that of the reconstructed DCT signal and that the skewness, kurtosis and variance are also closest to the input rainfall signal, it is recommended that the DWT also be used as a model input data reduction technique for hydrologic studies that have long time steps with rainfall as an input.

Appendix A

A1

Author contributions. Ashley Wright conducted the experimental work, contributed towards the theory and wrote the manuscript. Jeffrey Walker and David Robertson assisted in the writing process. Valentijn Pauwels contributed towards the theory and assisted in the writing process.

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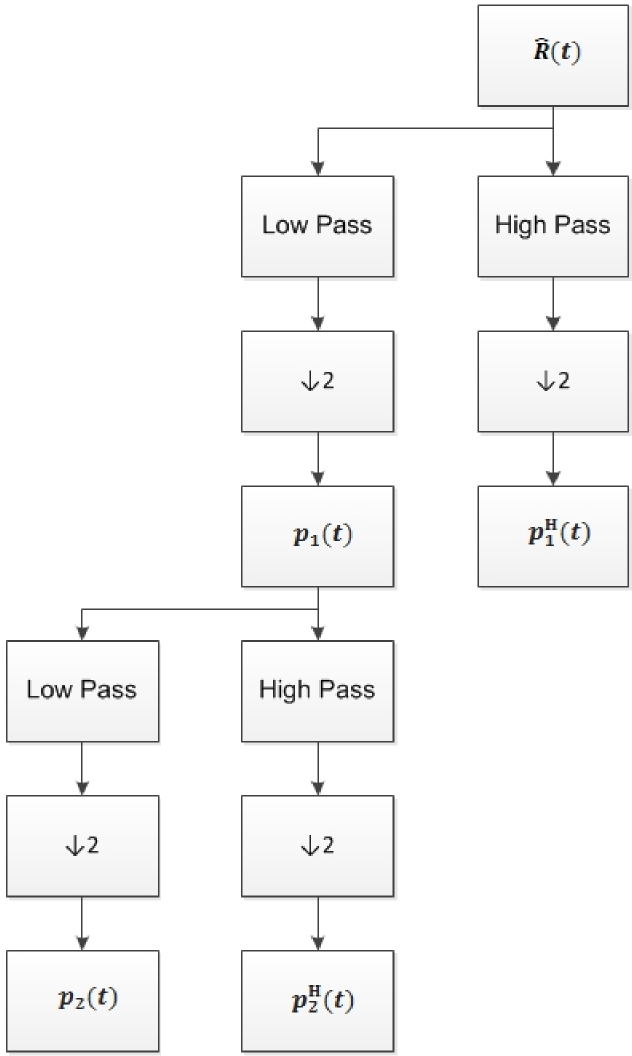

Figure 1. A schematic showing the pyramid algorithm used to decompose and down sample ($\downarrow 2$) an input signal ($\hat{\mathbf{R}}$) into high and low frequency components. The input signal is filtered using the high and low pass filters described in Equations 7 & 8 before being down sampled to produce the level one high and low pass parameters. The low pass parameters are now used as input for the high and low pass filters. This process of filtering and down sampling is repeated until the desired level of decomposition is met.

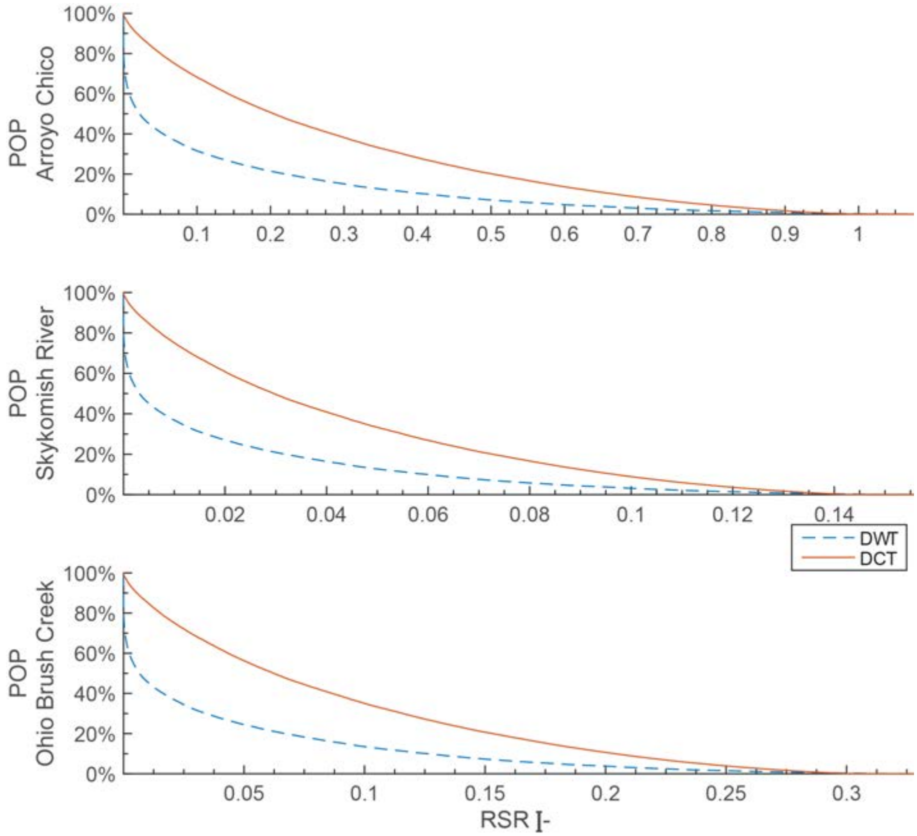

Figure 2. Empirical plots showing the relationship between RSR and the POP used for reconstructing an input rainfall signal using the DWT and DCT. The three catchments, from top to bottom of the figure, represent the smallest, largest and mean rainfall volumes throughout the 1990's for the MOPEX data set.

Table 1. The mean and standard deviation (StDev) of NSE for the DWT and DCT using a different POP.

	NSE DWT		NSE DCT	
POP	Mean	StDev	Mean	StDev
40%	0.988	0.007	0.918	0.010
30%	0.965	0.017	0.844	0.016
20%	0.905	0.036	0.729	0.025
10%	0.746	0.070	0.522	0.037

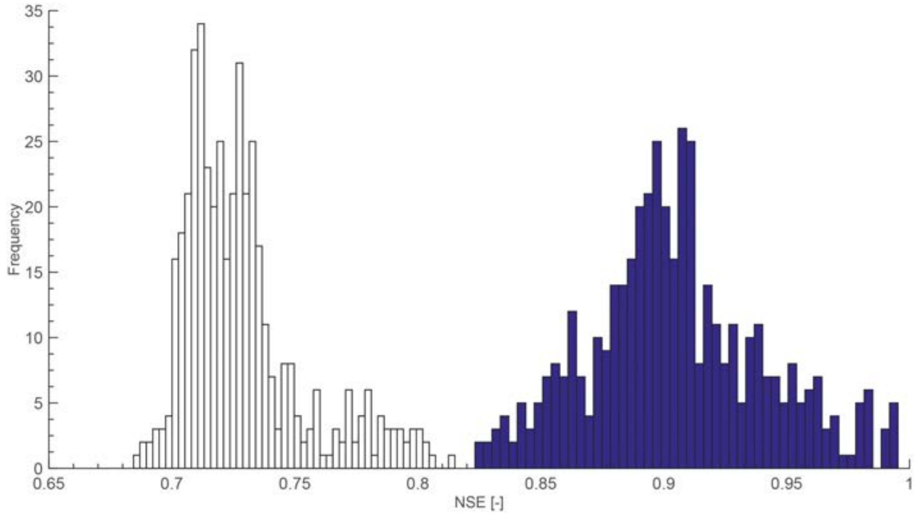

Figure 3. Histogram representing the reconstructed DWT (dark bins) and DCT (clear bins) NSE when compared to the observed rainfall signal. Rainfall is reconstructed after the input signal is compressed to 20 POP. Each frequency count represents a catchment from the MOPEX data set.

Table 2. The mean and standard deviation (StDev) for the number of missed rainfall events for the DWT and DCT using a different number of parameters.

	Number of missed rainfall events			
	DWT		DCT	
POP	Mean	StDev	Mean	StDev
40%	239.004	117.317	587.934	155.375
30%	398.005	138.793	645.495	159.378
20%	581.591	145.769	696.288	168.524
10%	852.340	168.590	748.075	184.910

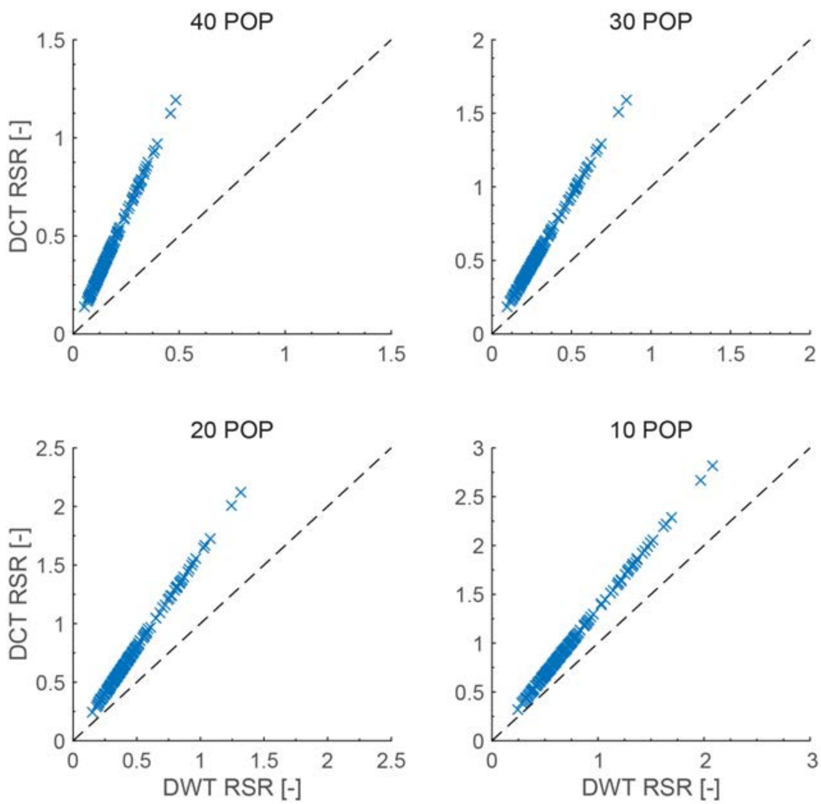

Figure 4. Comparative plots of RSR for the DCT and DWT using a different POP. Each data point represents a catchment.

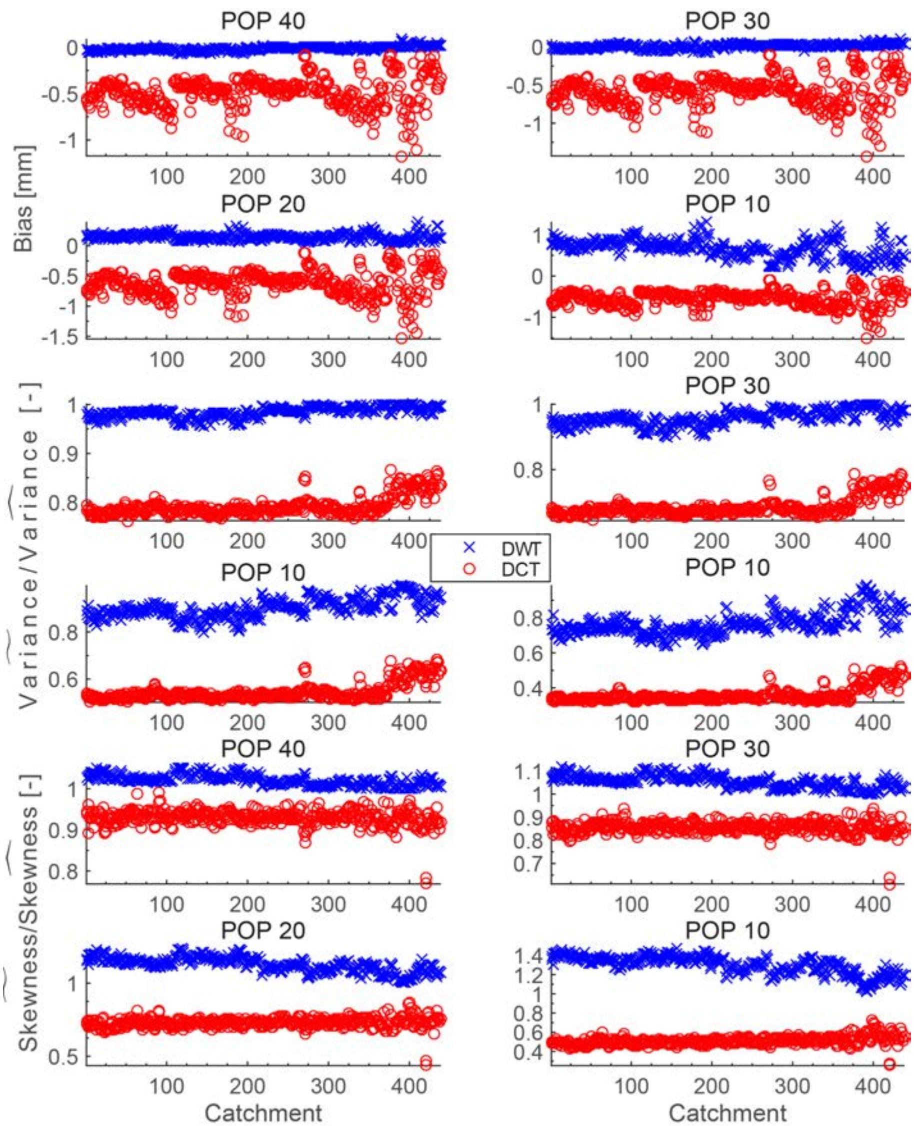

Figure 5. Bias and normalized variance and skewness of the reconstructed DWT and DCT signals for each catchment using a different POP.

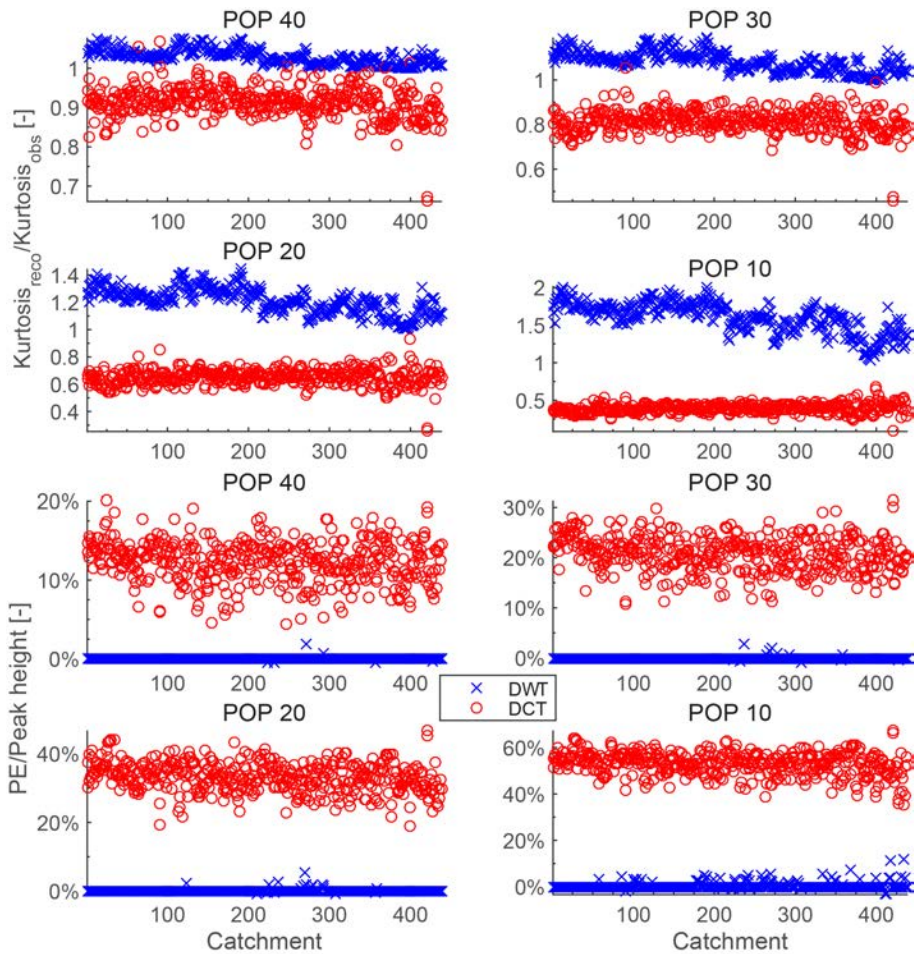

Figure 6. Normalized kurtosis of the reconstructed DWT and DCT signals and percentage PE for the reconstructed DWT and DCT signals for each catchment using a different POP.

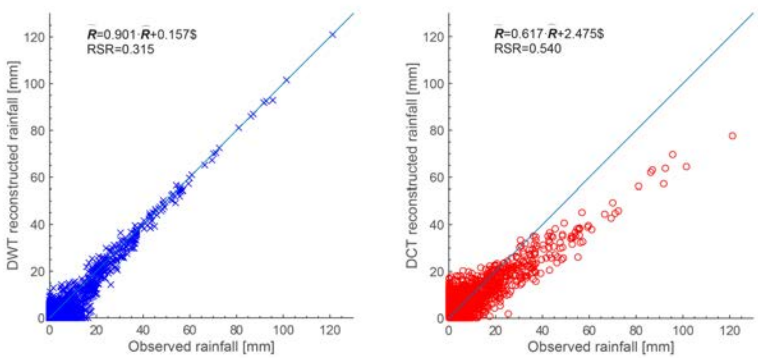

Figure 7. Comparison of the reconstructed DCT and DWT signal for the Leaf River (Collins) catchment using 20 POP.

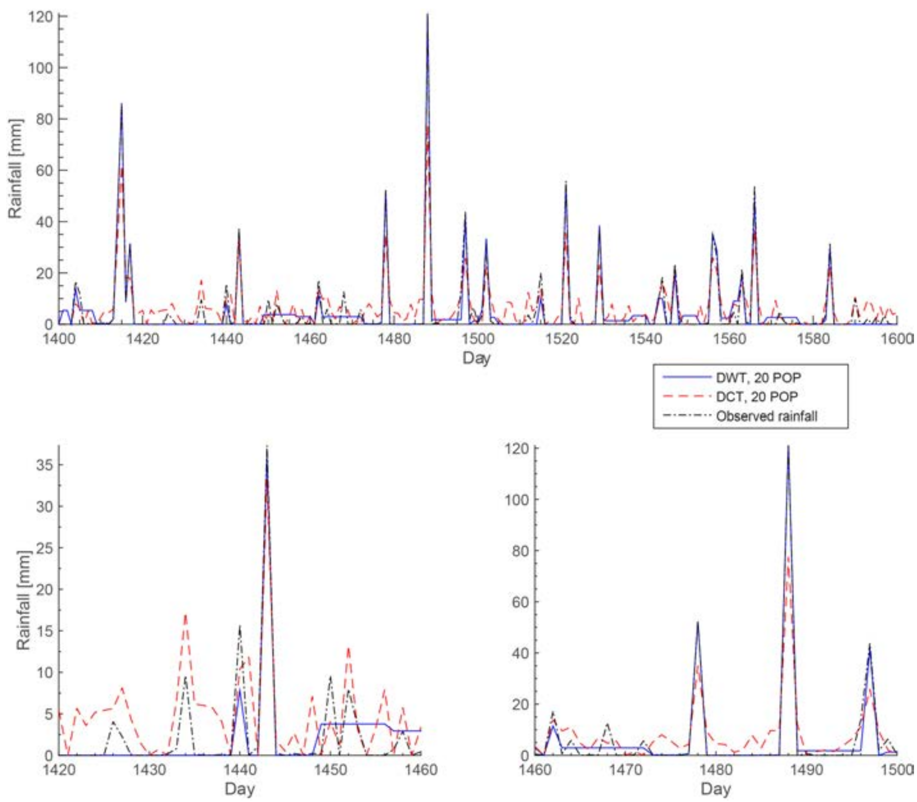

Figure 8. The top panel shows a time series comparison of the reconstructed DCT and DWT signal for the Leaf River (Collins) catchment using 20 POP for a period of 200 days. The bottom left and right panels are smaller windows of the same time series during both low and high rainfall periods.