Interactive comment on "A class of probability distributions for application to non-negative annual maxima" *by* Earl Bardsley

Anonymous Referee #1

Received and published: 8 June 2017

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General comments

The approximation of the distribution of extremes is of great interest for both hydrology and water management. Given the gap between theory and practice it is always a pleasure to see a somewhat more theoretical approach. The examination of alternatives to the standard GEV is welcome as well, it is known that GEV does not cover all possible ways to examine distributions of extremes, but theoretical alternatives do not seem to

get much attention in hydrology.

The paper under review tries to provide one such alternative. It takes a random sample of n elements from a random variable. The random variable has range $[0, \omega]$ with $0 < \omega \le \infty$. It defines the new random variable

$$X^* = \max(X_1, X_2, \ldots, X_N)$$

Moreover it selects a positive monotonically decreasing function g and defines new random variables $Y_i = g(X_i)$ and defines

$$Y^* = \min(Y_1, Y_2, \ldots, Y_n)$$

It then assumes that for Y^* a limiting distribution exists. It takes a variable W distributed according to that limiting distribution and defines a random variable $Z = g^{-1}(W)$. It then claims that the distribution of Z can be used to approximate that of X^* and that this has no less validity than the use of an extreme value distribution. Along the way several assumptions are made, mostly implicitly. As far as I can see, some of these do not hold in the full generality needed in the paper in its current form and therefore it does not yet show that the method "has no less validity than the use of an extreme value distribution".

Specific comments

I have three main comments.

- The text assumes that a limiting distribution exists for Y^*. However, not all distributions are in a domain of attraction of an extreme value distribution, there are examples of distributions that are not in a domain of attraction of an extreme value distribution.

- Even if X^* does have a limiting distribution, becaue of the potential non-linearity of g, this not necessarily imply that Y^* has a limiting distribution.

- The text assumes that Y^* has the Weibull distribution because the underlying random variable is bounded from below. This is not necesarily true, even for distributions that are in the domain of attraction of one of the extreme value dsitrbutions for minima. There are variables that are bounded from below where the limit is in the domain of attraction of the variation of Gumbel that applies to minima.

Technical comments

The comments refer to the pages (P) and lines (L) in the pdf as downloaded from the website.

P2 L8-12. The claim is made that "... it is more reflective of reality if probability distributions for design purposes are defined within the same bounds as the physical variable concerned ...". This is similar to an argument sometimes brought against the normal distribution for means of sums of random variables. In both cases one might use the counter argument that these are limit distributions as the number of random variables goes to infinity. The mathematical theory behind the limit process makes clear that, given the choice of limit process and the assumptions made, these limit distributions are the only ones that are "reflective of reality".

P2 L19. Lower case f is usually used to refer to a probability density function (pdf), not a (cumulative) distribution function (cdf). If there is a pdf then it implies that the cumulative distribution function is absolutely continuous and life becomes much easier. Amongst other things this rules out some pathological behaviour for the limit process that leads to the extreme value distributions.

<div align="center">C3</div>

P2 L19-20. Not all definitions of a random sample include independence. Do you assume that that the X_i are independent?

P2 L21. Two comments.

1. Not every monotonic function is everywhere continuous. To avoid unnecessary complications one might add the condition that g is continuous.
2. A note stating that monotonicity of g implies that if X is a random variable then $g(X)$ is a random variable might help readers.

P3 L2 The inverse function g^{-1} exists only if g is strictly monotonic on $(0, \omega)$. Usually the following definitions are used: a function f is monotonic decreasing when $x < y \Rightarrow g(x) \geq g(y)$ and strictly monotonic decreasing when $x < y \Rightarrow g(x) > g(y)$.

P3 L3 If Y^* has a Weibull distribution then it is not guaranteed that g^{-1} is defined on the whole of the range of Y^* which always runs up to $+\infty$. Take for instance

$$g(x) = \{\, 3 - \frac{1}{1+|x|}x < 02x = 01 + \frac{1}{1+x}x > 0$$

which has a range that lies in $[1, 3]$.

<div align="center">C4</div>