Hydrol. Earth Syst. Sci. Discuss.,
doi:10.5194/hess-2017-181-RC4, 2017

Hydrology and Earth System Sciences
Open Access
Discussions

Interactive comment on "Nonparametric lower bounds to mean transit times" *by* Earl Bardsley

Anonymous Referee #4

Received and published: 20 May 2017

This technical note proposes that linear programming methods can be used to fit discrete nonparametric transit time distributions to tracer data, and a lower bound for the mean transit time can be determined among all such distributions that achieve an acceptable goodness of fit to the data.

The proposed method may be a useful contribution if a) its underlying assumptions can be shown to be correct (at least to a good enough approximation), or if, nevertheless, b) its results can be shown to be robust and reliable under realistic benchmark tests, which will include both realistic data errors and deviations of the real world conditions from the idealized assumptions of the method.

Unfortunately, condition (a) is not met, because the behavior of real-world catchments is widely recognized to be inconsistent with the linearity assumptions that underlie the method presented, and the illustrative example presented in section 5 falls far short of the requirements of condition (b). Specific comments on each section follow.

Printer-friendly version

Discussion paper

Section 1 (introduction):

The manuscript does not indicate much familiarity with the extensive literature on transit time estimation, or with the fundamentals of transit time models. To take just one example, the introduction confuses conditions that are SUFFICIENT to generate a given transit time distribution, with conditions that are NECESSARY to do so:

"In this regard it could be noted the widespread use of the well-mixed model (exponential distribution of transit times) seems particularly inappropriate because the assumption is that in a given small interval of time all tracer particles must have equal probability of passing out the catchment exit.... This must apply regardless of tracer particle location on the land surface or below ground... Similarly, gamma distributions do not warrant special consideration as transit-time distributions outside of idealised situations, except for the special case where the catchment largely comprises a cascade of well-mixed lakes (gamma distributions obtained as sums of independent exponential random variables)."

While it is true that equal probability of exit is SUFFICIENT to generate an exponential transit time distribution, it is not the case that this is the ONLY way that such a distribution can arise (one can imagine, hypothetically, a series of flowpaths whose lengths are exponentially distributed, or an aquifer whose permeability decreases systematically with depth. In both cases one could obtain an exponential distribution without equal probabilities of exit for every particle in the system.

The statement about gamma distributions is likewise misplaced. Nash cascades can indeed generate gamma distributions, but only for integer shape factors >=1. In contrast, the empirically determined shape factors among the 20 catchments analyzed by Godsey et al. (2010) ranged from about 0.3 to 0.8. None were even close to 1. Approximate gamma distributions with shape factors near 0.5, consistent with the Godsey et al. findings, can potentially arise from advection and dispersion with spatially distributed inputs (Kirchener et al. 2001).

Sections 2 and 3 (definitions and steady-state case):

From a purely theoretical standpoint I do not see any problems here, but I am not an expert in linear programming techniques. From a practical standpoint the steady-state case is relevant only as a (potentially rather poor) approximation to the real world.

Section 4 (non-steady-state case):

The time-varying case presented here, in which the shape of the transit time distribution can change but its mean must stay the same, is inconsistent with the entire literature on catchment nonstationarity. Even theoretically, it is very difficult to imagine any catchment that could possibly work this way. This section may be interesting from a mathematical standpoint but is completely irrelevant to the real world.

Section 5 (illustration):

There are very substantial problems here:

a) The gamma PDF's (with shape factor of 5) that are used to generate the test time series bear no resemblance to real-world catchment PDF's (with shape factor well below 1). There is no reason to assume that a method that works with such an unrealistic test time series will necessarily work with a more realistic one.

b) The nonparametric TTD is unrealistically truncated at 23 months. Of course, given that the generating distributions – see (a) above – have trivial tails beyond 23 months, the truncation effects are small. But in the real world, where one cannot know this in advance, what could be the justification for not extending the nonparametric TTD to much longer lags? In that case, of course, the computations would become more difficult and, more importantly, the solutions would become much less constrained.

c) The truncation of the nonparametric TTD automatically imposes bounds on the mean – between 0 and 23 months in theory, but in practice, with any dispersion (either physical or numerical), the range will be narrower and the tendency will be more toward the center. Thus it is guaranteed that the solution will not be that far from 6 or 12 months, which are known in advance to be the correct answers.

d) The simulated time series are unrealistically long (not many sites have 21 years of tracer data).

e) The simulated time series are unrealistically PERFECT. There are no measurement errors in either the input or output time series. Random numbers are added to the square wave in order to create the X values, but then the Y values are convolved with zero error, and then the X and Y values are used – with zero error – in the proposed LP inversion technique. This bears no resemblance to the real-world inversion problem, where both the inputs and outputs will have errors. Furthermore, those errors are potentially catastrophic for an poorly constrained inversion technique like the proposed LP method.

f) In summary, is such a contrived test case that it gives no useful information about the robustness or the practical utility of the proposed method. There is no guarantee that, in a more realistic test case, the proposed method would give a meaningful lower bound estimator. Indeed, the result could potentially even exceed the true mean.

Interactive comment on Hydrol. Earth Syst. Sci. Discuss., doi:10.5194/hess-2017-181, 2017.