Response to the reviewers

I appreciate very much the comments that were very helpful in improving the manuscript. Below I have copied all the comments and have inserted my replies in italics. I believe that this revision adequately addresses most concerns. Nonetheless, since this is a major revision, minor revisions may still be needed. I hope that the Reviewers will be available for a subsequent revision round.

Before addressing the comments one-by-one, I would like to address thoroughly the main concern of the reviewers. It seems that the main concern of the reviewers is that the results and conclusions as presented in the manuscript are either not novel or irrelevant. Later on I will refer to the response to this main concern as the main response.

Main response:

To address main concern of the reviewers, I have revised the manuscript in the following ways. Firstly, I have changed the title. Following the comments, I realized that the original title is misleading (A consistent implementation of the dual node approach for coupling surface-subsurface flow and its comparison to the common node approach). Namely, it may imply that this implementation is a novelty which is not the case. The new title is: New insights into the differences between the dual node approach and the common node approach for coupling surface-subsurface flow. Secondly, I have revised the manuscript thoroughly to make clearer what those insights are. Also, I have decreased the number of model scenarios and figures. I hope that this will help to keep more focus on the most important insights provided in this study and to make the manuscript easier to read.

Nonetheless, I disagree that that most of the conclusions in this study are not novel. To convince the Reviewers, I here summarize the novel insights provided in the revised manuscript:

1)
According to commonly held views the common node approach is more physically-based. This view is based on the idea that the dual node approach introduces an additional parameter in the form of a coupling length. Typically, this coupling length is thought of as the thickness of layer in between the surface and the subsurface domain. I show that the dual node approach can be formulated such that this coupling length is fully defined by the grid topology. This results in what I call a consistent dual node approach. In this approach, the coupling length is not a non-physical model parameter.

2)
In comparison to the common node approach the head continuity as implemented in the consistent dual node approach is more correctly formulated in the consistent dual node approach

3)
The manuscript explains in detail the comparison between the common node approach and the consistent dual node approach. Such a comparison has not been published as far as I know. Instead, most comparison studies between the dual node approach and the common node

approach are based on alternative dual node approaches. The comparison in this study considers accuracy as well as numerical efficiency. I think that this comparison is a valuable contribution to the scientific literature.

4)
Although the scheme is not new, the scheme is typically not being recognized as a dual node approach. Instead, it is taken as being equivalent to the common node approach (An and Yu, Kumar et al.). Even if it is recognized as a dual node scheme, it is not recognized that this is a particularly advantageous scheme (Panday and Huyakorn). This study shows that models that already use this scheme should display some advantages in terms of accuracy as well as efficiency with respect to models that use the common node approach.

5)
The manuscript explains that to understand how the approaches work, it is important to consider the meaning of nodal values. Although, this may seems trivial to a numerical modeler, it seems that this point is often overlooked. Moreover, the consistent dual node approach is derived in the manuscript from basic flow equations using finite differences. Again, this may seem trivial to a numerical modeler, but I think it is important because it leads to a conceptualization of the dual node approach, which is very different from the one in existing literature. Namely, it is not an approach that needs an additional non-physical model parameter. The manuscript also provides an explanation how the dual nodes can be separated in a vertex-centered scheme to obtain a consistent dual node scheme.

Except for not presenting new results or conclusions, another comment is that the differences between the approaches are irrelevant because they all yield accurate results if the spatial resolution is chosen carefully.

It is indeed true that all coupling approaches can yield accurate results if the vertical discretization is sufficiently fine and if the coupling length (when using a dual node approach) is carefully chosen. I made revisions such that this point is acknowledged clearer and to be more objective. It is also true that the advantages in accuracy are limited to simulating excess infiltration using a relatively coarse grid. Thus it is true as mentioned by Reviewer 1 that problems related to a difference in accuracy are not a problem if the model setup is defined carefully. However, in my opinion this does not mean that the difference in accuracy when using a relatively coarse resolution is not interesting or irrelevant. In essence, from a practical point of view, I can understand the argument why this could be irrelevant. Namely, one would wish a very fine vertical discretization irrespective of the chosen coupling approach to simulate accurately the movement of saturation fronts. But there are different coupling schemes in use today to solve surface-subsurface flow. I think it is important to understand when and why these schemes yield different results. If scientists cannot answer this question, I think that that would be a problem.

Moreover, the idea that a dual node approach can be more accurate than a common node approach is new and the current consensus is very different. Namely, the commonly held view is that the maximum accuracy of the dual node approach is reached when it mimics a common node approach. Also, studies based on models that use a consistent dual node approach (i.e. An

and Yu, Kumar et al.) did not recognize that there is a difference in accuracy with respect to the common node approach. In the study of An and Yu it was found that their model is less sensitive to the vertical discretization in comparison to ParFlow (which uses the common node approach). However, they did not recognize that this difference is related to using a different coupling approach.

The difference in accuracy also shows the consequence of how the head continuity is formulated in the common node approach. Namely, this formulation is only correct if the discretization is very fine and thus if a coarser vertical resolution is used, the common node approach can become less accurate than the consistent dual node approach (i.e. the formulation of head continuity in the consistent dual node approach is correct irrespective of the vertical discretization).

Overall, it seems that the disagreement on whether the difference in accuracy is relevant or not, depends on whether one takes a practical or theoretical view point. Obviously, I look at it from a more theoretical perspective.

But suppose for the sake of argument that the difference in accuracy is completely irrelevant to the scientific community. (Although as argued I do not think that is true). Then I still do not see why this would make the manuscript irrelevant in its totality. Namely, there is also the case of numerical efficiency. And the manuscript indicates quite clearly that the consistent dual node approach can be advantageous in terms of efficiency. Moreover, the case of accuracy can also be regarded as simply being an important issue to be considered when comparing two different approaches.

Anonymous Referee #1

Received and published: 12 May 2017

I have carefully read the manuscript called "A consistent implementation of the dual node approach for coupling surface-subsurface flow and its comparison to the common node approach" by Rob De Rooij. This paper raises important issues regarding the application of integrated hydrological models through the examination of the possible influence of the coupling strategy and the vertical discretization. It especially investigates the following scientific questions (i) what is the proper coupling length to be used for the so-called dual node approach; (ii) how to formulate the dual node approach to conserve the physically based nature of the model; (iii) how does the coupling strategy influence the simulated dynamics when the vertical resolution is coarsened and (iv) how do the common node and the dual node approaches compare on synthetical test cases.

I appreciate the careful reading and interest of the reviewer.

Before going to my comments of the paper, I want to stress out that these issues are critical and barely discussed in the integrated hydrologic modeling literature. Integrated hydrologic models are more and more used to investigate hydrologic behaviors but the questions of the appropriate

scale, spatial resolutions (both horizontal and vertical), the crucial modeling choices that are to be made (coupling length for instance) and their effect on the simulated dynamics are too often forgotten although in my opinion of primary importance. I especially believe that there is a need to keep the physical meaning of integrated hydrological models through the use of appropriate spatial resolutions. This point is made very clear in the paper and is in a way the starting point of the research presented.

I agree with the reviewer that it is important to keep the physical meaning of a model trough appropriate spatial resolutions. Indeed, this becomes a critical issue if non-linearity is significant. And this is typically the case in integrated surface-subsurface models.

The consistent dual node approach proposed in the paper is clearly exposed and is a way to properly account for infiltration, especially in partially ponded cells. This approach for coupling allows preserving the physics of infiltration across the land surface if numerical parameters and spatial resolution are chosen adequately. A detailed analysis on the surface and subsurface pressure values, on the infiltration flux and on the time to ponding is provided. This analysis demonstrates the added-value of this method mainly (and only?) to describe the infiltration excess process. Although the issues tackled are of interest and the method proposed seems appropriate, I have serious concerns with the paper and I am not sure that the material presented is enough for a research paper. It seems that the added value of the approach proposed is not so important compared to the classical coupling approaches if the classical approaches are used in a relevant way. I hope that the following comments will somehow help improving the manuscript and maybe help in the publication process.

Major comments:

(1) One of my major concern deals with the fact that most of the conclusions of the research proposed in this paper are not novel and already documented in the literature. For instance, it has already been demonstrated that when using a proper discretization both coupling approaches gives very similar results and that a relatively small coupling length needs to be used with the dual node approach to conserve the physical meaning. It is true that integrated models tend to be used out of their proper application domain with coarse vertical discretization but it is more than intuitive that the vertical resolution should be small to properly capture the non-linear dynamics of infiltration fronts (especially when infiltration excess occurs). If the integrated models are properly applied, most of the questions that are tackled in the paper are not a problem anymore. In a way, the paper aims at determining which method is the less inaccurate (see line 554 to 556) when using a coarse vertical discretization, which is in a way irrelevant as both approaches are acceptable when using a proper resolution. These comments are illustrated through the conclusion that is short and not so much informative.

I have addressed this concern by acknowledging in the conclusions more clearly that the advantages in terms of accuracy of the consistent dual node approach versus the common node approach are indeed limited.

Although I understand the reviewer's concern, I do not agree that the most conclusions are already documented nor that the differences in between the approaches are in a way irrelevant. I refer to my main response as to why I disagree.

(2) The second main concern is linked to the tone and the phrasing of the paper that are not always adapted especially when reference models of the literature – i.e. Hydrogeosphere, MODHMS or Parflow – are criticized. I acknowledge that the coupling in Parflow is not well described in Kollet and Maxwell (2006) and that as a consequence some important aspects of Parflow turn out to be unclear. But I don't feel like there is a need to point out in details what the author think is not done properly by others. Once again, if an integrated model is used carefully with proper discretization and coupling length, it will produce consistent (with the physics) results regardless if it is a common node or a dual node approach. As a consequence, it is preferable to highlight what the consistent dual node approach brings than to denigrate the other approaches. I think that part 5 should be removed or at least strongly modified.

I have changed the explanation of the coupling in ParFlow. However, Section 5 (now section 4.3) is in my opinion essential. This section does not aim to denigrate other models. Namely, the shortcomings of inconsistent dual node approaches have already been discussed elsewhere. As such I do not heavily criticize other models here. I merely contrast the shortcomings with the consistent dual node. Nonetheless, I have tried to change the tone and the phrasing in this section.

(3) I have serious concern about the result regarding the numerical efficiency. First I don't understand the arguments presented at the beginning of the part 7.2 that directly link the infiltration rate and the gradient across land surface with the numerical efficiency. It is a problem for me as all the following discussion on the efficiency is related to that argument. I feel like this point should be explained better. Moreover, the efficiency of the resolution is highly linked to the numerical procedure (numerical scheme, time integration,….) that is used to solve the common node approach. In the paper by De Rooij (2013) it is explained that the model uses a dual node approach. But the common node approach is not described. Either I missed something or this should be detailed somewhere so that the reader can have all the needed information. Finally, for some test cases the difference in the number of Newton iteration is rather limited when using a proper discretization and coupling length making it difficult to say in a general way that the dual node approach is more efficient that the common node approach.

I have made major revisions in the discussions to explain better the differences in efficiency. Moreover, I also added an explanation about how the common node approach is implemented.

(4) Regarding the efficiency, I also believe that the tighter the coupling, the more difficult the resolution will be. Considering the experience I have in the domain, it is much harder to impose continuity through a common node type of approach than to impose a first order coupling through a dual node approach (if the numerical resolution is the same). As a consequence, it is for me logical that convergence is harder to obtain for some test cases with the common node approach.

In the literature the difference in efficiency havs indeed been explained in terms of tight or less tight coupling (i.e. Ebel et al.).I think that my explanations of why the consistent dual node approach can be more efficient are more detailed and add some significant understanding on why the efficiency can be different. Namely, it is shown that this difference can be tied to how fast water depths are changing at the moment of ponding. These rates are different depending on the approach.

(5) The paper is quite clear but some parts are too long. This makes the paper sometimes hard to read. Part 4 is an example. This part is very long and the first conclusions are deceiving – i.e the proper implementation has already been proposed by other (Line 240) and the proposition of a numerical trick to properly implement dual node in vertex-centered scheme (line 256 to 259). Maybe this can be improved.

I shortened this section considerably. I have tried to re-phrase this part a bit to be clearer.

(6) The part that presents the results is also hard to follow. I believe that there are too many test cases presented and that all of them are not needed. The saturation excess test cases may be removed as they are only illustrative for the efficiency. Maybe only the infiltration excess should be kept as it is for this process that the added-value of the method proposed is the most important. The consequence of multiple test cases per hydrological processes is that the reader has to jump from one figure to another which is not convenient at all. The number of figure presenting the results is also quite high.

Instead of removing the saturation excess cases, I have removed the column experiments. In fact, the hillslope experiments are enough to make my points.

(7) Regarding hydrological processes, it seems that the differences between both approaches are very small when dealing with the saturation excess process, which is the dominant process of streamflow generation in most temperate region. The main problems/conclusions are linked to the infiltration excess process. The findings for both processes are rather limited as (i) for saturation excess both approaches are OK and (ii) it is well-known that using the Richards equation infiltration excess cannot be properly capture with a 20 cm or a 50 cm resolution.

I acknowledge more clearly that indeed, the advantage in accuracy is limited. But the fact that a dual node approach can be at least or more accurate in comparison to the common node approach is significant in my opinion as it illustrates that it matters how the head continuity is implemented. Moreover, the findings in this study contrast to commonly held views, according to which the dual node approach is only more efficient with respect to the common node approach at the expense of accuracy. Namely, the consistent dual node approach can be more efficient as well as more accurate for certain simulation scenarios. See also my main response for additional arguments why I think the difference in accuracy is actually quite relevant.

The point of the manuscript is not only that the consistent dual node approach can be more accurate. Efficiency is also considered. In a more general sense, the manuscript simply compares in detail the consistent dual node approach with a common node approach. The fact

that both approaches can yield similar results and that the differences in accuracy or efficiency are not extreme does not make this comparison study irrelevant.

(8) The coupling between surface and subsurface strongly depends on the numerical schemes use for resolution. This point is clear on the paper (especially through the explanations related to figure 1) but the paper – although using 2 different schemes – is not exhaustive. Some published models using other resolution schemes are built using a properly implemented dual node approaches and this point should be fairly mentioned somewhere.

I only found that the model of Kumar et al. is also in essence based on a consistent dual node approach. In addition, I also point out that CATHY as well as the model of Morita and Yen share characteristics with the consistent dual node approach.

(9) I am a bit uneasy with the concepts of elegance and generality when considering physically-based modelling. In my opinion, the main question is whether the modelling approach chosen allows for a proper description of the physics considered. I believe that it is an endless debate to determine which approach is the more elegant or the more general and I would suggest the author to remove the sentences related to that and focus on the accuracy and/or the efficiency that are can be somehow measured.

Corrected in the revised manuscript.

Other comments:

- Some parts of the paper are only about interpretation and as a consequence are very subjective. See for instance from line 274 to line 283.

Removed

- Line 45: hillslopes not hill slopes

Corrected

- Line 50: the reference paper for CATHY is rather Camporese et al, WRR, 2010 than Weill et al, AWR, 2011.

Corrected

- Line 60: the interface is not always saturated. Its property is constant but saying that it is always saturated can be misunderstood regarding the infiltration process.

Corrected

-From line 191 to line196: this part is not clear and needs to be improved. To my knowledge and in most of the integrated models mentioned in the paper, when a cell is not ponded, all the rainfall infiltrates. When the cell is ponded or partially ponded, infiltration occurs under the ponded area. I agree that infiltration under the non-ponded fraction of a partially ponded area should be theoretically accounted for, but the sentences in the paper could lead to misunderstandings.

Rephrased paragraph

- Line 223: I don't understand why it is mentioned here that the surface head can be used as a Dirichlet boundary condition. I agree that it can be done but not in the context of a coupling through a dual node approach. Maybe this is linked to the implementation of the common node approach.

Corrected

- Line 326: typo - Figure 1c

Corrected

- Line 365-368: Repetition of things already said from line 274 to 283

Removed

- Line 395-397: I quickly checked in de Rooij et al (2013) and this paper only describe the dual node approach for coupling. Some results with the common node approach are presented later in the paper. The way the common node approach is implemented should be presented somewhere.

Corrected in revised manuscript, added explanation in section 5 (numerical experiments)

-Line 464 to 478: this part does not bring anything to what is already well known and described in the literature. Just say that the reference is computed using a fine resolution.

This is not completely true (already known), because I compare with a consistent dual node approach which is different. But the overall idea does indeed remain the same. I have shortened the paragraph.

- Line 498-500: Please explain before in the paper how the inconsistent dual node approach was implemented.

I have removed this approach from the experiments (Note that it would be quite simple. Namely a simple change in elevation heads of the surface nodes).

- It is strange that figure 2 d and 4d shows so different results. We would expect that the behavior between different coupling approach/resolution provides same trends regarding the reference and it's not the case. Can you explain?

These figures are removed. But the difference is related to a difference in the effective rainfall rate, Namely, when simulating excess infiltration, the inconsistent dual node approach requires a water depth greater than the coupling length for top-down saturation to occur. Thus, if the effective rainfall rate is large enough then this is more likely to be the case.

- Test cases with excess infiltration: even though the dual node approach displays "more desirable behavior" (line 521), the results with coarse discretizations are far from the reference. Meaning that a consistent implementation of the dual node approach is not sufficient enough if the resolution is not well chosen.

I have changed the phrasing. But, the finding that the consistent dual node approach is less sensitive to the vertical discretization remains a significant insight. Indeed, I think that it can be argued well that the dual node approach displays more desirable behavior. Namely, ponding starts before the topmost subsurface node is saturated. Since this represents a value at some depth below the surface, it is logical to assume that this node should reach fully saturated conditions some time after reaching fully saturated conditions at the surface. Of course, this does not mean that the consistent dual node approach is accurate for any discretization. More in terms of comparing the two approaches I think it is fair to say that the consistent dual node approach displays more desirable behavior.

- Figure 10 c and 10 d: it is hard to say who the best is between the common node and the dual node. Needs to be discussed.

I have removed the simulations with the coarsest discretizations. Also because Reviewer 2 stated that such a coarse discretization is rarely used.

- Figure 13: why is there so much difference for this test case only? When the discharge are so close and match pretty well, the efficiency seems very different between the coupling approaches.

Added further and better explanation.

- Line 538-539 (excess infiltration): all the simulations are far from the reference. The argument presented in this sentence is not valid in my opinion.

Corrected

- Line 553: typo "understimates or overestimates"

Corrected

- Line 671: Figure 9 not 10

Corrected

- Line 635: Figure 6 not 7

Corrected

Anonymous Referee #2

Received and published: 16 May 2017

R. deRooij (RdR) presents the dual node approach for coupling surface and groundwater flow including a comparison to the common node approach and other dual node approximations based

on synthetic numerical experiments and also numerical measures (i.e. number of non-linear iterations).

I have two major points of concern with the manuscript. While I like and appreciate the effort by RdR to clarify general misperceptions and confusion of different common and dual nodes approaches, the manuscript reads more like a reckoning with numerical, hydrologic scientific software than a research paper. It is important to keep in mind that we are dealing with a highly non-linear problem ultimately cast in discrete mathematics that a computer can understand. As such there will always be ambiguities and errors. For example, I was always wondering, how these models handle the following situation. Imagine the following thought experiment of model with a cell-centered grid,where the top layer is just under tension saturation. Adding an incremental amount of water will switch the pressure value at the cell center from some negative value to $\sim dz/2$. A dual node right at the land surface interface would switch from some negative value to ~ 0. In both cases surface runoff is initiated. Thus, there is something like a discontinuity in pressure due to the discrete mathematics, which will lead to errors under both excess infiltration and saturation conditions for both the dual and common node approach, which can only be resolved with very high spatial discretization. This can be nicely seen, in my opinion in the results of the numerical experiments presented here and have been shown before in publications related to the simulation of coupled groundwater-surface water flow and the development of integrated hydrologic scientific software. Looking at the results presented here, these types of problems are still not resolved by the proposed dual node approach, and probably never will be because of the limitations of discrete mathematics.

The idea or objective of this paper is not to find a panacea for all these problems. Instead I show how the consistent dual node approach compares to the common node approach, which has not been done to the best of my knowledge. While I use similar experiments as previous studies, the results are thus novel. I use similar experiments as it is common practice to make comparisons on benchmark tests if available.

I have changed the tone of the paper to make it look less than a reckoning with other models. Nonetheless, to make clear that this paper contains novel insight, I do need to discuss the differences with respect to other numerical models.

I hope that the new figures in the manuscript will help the reviewer in finding an answer to his thought experiment. In general, the pressure head will never make an abrupt jump as long there is a specific storage greater than zero. Instead the pressure head can change very fast from 0 to a value equal to half the thickness of the topmost cell. How fast this change occurs depends on the coupling scheme as it is now explained in more detail in the manuscript. Also, when using the consistent dual node approach ponding will only start if the infiltrability is exceeded. The computation of this infiltrability depends on the vertical discretization. But in case of excess saturation, this does not matter since the ponding is merely governed by the time it takes to saturate the subsurface which depends on the initial water content and the applied flux rate.

Therefore, because of numerical aspects, it is also not appropriate to compare directly the non-linear iterations for both coupling schemes. The common and dual node implementation are

different discrete approaches that of course will exhibit different non-linear convergence, and, second, it is not clear from the presentation how the common node approach has been implemented by RdR.

I have added an explanation of how the common node approach is implemented in the model code. However, since all the flow computations (except for the exchange flow) are identical, I think it is fair to compare the non-linear iterations. I have added a remark that the iterations do depend on how the model code is constructed. But that dependency is equal for both approaches as they are implemented in the same code. Moreover, I explain in greater detail why there are differences in efficiency. Since they can be tied to how abrupt the pressure heads are changing near the surface at the moment of ponding, I think that any model will encounter similar problems (i.e. more iterations and smaller time steps if the changes are more abrupt). Considering the concern of comparing the number of iterations, I would be interested if the Reviewer has alternative ideas of measuring the efficiency.

My second concern is related to the RdR's dual node approach, which is not novel. As the author acknowledges himself that "Nonetheless, their [An, H., and S. Yu (2014)] approach is actually a properly implemented dual node approach practically similar to the one proposed in this paper." Thus, it appears that main contribution of the manuscript is the discussion of the difference between the common and dual node approach and clarification of some of the applied concepts in different scientific hydrologic software.

While I feel this is a valuable contribution to the scientific literature, the manuscript requires major revisions and a more objective discussion. After all, for example, figure 2 suggests that for coarse spatial resolution both the common and dual node approach are quite far off the reference simulation. But in the past ten years or so, model implementations improved and a spatial discretization of 0.5m at the land surface is rarely used in todays models that I read about.

I have tried to strike a more objective tone and to make clearer what the paper is about. I acknowledge in the revised conclusion section that the advantage in accuracy is limited. The point that is being made in the paper is that the dual node approach should be perceived more positively in comparison to the common node approach. Namely, the common view is that a) the common node approach is more physically based, b) the common node approach is more accurate (i.e. the common view is that a dual node approach is most accurate when it mimics a common node approach), c) the dual node approach can be more efficient but at the expense of accuracy vis-à-vis the common node approach. This paper shows that this is very different when using a consistent dual node approach. Namely, in the dual node approach the head continuity is actually more properly formulated, the approach is at least at accurate as the common node approach and is often more efficient without a trade-off in accuracy. That the approach is not new (which is acknowledged in the paper) does not change the fact that these are significant new insights.

I also removed the spatial discretization of 0.5 m and now set the coarsest discretization to 0.2 m. Again, I do not pretend that the consistent dual node approach is always better or that it can be used with a very coarse vertical discretization. But it is interesting that the consistent dual

node approach is less sensitive to the vertical discretization when simulating excess infiltration and that it can be more efficient for excess saturation as well as excess infiltration.

1 ~~A consistent implementation of~~ <u>New insights into the differences</u>

2 <u>between</u> the dual node approach <u>and the common node approach</u>

3 for coupling surface-subsurface flow ~~and its comparison to the~~

4 ~~common node approach~~

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21   Fax: 1-352-392-6855

22   **Key points**

23   Surface-subsurface flow coupling

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43 **Abstract**

44 ~~Commonly,~~The common node approach and the dual node approach ~~for coupling~~ are two widely

45 applied approaches to couple surface-subsurface flow ~~is conceptualized as a hydraulic separation~~

46 ~~of the surface and the subsurface by a distinct interface with a given thickness. Since such an~~

47 ~~interface is not supported by field observations, it has been argued that the dual node depends on~~

48 ~~a non-physical parameter in the form an ill-defined interface thickness. As such, the alternative~~

49 ~~common node approach is considered to be a more general and a more elegant approach since it is~~

50 ~~based on the physical principle of head continuity along the surface-subsurface interface.~~. In this

51 study~~, however, it is argued that if properly implemented, then the dual node approach is actually~~

52 ~~the more general, the more elegant as well as the more accurate approach. This insight is obtained~~

53 ~~by considering that the topmost subsurface nodal values represent the mean values within discrete~~

54 ~~control volumes and by deriving the dual node approach from equations that govern infiltration~~

55 ~~and infiltrability.~~ both approaches are analyzed for cell-centered as well as vertex-centered finite

56 difference schemes. It is shown that the dual node approach should be conceptualized and

57 implemented as a ~~simple~~ one-sided first-order finite-difference to approximate the vertical

58 subsurface hydraulic gradient at the land surface ~~and that there is no need to assume a hydraulic~~

59 ~~separation between the two flow domains by a distinct interface. Whereas a consistent properly~~

60 ~~implemented~~. This results in a consistent dual node approach in which the coupling length is

61 related to grid topology. In this coupling approach the coupling length is not to be interpreted as a

62 non-physical model parameter. Although, this particular coupling approach is technically not new,

63 the differences between this consistent dual node approach and the common node approach have

64 not been studied in detail. In fact, this coupling scheme is often believed to be similar to the

common node approach. In this study it is illustrated that in comparison to the common node approach, the head continuity at the surface-subsurface interface is formulated more correctly in the consistent dual node approach. Numerical experiments indicate that the consistent dual node approach is ~~in agreement with the physical principle of head continuity at the land surface, it is shown that the common node approach is not. Studies that have compared the two coupling approaches have been based on improperly implemented dual node approaches. As such, this study presents a re-evaluation of how the common node compares to the~~less sensitive to the vertical discretization when simulating excess infiltration. It is also found that the consistent dual node approach~~. Cell-centered as well as vertex-centered schemes are considered.~~ can be advantageous in terms of numerical efficiency.

1   Introduction

There exists a variety of hydrogeological problems, such as the hydrologic response of ~~hill slopes~~hillslopes and river catchments, which requires an integrated analysis of surface and subsurface flows. This has led to the development of physically-based, distributed parameter models for simulating coupled surface-subsurface flows. Well-known examples of such models include MODHMS ~~[*Panday and Huyakorn*, 2004]~~[*Kollet and Maxwell*, 2006; *Panday and Huyakorn*, 2004] , InHM [*Ebel et al.*, 2009], HydroGeoSphere [*Therrien et al.*, 2010], CATHY

87 ~~[*Weill et al.*, 2011]~~,[*Camporese et al.*, 2010], WASH123D [*Yeh et al.*, 2011], ParFlow [*Kollet and*

88 *Maxwell*, 2006] and OpenGeoSys [*Kolditz and Shao*, 2010]. Typically, subsurface flow is

89 governed by the Richards' equation whereas surface flow is either governed by the kinematic wave

90 or the diffusive wave equation.

91   The coupling between subsurface and surface flow may be either based on the common

92 node approach [*Kollet and Maxwell*, 2006] or on the dual node approach [*Ebel et al.*, 2009; *Panday*

93 *and Huyakorn*, 2004; *VanderKwaak*, 1999]. In the common node approach coupling is formulated

94 by a continuity in head between surface and subsurface nodes. The dual node approach is based

95 on formulating an exchange flux between the surface and subsurface nodes. Typically, the dual

96 node approach is conceptualized as a hydraulic separation of the surface and the subsurface by ~~a~~

97 ~~saturated~~an interface with a given thickness [*Liggett et al.*, 2012]. The thickness of this interface

98 defines a coupling length between the dual nodes to formulate the discrete exchange flux between

99 the dual nodes.

100   It has been argued that the coupling length ~~is~~represents a non-physical model parameter,

101 because there is often no evidence to support the existence of a distinct interface between the two

102 flow domains [*Kollet and Maxwell*, 2006]. ~~As such it appears that the common node approach is~~

103 ~~a more general coupling approach [*Kollet and Maxwell*, 2006]. Considering that smaller coupling~~

104 ~~lengths tend to improve the accuracy of the dual node approach~~As such it appears that the common

105 node approach is a more physically based coupling approach [*Kollet and Maxwell*, 2006; *Liggett*

106 *et al.*, 2012]. It has also been found that accurate simulations based on the dual node approach

107 typically require a very small coupling length [*Ebel et al.*, 2009; *Liggett et al.*, 2012; *Liggett et al.*,

108 2013]~~, it also seems that the common node approach is generally more accurate. Namely, in the~~

109 ~~limit as the coupling length goes to zero, the dual node approach mimics the common node~~

110 ~~approach.~~ Since it is known that the dual node approach mimics the common node in the limit as

111 the coupling length goes to zero [*Ebel et al.*, 2009], it thus seems that the dual node approach is

112 most accurate if it mimics the common node approach. Nonetheless, it has been argued that the

113 dual node approach remains an attractive alternative coupling approach since it offers more

114 flexibility than the common node approach. Namely, while it can mimic the common node

115 approach, the dual node approach offers the possibility to simulate a less tight coupling of surface-

116 subsurface flow which results in increased computational efficiency [*Ebel et al.*, 2009]. ~~It has been~~

117 ~~illustrated that both the dual node approach as well as the common node approach are sensitive to~~

118 ~~the vertical discretization near the surface [*Liggett et al.*, 2012; *Sulis et al.*, 2010].~~

119   In this study ~~it is illustrated that if the dual node approach is properly implemented as well~~

120 ~~as properly conceptualized, then the dual node approach is actually the more general, more elegant~~

121 ~~as well as the more accurate approach. This is a significant finding particularly since this~~

122 ~~contradicts the findings~~a detailed analysis of ~~other studies in which the common node is commonly~~

123 ~~regarded as a more general and more elegant approach [*Dawson*, 2008; *Kollet and Maxwell*, 2006;~~

124 ~~*Liggett et al.*, 2012; *Liggett et al.*, 2013]. To arrive at a properly implemented or consistent dual~~

125 ~~node approach the dual node approach is derived from basic flow equations. Moreover, to develop~~

126 ~~and understand the consistent approach, it is crucial to realize that the topmost subsurface nodes~~

127 ~~should ideally represent values at the centroids of discrete control volumes. It is shown that the~~

128 ~~dual node approach should not be conceptualized as a distinct interface across which an exchange~~

129 ~~flux is computed. Instead the dual node approach should be interpreted as a one-sided finite~~

130 ~~difference approximation of the vertical hydraulic gradient at the land surface in which the~~

131 ~~coupling length is defined by the grid geometry. Moreover, whereas the consistent dual node~~

132 ~~approach is in agreement with the principle of head continuity at the surface-subsurface interface,~~

133 ~~it can be shown that the common node approach is not.~~

134 ~~In this study the~~ both coupling approaches ~~are considered~~is provided for cell-centered as

135 well as vertex-centered finite difference schemes. ~~Theoretical considerations as well as~~ This

136 analysis starts with the crucial observation that that the topmost subsurface nodal values as

137 computed by the finite difference schemes represent the mean values within the topmost discrete

138 control volumes. Numerical experiments ~~indicate that the dual node approach when properly~~

139 ~~implemented is often more accurate as well as more computationally efficient than the common~~

140 ~~node approach, particularly if the vertical discretization is relatively coarse. This is an important~~

141 ~~finding because using a relatively coarse vertical discretization is common practice in regional~~

142 ~~coupled surface-subsurface models [*Jones et al.*, 2008; *Kollet and Maxwell*, 2008; *Srivastava et~~

143 ~~al.*, 2014]. The numerical experiments~~to compare the coupling approaches are carried out with the

144 model code DisCo [*de Rooij et al.*, 2013]. It is shown that the dual node approach should be

145 interpreted and implemented as a one-sided finite difference approximation of the vertical

146 hydraulic gradient at the land surface. This yields a consistent dual node scheme in which the

147 coupling length is defined by the half the thickness of the topmost subsurface cells. The scheme

148 of An and Yu [*An and Yu*, 2014] as well as the scheme of Kumar et al. [*Kumar et al.*, 2009] are

149 essentially very similar to this consistent dual node scheme. In the work of Panday and Huyakorn

150 [*Panday and Huyakorn*, 2004], one of the suggestions to define the coupling length is to use half

151 the thickness of the topmost subsurface cells, which yields a consistent dual node scheme. While

152 the idea that the coupling length can be based on the grid topology is not new [*Panday and*

153 *Huyakorn*, 2004], the idea that it must be related to grid topology to obtain a consistent approach

154 is a significant new insight. Namely, since the coupling length in the consistent dual node approach

155 is not to be interpreted as the thickness of a layer that separates the subsurface from the surface,

156 the consistent dual node approach is not automatically less physically based than the common

157 node. In fact, as explained in this study in comparison to the common node approach the

158 implementation of a head continuity at the surface-subsurface interface is formulated more

159 correctly in the consistent dual node approach.

160   The current consensus about how the dual node approach compares to the common node

161 approach is based on alternative dual node approaches which as explained in this study are

162 different from the consistent dual node approach. In this study the consistent dual node approach

163 is compared in detail with the common node approach. It is shown that if the vertical discretization

164 is sufficiently fine, then the common node approach and the consistent dual node approach are

165 equally accurate. However, when simulating excess infiltration the consistent dual node approach

166 is found to be less sensitive to the vertical discretization in comparison to the common node

167 approach. This advantage in accuracy is related to the fact that head continuity is more correctly

168 formulated in the consistent dual node approach. Moreover, it is also shown that the consistent

169 dual node approach can be advantages in terms of numerical efficiency when simulating runoff

170 due to both excess saturation as well as excess infiltration. The finding of this study show that the

171 consistent dual node approach compares more positively with respect to the common node

172 approach than other dual node approaches.

173 **2   Interpretation of nodal values**

174 As explained later on, a correct interpretation of nodal values is crucial for understanding the dual

175 and common node approach for coupling surface-subsurface flow. Moreover, both coupling

176 approaches depend on the configuration of surface and topmost subsurface nodes near the land

177   surface. This configuration depends on whether cell-centered or vertex-centered schemes are used.

178   In this study both type of schemes will be covered, but for simplicity only finite difference schemes

179   are considered.

180   In both cell-centered as vertex-centered schemes the flow variables such as the heads and

181   the saturation are computed on nodes. In vertex-centered schemes these nodes coincide with the

182   vertices of mesh, whereas in cell-centered schemes the nodes coincide with the cell centers. When

183   employing a finite difference scheme, nodal values correspond to the mean value within

184   surrounding discrete control volumes. In cell-centered finite difference schemes these discrete

185   volumes are defined by the primary grid cells. In vertex-centered finite difference schemes these

186   discrete volumes are defined by the dual grid cells. Ideally, the mean values in the discrete control

187   volumes are derived by applying the midpoint rule for numerical integration such that their

188   approximation is second-order accurate. Therefore, the nodal values should ideally represent

189   values at the centroid of the surrounding discrete control volume [*Blazek*, 2005; *Moukalled et al.*,

190   2016]. In that regard, a cell-centered finite difference scheme is thus more accurate than a vertex-

191   centered finite difference scheme. Namely, in cell-centered finite difference schemes the nodal

192   values always correspond to the centroids of the cell whereas in vertex-centered finite difference

193   schemes nodes and centroids (of the dual cells) do not coincide at model boundaries and in model

194   regions where the primary grid is not uniform. It is well-known that this mismatch between nodes

195   and centroids can lead to inaccuracies since the mean values within affected discrete volumes are

196   not computed by a midpoint rule [*Blazek*, 2005; *Moukalled et al.*, 2016].

197   Typically, vertex-centered schemes for simulating coupled surface-subsurface flow are

198   based on mass-lumped finite element schemes [*Liggett et al.*, 2012]and not on finite difference

199   schemes. However, with respect to coupling surface-subsurface flow there is actually no difference

200   between a mass-lumped finite element scheme and a vertex-centered finite difference scheme.

201   Similar as in vertex-centered finite difference schemes, the nodal values in mass-lumped finite

202   element schemes define the mean values inside dual grid cells [*Zienkiewicz et al.*, 2005].

203   Moreover, the coupling approaches establish one-to-one relations between surface and topmost

204   subsurface nodes which do not depend on whether a finite difference or a finite element approach

205   is being used. ~~Thus, a less complicated vertex-centered finite difference scheme may be used to~~

206   ~~provide insights in the coupling approaches as used in mass-lumped finite element schemes.~~

207   **3   Common node approach**

208   The common node approach defines a head continuity between the topmost subsurface nodes and

209   the surface nodes. This continuity requires that the topmost subsurface nodes and the surface nodes

210   are co-located at the land surface such that there exists a continuity in the elevation head. This

211   requirement is automatically full-filled in vertex-centered schemes. Figure 1a illustrates the

212   configuration of common nodes in ParFlow, a cell-centered scheme [personal communication

213   Maxwell, R. in relation to previous work of the author [*De Rooij et al.*, 2012]]. Figure 1c illustrates

214   the configuration of common nodes for vertex-centered schemes. This configuration is similar to

215   the configuration as used in HydroGeoSphere [*Therrien et al.*, 2010]. ~~However, in cell-centered~~

216   ~~schemes such as ParFlow the co-location of nodes is less straightforward. Also, the basic~~

217   ~~explanation that the pressure head continuity is assigned at the top cell of the subsurface domain~~

218   ~~at the boundary between the two domains [*Kollet and Maxwell*, 2006; *Maxwell et al.*, 2009; *Sulis*~~

219   ~~*et al.*, 2010] is ambiguous since the location of the land surface with respect to the top cell is not~~

220   ~~specified. Nonetheless, since ParFlow is a cell-centered scheme where the topmost subsurface~~

221   ~~node is located at the center of the top cell, it follows that the surface node is located at the center~~

222   ~~of the topmost subsurface cells as depicted in Figure 1a such that the land surface is located at the~~

223 ~~center of the topmost subsurface cell. This is the correct configuration as applied in ParFlow~~

224 ~~[personal communication Maxwell, R. in relation to previous work of the author [*De Rooij et al.*,~~

225 ~~2012]]. It can be argued that the additional subsurface volumes that extent above the land surface~~

226 ~~do not drastically affect the timing of runoff. Namely, once the topmost subsurface node reaches~~

227 ~~fully saturated conditions, the amount of additional water that can be stored in those volumes is~~

228 ~~relatively small as long as the specific storage assigned to the topmost cell is relatively small.~~

229 ~~Since the location of the land surface in ParFlow is somewhat unclear, some studies have inferred~~

230 ~~that ParFlow uses a completely different nodal configuration. For example, it has been inferred~~

231 ~~that the topmost subsurface nodes in the ParFlow model are placed on top of the topmost~~

232 ~~subsurface cell such that they are co-located with the surface nodes [*Liggett et al.*, 2013]. An and~~

233 ~~Yu [*An and Yu*, 2014] infer that the surface and subsurface nodes are not co-located at all and the~~

234 ~~surface nodes are located at the top face of the topmost subsurface cells and that the topmost~~

235 ~~subsurface nodes are located at the center of the topmost subsurface cells.~~

236 Considering that nodal values represent ideally the mean values within discrete control

237 volumes as described in Section 2, it can be argued that the head continuity as implemented in the

238 common node approach is not in agreement with the physical principle of head continuity at the

239 land surface. Namely, the common node approach enforces a continuity between surface heads at

240 the land surface and the mean subsurface heads within the topmost subsurface discrete control

241 volumes which have a finite thickness. This is different from enforcing a continuity between

242 surface heads and subsurface heads within an infinitesimal thin subsurface layer directly below the

243 land surface. As such ~~inconsistent behavior is expected when using~~ the common node approach~~.~~

244 ~~To effectively remove this inconsistency a~~ is only numerically correct if the topmost subsurface

245 cells are very ~~fine vertical discretization is required near the land surface.~~thin.

246 **4   ~~Consistent~~ Dual node ~~implementation~~approach**

247 **4.1   Basics**

248 Figure 1b and 1c illustrate the classical arrangement of surface and subsurface nodes in cell-

249 centered and vertex-centered finite difference schemes, respectively. Commonly, the dual node

250 approach is expressed in terms of an exchange flux ~~q_e~~ q_e [LT^{-1}] computed as [*Liggett et al.*, 2012;

251 *Panday and Huyakorn*, 2004]:

252
$$\cancel{q_e = f_p \frac{K_z}{l}(h_s - h_{ss})} \; q_e = f_p \frac{K_z}{l}(h_s - h_{ss}) \tag{1}$$

253 where h_s and h_{ss} are the hydraulic heads [L] associated with the surface node and the topmost

254 subsurface node, respectively, ~~f_p~~ f_p [-] the fraction of the interface that is ponded and l the

255 coupling length [L]. The ponded fraction of the interface is typically defined by a function that

256 varies smoothly between zero at the land surface elevation and unity at the rill storage height which

257 defines the minimum water depth for initiating lateral overland flow [*Panday and Huyakorn*,

258 2004]. In equation ~~(1)~~(1) the term ~~$f_p K_z / l$~~ $f_p K_z / l$ is commonly referred to as the first-order

259 exchange parameter, where first-order means that the exchange flux depends linearly of the

260 hydraulic head difference.

261 Typically, equation ~~(1)~~(1) is not derived as a numerical approximation of basic flow

262 equations that govern the exchange flux, but is merely presented a numerical technique to couple

263 two different flow domains [*Ebel et al.*, 2009; *Liggett et al.*, 2012]. Subsequently, the dual node

264 approach is conceptualized by interpreting equation ~~(1)~~(1) as an expression that describes

265 groundwater flow across a distinct interface separating the two flow domains [*Ebel et al.*, 2009;

266 *Liggett et al.*, 2012; *Liggett et al.*, 2013]. ~~Evidently, if the coupling length is assumed to be a non-~~

267 ~~physical parameter, then it follows that equation (1) cannot be derived from basic flow equations.~~

268 **4.2   Consistent dual node approach**

269 In the following, ~~however,~~ it is illustrated that the dual node approach can and should be

270 derived from basic equations that describe infiltration into a porous medium. ~~This derivation is~~

271 ~~inspired by but slightly different from the work of Morita and Yen [*Morita and Yen*, 2002].~~

272 ~~Before deriving the dual node approach from equations that describe infiltration, it is~~

273 ~~worthwhile to point out that above formulation of an exchange flux implies that infiltration only~~

274 ~~occurs across the ponded fraction of the surface-subsurface interface. This is not correct, because~~

275 ~~rainfall typically results in infiltration across non-ponded areas. Although this issue is not a crucial~~

276 ~~problem since the ponded fraction will typically increase during rainfall, it is more elegant to~~

277 ~~account explicitly for infiltration across non-ponded areas. This is relatively straightforward since~~

278 ~~before ponding occurs the infiltration rate equals the rainfall rate if the rainfall rate is smaller than~~

279 ~~the infiltrability and is limited to the infiltrability otherwise [*Hillel*, 1982] and such a computation~~

280 ~~is also used by others [*Morita and Yen*, 2002]. In the approach presented here the surface cell can~~

281 ~~be partially ponded whereas in the work of Morita and Yen [*Morita and Yen*, 2002] a surface cell~~

282 ~~is either ponded or non-ponded.~~

283 Using Darcy's Law, the infiltration rate at the ponded land surface ~~$q_{s \to ss}$~~ $q_{s \to ss}$ [LT^{-1}] can be written

284 as a function of the vertical subsurface hydraulic gradient at the land surface:

285
$$\cancel{q_{s \to ss} = \left(k_r K_z \frac{\partial h}{\partial z} \right)\bigg|_{z=z_s} = K_z \frac{\partial h}{\partial z}\bigg|_{z=z_s}} \quad q_{s \to ss} = \left(k_r K_z \frac{\partial h}{\partial z} \right)\bigg|_{z=z_s} = K_z \frac{\partial h}{\partial z}\bigg|_{z=z_s} \tag{2}$$

286   where h the hydraulic head [L], z the elevation head [L], k_r the relative hydraulic conductivity [-]

287   K_z the saturated vertical hydraulic conductivity [LT^{-1}] and z_s the elevation head at the land surface.

288   The relative hydraulic conductivity is unity because equation ~~(2)~~(2) applies to the ponded land

289   surface which implies fully saturated conditions at the land surface (i.e. ponding means $p_s > 0$,

290   where p_s is the pressure head at the surface). Similarly, the infiltrability [LT^{-1}], defined as the

291   infiltration rate under the condition of atmospheric pressure [*Hillel*, 1982], can be written as:

$$\cancel{I = \left(k_r K_z \frac{\partial h}{\partial z} \right)\Big|_{z=z_s, p_s=0} = K_z \frac{\partial h}{\partial z}\Big|_{z=z_s}} \quad I = \left(k_r K_z \frac{\partial h}{\partial z} \right)\Big|_{z=z_s, p_s=0} = K_z \frac{\partial h}{\partial z}\Big|_{z=z_s} \qquad (3)$$

293   The relative hydraulic conductivity is again unity because the saturation equals unity under

294   atmospheric conditions ($p_s = 0$). The infiltration rate at non-ponded land surface $\cancel{q_{\mathrm{atm}\to ss}}$ $q_{\mathrm{atm}\to ss}$ [LT^{-}

295   ^{1}] can be expressed as:

$$\cancel{q_{\mathrm{atm}\to ss} = \min\left(\max(I,0), q_R\right)} \quad q_{\mathrm{atm}\to ss} = \min\left(\max(I,0), q_R\right) \qquad (4)$$

297   where q_R is the effective rainfall rate (i.e. the infiltration rate is limited by either the infiltrability

298   or the available effective rainfall rate). The total exchange flux across the surface-subsurface

299   interface can now be written as:

$$\cancel{q_e = f_p q_{s\to ss} + (1 - f_p) q_{\mathrm{atm}\to ss}} \quad q_e = f_p q_{s\to ss} + \left(1 - f_p\right) q_{\mathrm{atm}\to ss} \qquad (5)$$

301   To approximate the vertical subsurface hydraulic gradient in equations ~~(2) and (3),~~(2) and

302   (3)_, it is crucial to recognize that according to the principle of head continuity at the land surface,

303   the surface hydraulic head at a surface node must also represent the subsurface head at the land

304   surface at that location. ~~Thus, the surface hydraulic head can be used as a Dirichlet boundary~~

305   ~~condition for the subsurface flow domain. Moreover, it is also crucial to recognize that~~Moreover,

306   since the subsurface hydraulic heads at the topmost subsurface nodes are ideally associated with

307 the centroids of the topmost subsurface discrete control volumes, these head values do not

308 represent values at the land surface but at some depth below the land surface. Because the

309 subsurface hydraulic heads at the dual nodes can be and should be associated with a different

310 elevation, the vertical subsurface head gradient between the dual nodes can be approximated by a

311 standard finite difference approximation. If this approximation is being used to approximate the

312 gradient at the land surface in equations ~~(2) and (3)~~(2) and (3), then this approximation is by

313 definition a one-sided first-order finite difference. Defining the coupling length by ~~$l = \Delta z$~~$l = \Delta z_{dn}$

314 where ~~Δz~~ Δz_{dn} is the difference in the mean elevation head associated with the dual nodes, the

315 infiltration rate and infiltrability can thus be computed with the following one-sided finite

316 difference approximation:

317
$$-K_z \left.\frac{\partial h}{\partial z}\right|_{z=z_s} \approx \frac{K_z}{l}\left(h_s - h_{ss}\right) \qquad K_z \left.\frac{\partial h}{\partial z}\right|_{z=z_s} \approx \frac{K_z}{l}\left(h_s - h_{ss}\right)$$

318   (6)

319 The above definition of the coupling length ~~$l = \Delta z$~~ $l = \Delta z_{dn}$ ensures a proper approximation of the

320 vertical gradient in elevation head at the land surface:

321
$$\left.\frac{\partial z}{\partial z}\right|_{z=z_s} = \frac{\Delta z}{l} = 1 \qquad \left.\frac{\partial z}{\partial z}\right|_{z=z_s} = \frac{\Delta z_{dn}}{l} = 1$$

322   (7)

323 The above derivation of the consistent dual node approach from basic flow equations has

324 implications for how the dual node approach is conceptualized and how it should be implemented.

325 The idea that the coupling length must be directly related to the spatial discretization is an

326 important new insight. Namely, as the coupling length is related to grid topology, it does not

327   represent a non-physical parameter associated with a distinct interface separating the two domains.

328   It is also crucial to observe the difference between the consistent dual node approach and the

329   common node approach regarding how the head continuity at the surface-subsurface interface is

330   formulated. As explained in Section 2, the formulation in the common node approach is only

331   correct if the topmost subsurface discrete volumes are very thin. In comparison, the formulation

332   in the dual node approach is correct irrespective of the vertical discretization. Namely, irrespective

333   of the vertical discretization the surface hydraulic heads equal the subsurface heads at the interface.

334   Since nodal values in cell-centered scheme are located at the centroids of the cells, the

335   coupling length is simply given by ~~$l = z_s - z_{ss}$~~. $l = z_s - z_{ss}$ where z_s and z_{ss} are the elevation heads

336   [L] associated with the surface node and the topmost subsurface node, respectively. This value ~~has~~

337   for the coupling length in cell-centered schemes has also been ~~proposed~~suggested by ~~others~~Panday

338   and Huyakorn [*Panday and Huyakorn*, 2004]~~. However, in vertex-centered schemes the~~

339   ~~commonly used nodal configuration near the surface is such that $z_s = z_{ss}$. If these elevation heads~~

340   ~~are used as the elevation heads at the dual nodes then $\Delta z = z_s - z_{ss} = 0$. Since the coupling length~~

341   ~~must be greater than zero, the coupling length cannot be defined as $l = \Delta z$. Indeed, the coupling~~

342   ~~length in vertex-centered schemes is typically not related to grid structure [*Liggett et al.*, 2013].~~

343   ~~However, if $\Delta z = 0$ and the coupling length is some lumped-parameter greater than zero, then the~~

344   ~~dual node approach is inconsistent. Namely, if $\Delta z = 0$ then the gradient in elevation head between~~

345   ~~the dual nodes equals zero. This may seem correct as the nodes are co-located. However, if $z_{ss} = z_s$~~

346   ~~, then the physical principle of head continuity implies that $p_{ss} = p_s$ must also hold. Moreover~~.

347   However, in their work, the particular advantage of choosing this value (i.e. maintaining a unit

348   gradient in elevation head) is not recognized. The coupling schemes as used by An and Yu [*An*

349 *and Yu*, 2014] and Kumar et al. [*Kumar et al.*, 2009] are also in essence consistent dual node

350 schemes. However, these schemes are not recognized as a dual node scheme. Instead, An and Yu

351 [*An and Yu*, 2014] argue that their scheme is similar to the common node approach of Kollet and

352 Maxwell [*Kollet and Maxwell*, 2006]. Kumar et al. [*Kumar et al.*, 2009] argue that their scheme is

353 similar to the dual node approach if the coupling length goes to zero which implies that their

354 scheme would be similar to the common node approach. However, contrary to the common node

355 approach the schemes of An and Yu [*An and Yu*, 2014] and Kumar et al. [*Kumar et al.*, 2009]

356 compute exchange fluxes between surface and topmost subsurface nodes and therefore these

357 schemes are technically dual node schemes. As explained in this study, it is crucial to observe that

358 the schemes of An and Yu [*An and Yu*, 2014] and Kumar et al. [*Kumar et al.*, 2009] are actually

359 quite different from the common node approach. As already mentioned, the consistent dual node

360 scheme differs from the common node approach with respect to how the head continuity is

361 formulated at the surface-subsurface interface. As discussed later on, this difference has crucial

362 consequences in terms of accuracy as well as numerical efficiency.

363 In vertex-centered schemes the commonly used nodal configuration near the surface is such that

364 $z_\mathrm{s} = z_\mathrm{ss}$. However, even though the topmost subsurface node is located at the land surface in a

365 vertex-centered scheme, the elevation head at this node should ideally correspond to the mean

366 elevation head within the topmost subsurface discrete ~~control volume such that $z_\mathrm{ss} < z_\mathrm{s}$.~~volume.

367 This suggests that the topmost subsurface node should be moved to the centroid of the topmost

368 subsurface discrete volume. Although this is a possible solution, the drawback of this solution is

369 that the subsurface model ceases to be a purely vertex-centered scheme. Moreover, such an

370 operation cannot be performed in finite element schemes since ~~changing~~ the nodal positions ~~would~~

371 ~~change~~define the geometry of the elements. Therefore, an alternative solution is proposed. ~~To~~

372 ~~enforce $l = z_s - z_{ss}$ without affecting the relative positions of nodes~~Namely, in ~~the subsurface~~

373 ~~grid,~~vertex-centered schemes the elevation of the surface nodes are changed according to

374 ~~$z_s = z_{ss} + l$~~ $z_s = z_{ss} + l$ where l is equals half the thickness of the topmost subsurface dual cell. ~~This~~

375 ~~change does also not affect the relative position of the nodes in the surface grid.~~ The resulting

376 nodal configuration is illustrated in Figure 1d. When applying this solution, all the topmost

377 subsurface cells must have the same thickness, such that the topography is increased with the same

378 value everywhere. In essence, the motivation behind this solution is that a more accurate

379 approximation the hydraulic gradient (i.e. enforcing a unit gradient in elevation head) is more

380 important than the actual elevation of the land surface. ~~Indeed it can be argued that~~ Similar to the

381 ~~change in land elevation will not drastically affect~~nodal configuration in ParFlow, the ~~timing of~~

382 ~~runoff.~~resulting nodal configuration may not seem ideal. Namely, ~~once~~ the ~~topmost~~ surface

383 elevation does not coincide with the top of the subsurface ~~node reaches fully saturated conditions,~~

384 ~~the amount of additional water needed to reach the elevated land surface is minor~~grid. Nonetheless,

385 as ~~long as the specific storage assigned to the topmost dual cell is relatively small.~~

386 ~~It is crucial to observe that the proposed dual node implementation is not based~~illustrated

387 later ~~on assuming a distinct interface with a certain thickness between the subsurface and the~~

388 ~~surface. Instead, the coupling length is to be interpreted as a distance between dual nodes that~~

389 ~~accounts for the fact that the topmost subsurface nodal value ideally corresponds to a value below~~

390 ~~the land surface. This distance is related to the vertical discretization near the land surface and as~~

391 ~~such does not represent a non-physical parameter associated~~, simulation results obtained with ~~a~~

392 ~~distinct interface separating the two domains.~~

393 ~~The common conceptualization of the dual node approach as a hydraulic separation by a~~

394 ~~interface with a given thickness [Kollet and Maxwell, 2006; Liggett et al., 2012; Liggett et al.,~~

395 2013], may arise if dual node approach is interpreted as a second-order central finite difference

396 approximation evaluated at the centre of a saturated layer with a thickness equal to the coupling

397 length. If in addition the topmost subsurface head values are taken as values at the land surface,

398 then it follows that the dual node approach introduces a distinct interface between the two flow

399 domains. However, as explained the topmost subsurface head values should not be taken as values

400 at the land surface but as values at some distance from the land surface, such that the interface

401 defined by the coupling length occupies the upper half of the topmost subsurface discrete control

402 volumes.

403 It is also worthwhile to explain in further detail that the dual node approach does not

404 account for the relative hydraulic conductivity near the land surface. This does not imply that the

405 subsurface near the land surface is saturated. Namely, saturation in the topmost subsurface discrete

406 volume is computed with the pressure head at the topmost subsurface node which may well be

407 below zero. It may appear that the vertical hydraulic conductivity between the dual nodes should

408 be computed by weighting the vertical hydraulic conductivities at the dual nodes, which would

409 result in a dependency on the relative hydraulic conductivity as long as the topmost subsurface

410 node is not fully saturated. However, no weighting is needed if the dual node approach is

411 understood as a one-sided finite difference evaluated at the land surface. Namely, the vertical

412 hydraulic conductivity at the land surface is readily available. This is a difference with respect to

413 the approach of Morita and Yen [*Morita and Yen*, 2002] who do use a weighting the resulting

414 scheme. Moreover, models typically apply upstream weighting to approximate the relative

415 hydraulic conductivities between nodes to avoid numerical instabilities [*Forsyth and Kropinski*,

416 1997]. Thus even if weighting is applied, then the dependency of the computations between the

417 ~~dual nodes on the relative hydraulic conductivity will automatically disappear as the upstream~~

418 ~~node is always saturated.~~ are reasonable.

419   To illustrate that the presented dual node approach exhibits consistent behaviour, the

420 necessary conditions for ponding due to excess infiltration and exfiltration are considered. In

421 general ponding starts when $q_R > I$ [*Hillel*, 1982]. Setting $q_R = I$, $p_s = 0$ and using $h = p + z$, it

422 follows from equation ~~(6)~~(6) and (7) that at the moment of ponding:

423 $$ \qquad\qquad p_{ss} = l\left(1 - \frac{q_R}{K_z}\right) \qquad\qquad\qquad\qquad p_{ss} = l\left(1 - \frac{q_R}{K_z}\right) $$

424   (8)

425 Ponding due to excess infiltration occurs if ~~$q_R/K_z > 1$~~ $q_R/K_z > 1$ and implies that saturation in the

426 subsurface starts from the top down [*Hillel*, 1982]. Using ~~$q_R/K_z > 1$~~ $q_R/K_z > 1$ it follows from

427 equation ~~(8)~~(8) that ponding due to excess infiltration occurs while ~~$p_{ss} < 0$~~ $p_{ss} < 0$. This is

428 reasonable since this value represents the pressure head at a certain depth below the land surface.

429 Namely, if saturation occurs from the top-down then the saturation at a certain depth occurs later

430 than saturation at the land surface. It is noted that if the ratio ~~q_R/K_z~~ q_R/K_z is greater than but close

431 to unity or if the coupling length is very small, then this condition becomes ~~$p_{ss} \approx 0$~~ $p_{ss} \approx 0$. Once

432 ponding starts the total flux rate between the dual nodes equals ~~$K_z\left((p_s - p_{ss})/l + 1\right)$~~

433 $K_z\left((p_s - p_{ss})/l + 1\right)$. Top-down saturation requires that this flux exceeds the vertical hydraulic

434 conductivity. Reaching saturation at the topmost node (~~$p_{ss} = 0$) thus~~ $p_{ss} = 0$) therefore requires

435 ~~$p_s \geq 0$~~ $p_s \geq 0$. Thus, top-down saturation will occur after ponding is initiated. Ponding due to

436 excess saturation occurs if ~~$q_R/K_z < 1$~~ $q_R/K_z < 1$ and implies that saturation in the subsurface starts

437 from the bottom up [*Hillel*, 1982]. ~~Using $p_s = 0$,~~ It follows from equation ~~(8)~~(8) that ponding due

438 to excess saturation occurs while ~~$0 < p_{ss} < l$.~~ $0 < p_{ss} < l$. Thus ponding starts after reaching fully

439 saturated conditions at the topmost subsurface node, which is again reasonable. ~~Namely, the~~

440 ~~topmost subsurface node represents a value at a certain depth below the surface and thus bottom-~~

441 ~~up saturation implies that this node reaches saturation earlier than the surface. It is noted that if the~~

442 ~~ratio q_R/K_z~~ It is noted that if the ratio q_R/K_z is smaller than but close to unity or if the coupling

443 length is very small, then ponding occurs when ~~$p_{ss} \approx 0$~~ $p_{ss} \approx 0$.

~~4.1~~4.3 **Comparison to ~~other dual node implementations~~alternative coupling approaches**

445 To illustrate that it is crucial to account for the meaning of the values at the topmost subsurface

446 nodes, it is instructive to consider what happens if these values are not taken as the mean values

447 within discrete control volumes. As a first example, consider vertex-centered schemes where the

448 dual nodes are defined such that ~~$z_{ss} = z_s$~~ $z_{ss} = z_s$ as illustrated in Figure ~~2c. As discussed in Section~~

449 ~~4~~1c. This is inconsistent because it defines a zero gradient in elevation head between the dual

450 nodes. ~~Nonetheless such schemes have been used in several models [*Ebel et al.*, 2009; *Liggett et*~~

451 ~~al., 2012].~~ Since the vertical gradient in elevation head between the dual nodes is zero the total

452 flux rate after ponding now equals ~~$K_z(p_s - p_{ss})/l$.~~ $K_z(p_s - p_{ss})/l$. Top-down saturation requires

453 that this flux exceeds the vertical hydraulic conductivity. Thus, reaching saturation at the topmost

454 subsurface node (~~$p_{ss} = 0$~~ $p_{ss} = 0$) requires ~~$p_s > l$~~ $p_s > l$. Therefore, top-down saturation will not

455 occur if runoff occurs and if the surface water depths remains smaller than the chosen coupling

456 length. Indeed, it has been pointed out in other studies that the coupling length should be smaller

457 than the rill storage height [*Delfs et al.*, 2009; *Liggett et al.*, 2012]. The zero vertical gradient in

458 elevation head between the dual nodal also means that the required condition for ponding now

459 becomes ~~$p_{ss} = lq_R/K_z$~~ $p_{ss} = -lq_R/K_z$. This implies that ponding due to excess saturation occurs

460 while the topmost subsurface node is not yet saturated. This dual node approach has been compared

461 to the common node approach in vertex-centered schemes [*Liggett et al.*, 2012].

462   A second example is the dual node approach for cell-centered schemes as implemented in

463 MODHMS which uses an adapted pressure-saturation relationship for the topmost subsurface

464 nodes such that the topmost subsurface node only becomes fully saturated if hydraulic head at the

465 node rises above the land surface [*Liggett et al.*, 2013]. Since the topmost subsurface heads are

466 associated with the cell centroid, this dual node scheme defines a unit gradient in elevation head

467 at the land surface. However, the saturation value at the topmost node is associated with a location

468 at the land surface and not with the centroid of a discrete control volume. This has undesirable

469 consequences. Namely, saturating the topmost subsurface node ($p_{ss} = l$) due to excess infiltration

470 requires that ~~$p_s > l$~~ $p_s > l$. Indeed, when simulating excess infiltration with MODHMS, a very

471 small coupling length is needed to simulate top-down saturation due to excess infiltration.

472 [*Gaukroger and Werner*, 2011; *Liggett et al.*, 2013]. It can also be shown that ponding due to

473 excess saturation occurs while ~~$0 < p_{ss} < l$~~ $0 < p_{ss} < l$. But, because of the adapted pressure-

474 saturation relationship this means that ponding starts while the topmost subsurface node is not yet

475 saturated. ~~Comparing these results with the results for the consistent~~This dual node

476 ~~implementation, it is clear that the adapted pressure-saturation relationship~~approach has

477 ~~undesirable consequences.~~been compared to the common node approach in cell-centered schemes

478 [*Liggett et al.*, 2013].

479 ~~The above inconsistent implementations of the dual node approach have been used in several~~

480 ~~studies to compare the dual node approach with the common node approach~~ The two

comparison studies of Liggett et al. [*Liggett et al.*, 2012; *Liggett et al.*, 2013]. Such studies indicate that the dual node approach is typically only competitive with the common node approach in terms of accuracy once the coupling lengths arelength is very small. However, the requirement for a very small coupling lengths, however, are a directlength, is a logical consequence of using inconsistent dual node approaches. Namelyif the topmost subsurface nodal values are not taken as the mean values within discrete volumes. In essence, by choosing a very small coupling lengths these inconsistencies are to some extent length this inconsistency is minimized. At best this minimization results in schemes that mimic the common node approach. However, as discussed, the common node approach is also inconsistent since it is not in agreement with the physical principle of a head continuity at the surface-subsurface interface. Since current views on how This contrasts with the consistent dual approach in which decreasing the coupling approaches compare are based on inconsistent dual node approaches, it is imperative to re-evaluate how the dual and common node approaches compare if the dual node approach is properly implemented.

Considering how the dual and the common node approach compare it is also crucial that the dual node approach is not to be conceptualized as a hydraulic separation between the flow domains in the form of length for a saturated interface. Namely, this conceptualization is often deemed a serious drawback of the dual node approach, since there is no evidence of such a distinct interface. Moreover, misconceptions about the coupling approaches can result in confusion. For example, in their paper An and Yu [*An and Yu*, 2014] reject the idea of using the dual node based on its classical conceptualization as a saturated interface and argue that their model is based on the approach proposed by Kollet and Maxwell [*Kollet and Maxwell*, 2006]. However, in their finite volume model the surface and subsurface nodes are not co-located. As such their coupling approach is, contrary to the claim of the authors, a dual node approach. This misunderstanding is probably also

related to aforementioned difficulties in inferring the nodal configuration as used in ParFlow. Nonetheless, their approach is actually a properly implemented dual node approach practically similar to the one proposed in this paper. Interestingly, the model of An and Yu [*An and Yu*, 2014] is less sensitive to thegiven vertical discretization near the land surface in comparison to ParFlow However, since An and Yu were convinced that they followed the same coupling approach as ParFlow they hypothesized that the difference in performance was probably related to using irregular grids instead of orthogonal grids as in ParFlow [*An and Yu*, 2014]. However, if this difference is instead due to using a different coupling approach, then will result in more inaccurate simulation results as this would be an indication that a dual node approach is less sensitive to the vertical discretization near the land surface. This reinforces the idea that it is desirable to reconsider the comparison between the two coupling approachesnumerically incorrect.

CATHY [*Camporese et al.*, 2010] as well as the model of Morita and Yen [*Morita and Yen*, 2002] are examples of models which are neither based on the common node approach, nor a dual node approach. Both these models are conjunctive models in which the surface and subsurface flow are computed separately in a sequential fashion and in which coupling is established by matching the flow conditions along the surface-subsurface interface. A complete discussion is outside the scope of this paper, but it is worthwhile to mention that these models share some crucial characteristics with the consistent dual node approach. Although the two models are different, both models switch between appropriate boundary conditions along the surface-subsurface interface, such that infiltration fluxes are limited to the infiltrability. In both models the infiltration fluxes are computed while accounting for the unit vertical gradient in elevation head near the surface-subsurface interface. In addition, in both models ponding occurs when the infiltrability is exceeded.

5   Numerical experiments

5.1   Numerical model

To compare the ~~coupling schemes~~consistent dual node approach with respect to the common node approach in terms of accuracy and computational efficiency numerical experiments are presented. These experiments are carried out with the model code DisCo ~~which~~. This model code can simulate coupled surface-subsurface flow with the dual node approach using a fully implicit or monolithic scheme [*de Rooij et al.*, 2013]. ~~This means that~~Subsurface flow is governed by the ~~linearized~~Richards' equation while surface flow is governed by the diffusive wave equation.

Starting from a dual node scheme, the implementation of a common node scheme is relatively straightforward. If the surface nodes are numbered last, a permutation vector can be constructed which gives the corresponding topmost subsurface node for each surface node. Then, the node numbering as used in the original dual node scheme can still be used to compute the surface and subsurface flow ~~equations are~~terms. Subsequently, using the permutation vector the surface and subsurface flow terms associated with a common node can be combined into ~~a single~~the same row of the global matrix system. In addition, when using the common node approach, there is no need to evaluate exchange flow terms between the two flow domains. It is noted that the surface flow and subsurface flow computations are exactly the same irrespective of the coupling approach. As such the model permits to compare the two approaches in terms of accuracy as well as numerical efficiency.

An adaptive error-controlled predictor-corrector one-step Newton scheme [*Diersch and Perrochet*, 1999] is used in which a single user-specified parameter controls the convergence as well the time stepping regime. ~~It is assumed that by using the same error norms and the same model parameters that control the time-stepping, the simulations results as obtained by different~~

550 ~~coupling approaches can be compared fairly in terms of accuracy and efficiency.~~Although, this

551 scheme may not be necessary the most efficient scheme, it ensures that time discretization error is

552 the same irrespective of the applied coupling approach. For brevity further details about the model

553 are not discussed here and can be found elsewhere [*de Rooij et al.*, 2013].

5.2   Hillslope scenarios

555 The model code is applied to a set of three hillslope scenarios. Table 1 lists the abbreviations used

556 in the figures to distinguish between the coupling approaches, and to distinguish between cell-

557 centered and vertex-centered schemes ~~and to distinguish between models based on a~~. Each

558 scenarios is solved using different but uniform ~~primary grid and grids that use a very thin primary~~

559 ~~top cell. The thickness of this top cell equals the thickness of the primary cells in the finest uniform~~

560 ~~grids. In models containing this thin layer of cells the~~ vertical discretizations and Δz specifies the

561 discretization ~~below the thin layer is based on the coarsest uniform grids. Further details about the~~

562 ~~discretizations are given in the figures.~~

563 ~~The presented experiments focus mainly on the comparison between the consistent dual~~

564 ~~node approach and the common node approach. Inconsistent dual node implementations based on~~

565 ~~a zero hydraulic head gradient between the dual nodes are only considered for relatively coarse~~

566 ~~vertical discretizations to illustrate their short-comings vis-à-vis the consistent dual node approach.~~

567 ~~It is noted, that although these schemes are commonly used in vertex-centered schemes, for the~~

568 ~~purpose of this study they have also been implemented in the cell-centered schemes by using the~~

569 ~~nodal configuration depicted in Figure 1a. The scheme with an adapted pressure-saturation~~

570 ~~relationship is not considered.~~

5.1 Soil column problems

These simulation scenarios consider infiltration into a vertical soil column and are inspired by scenarios as studied by Liggett et al. [*Liggett et al.*, 2012; *Liggett et al.*, 2013]. In the simulation scenarios rainfall is applied to a soil column with a height of 5 m. Initial conditions are defined by $h = 0$ m. The saturated conductivity is 1.0608 md^{-1}. The porosity is 0.41 and the specific storage is 10^{-4} m^{-1}. The van Genuchten parameters are given by $s_r = 0.387$, $s_s = 1.0$, $\alpha = 7.5$ m^{-1} and $n = 1.89$.

For the first two scenarios a constant head boundary of $h = 0$ m is applied at the bottom of the column and the flux rate applied to the top of the soil column exceeds the saturated conductivity of the soil column, resulting in runoff due to excess infiltration. In the first scenario the applied flux rate is 1.1 md^{-1}. Figure 2 and 3 illustrates the simulated runoff and the number of Newton steps for this scenario, respectively. Figure 4 illustrates the simulated runoff for the second scenario in which the flux rate is 10.608 md^{-1}. It is noted that figure 4 does not display the results at later times when a steady-state is reached. However, to show the differences in results around the timing of ponding only a limited time period is displayed. Figure 5 illustrates the number of Newton steps for the second scenario. For the second scenario, Figure 6 compares the evolution in water depth between the common node approach and the dual-node approach when using a relatively coarse vertical discretization and a cell-centered scheme.

To compare the different coupling approaches when simulating excess saturation, a third scenario is considered. The model setup is exactly the same as before, except that the effective rainfall rate is set to 0.5 md^{-1} and that the bottom boundary is changed into a no-flow boundary. The simulated runoff is depicted in Figure 7. Figure 8 shows the total number of Newton steps during the model runs. Figure 9 compares the evolution in water depth between the common node

594 ~~approach and the dual node approach when using a relatively coarse vertical discretization and a~~

595 ~~cell-centered scheme.~~

596 ~~**5.2 Hillslope problems**~~

597 ~~In the following~~of the primary grid. The first two simulation scenarios consider hillslope problems

598 as designed by Sulis et al. [*Sulis et al.*, 2010]. For the purpose of this study, a third scenario is

599 considered in which the initial and boundary conditions are different to create a flooding wave

600 across an unsaturated hillslope. The problems consist of a land surface with a slope of 0.05 which

601 is underlain by a porous medium. The domain is 400 m long and 80 m wide. The subsurface is 5

602 m thick. In the direction of the length and in the direction of the width the discretization is 80 m.

603 Different vertical discretizations are considered. The van Genuchten parameters are given by s_r =

604 0.2, s_s = 1.0, α = 1 m^{-1} and n = 2. The porosity is 0.4 and the specific storage is 10^{-4} m^{-1}. The

605 manning's roughness coefficients are given by 3.3 x 10^{-4} m$^{-1/3}$min. The surface flow domain has a

606 zero-gradient outflow condition. For the first two simulation scenarios the domain is recharged

607 with an effective rainfall rate of 3.3 x 10^{-4} m/min for a duration of 200 minutes and the initial water

608 table depth is at a depth of 1.0 m below the land surface.

609 The first scenario considers excess ~~infiltration and the saturated hydraulic conductivity~~

610 ~~equals 6.94 x 10^{-6} m/min. Figure 10 and 11 show the simulated runoff and the number of Newton~~

611 ~~steps, respectively. For the second scenario which considers excess~~ saturation, the saturated

612 conductivity equals 6.94 x 10^{-4} m/min. Figure ~~12~~2 and ~~13~~3 illustrates the simulated runoff and the

613 number of Newton steps, respectively. Figure 4 and 5 illustrate the subsurface pressure heads at

614 the topmost subsurface nodes and the water depths on the surface nodes. For the second scenario

615 which considers excess infiltration the saturated hydraulic conductivity equals 6.94 x 10^{-7} m/min.

616 Figure 6 and 7 show the simulated runoff and the number of Newton steps, respectively. Figure 8

617 and 9 illustrate the subsurface pressure heads at the topmost subsurface nodes and the water depths

618 on the surface nodes for the finest and the coarsest vertical discretization, respectively. In the third

619 scenario a surface water flood wave crossing the hillslope in the downhill direction is simulated

620 by applying a Neumann boundary condition of 1.0 m^3/s for a duration of 200 minutes to the surface

621 nodes with the highest elevation. The initial water table is located at a depth of 1.5 m. The vertical

622 saturated hydraulic conductivity equals 6.94 x 10^{-6} m/min. Figure ~~14~~10 illustrates the differences

623 in simulated runoff and Figure ~~15~~11 illustrates the number of Newton steps of the model runs.

624 Figure ~~16 compares~~12 and 13 illustrate the ~~evolution in~~ subsurface pressure heads at the topmost

625 subsurface nodes and the water ~~depth~~depths on the surface nodes ~~as well as the time step sizes~~

626 ~~between the common node approach and~~ for the ~~dual node approach when using a relatively~~

627 ~~coarse~~finest and the coarsest vertical discretization ~~and a cell-centered scheme~~, respectively.

628 **6   Discussion**

629 **6.1   Accuracy**

630 ~~Considering the simulation of vertical flow through the unsaturated zone, a relatively fine vertical~~

631 ~~discretisation is needed to simulate sharp saturation fronts with the Richards' equation [*Pan and*~~

632 ~~*Wierenga*, 1995; *Ross*, 1990]. A relatively fine vertical discretisation also implies that the common~~

633 ~~node approach will be in close agreement with the physical principle of head continuity along the~~

634 ~~surface-subsurface interface. Finally, if the vertical discretisation is relatively small then the~~

635 ~~coupling length for the consistent dual node approach is also small and this implies that the dual~~

636 ~~node approach mimics the common node approach. Therefore, it is expected that the coupling~~

637 ~~approaches will give similar and accurate results if the vertical discretization is sufficiently fine.~~

638 ~~Indeed, the simulations results indicate that a relatively fine and uniform vertical discretization~~

639 ~~yields similar results for the common node approach as well as for the consistent dual node~~

640 ~~approach (Figure 2a, 4a, 5a, 7a, 10a, 12a and 14a). The simulation results based on the finest~~

641 ~~vertical discretization may thus be taken as reference solutions that enables a comparison of the~~

642 ~~coupling approaches when a coarser vertical discretization is used. This is an important issue,~~

643 ~~because using a relatively coarse vertical discretization is common practice in regional coupled~~

644 ~~surface-subsurface models [*Jones et al.*, 2008; *Kollet and Maxwell*, 2008; *Srivastava et al.*, 2014].~~

645 ~~6.1.1 Excess saturation~~

646 As discussed by Ebel et al. [*Ebel et al.*, 2009] and confirmed by others [*Liggett et al.*, 2012] the

647 dual node approach mimics the common node approach if the coupling length becomes sufficiently

648 small. When comparing the consistent dual node approach and the common node approach a very

649 similar observation applies. If the topmost subsurface cells are very thin, then the coupling length

650 in the consistent dual node approach is very small. Also, if the topmost subsurface cells are

651 sufficiently thin then the formulation of head continuity at the surface-subsurface interface in the

652 common node approach is correct. Thus, the common node approach will mimic the consistent

653 dual node approach. Indeed, the simulations results indicate that a relatively fine vertical

654 discretization yields similar results for the common node approach as well as for the consistent

655 dual node approach (Figure 2a, 4a, 6a, 8a, 10a and 12a).

656 A relatively fine uniform vertical discretisation also enables to simulate sharp saturation

657 fronts with the Richards' equation [*Pan and Wierenga*, 1995; *Ross*, 1990]. As such the simulation

658 results based on the finest vertical discretization can be taken as reference solutions that enables

659 comparisons of the coupling approaches when a coarser vertical discretization is used.

6.1.1 Excess saturation

The simulation results of runoff due to excess saturation as obtained by the common node approach and the consistent dual node approach as depicted in Figure 2 illustrate that simulating excess saturation runoff is not significantly affected by the vertical discretization ~~(Figure 7 and 12).~~. This is because the time needed to reach fully saturated conditions in the subsurface is a simple function of the flow boundary conditions and the initial water content. It is thus expected that the vertical discretization does not significantly affect the simulation of excess saturation. Although the vertical discretization may affect the computed initial water content, this effect is usually negligible. It has been found in other studies that the vertical discretization has little effect on simulated runoff due to excess saturation [*Sulis et al.*, 2010].

~~As described in Section 4, when using the consistent dual node approach, ponding due to excess saturation occurs when $0 < p_{ss} < l$. Thus at the moment of ponding the hydraulic head at the topmost subsurface node is generally below the land surface. When using the common node approach, the hydraulic head at the topmost subsurface node is at the land surface at the moment of ponding. However, if the specific storage is relatively small, then the timing of runoff will be similar for both coupling approaches. Both approaches are thus expected to yield similar and reasonably accurate results even when the vertical discretization is relatively coarse. Indeed, the simulation results indicate that there is little difference between the common node approach and the consistent dual node approach (Figure 7 and 12).~~

~~As indicated in figure 7d, when using an inconsistent dual node approach, the timing of runoff may be underestimated unless a very small coupling length is being used. As discussed in section 5 this is expected.~~

6.1.2 Excess infiltration

When simulating excess infiltration the common node approach requires fully saturated conditions at the topmost subsurface node for ponding to occur. ~~This is a direct consequence of the head continuity between the surface nodes and the topmost subsurface nodes.~~ However, top-down saturation associated with excess infiltration implies that reaching fully saturated conditions in the topmost subsurface discrete volumes should requires more time than reaching fully saturated conditions ~~in~~at the ~~very near~~land surface, especially if the vertical discretization is relatively coarse. It is thus expected that the common node approach delays runoff and that this delay increases for a coarser vertical discretization. In addition, if the saturation fronts are less sharp due to a relatively coarse vertical discretization, it takes more time to reach saturated conditions at the common node. This will further delay runoff. Indeed, the simulation results indicate clearly that runoff is delayed when using the common node approach, particularly if the vertical discretization is relatively coarse (Figure ~~2, 4~~6, 9a, 10 and ~~14~~13a). It has also been found in other studies that the common node approach delays runoff due to excess infiltration if the vertical discretization is relatively coarse [*Sulis et al.*, 2010]. ~~The overestimation of the infiltration associated with the delay in runoff may result in runoff due to excess saturation even if the applied flux rate should result in runoff due to excess infiltration. This is illustrated in Figure 10c for the model run based on a cell-centered scheme and the common node approach. This Figure illustrates that overestimating the infiltration can yield a distinctive higher peak in runoff. Comparing this peak with the runoff responses in Figure 12, it is clear that this model run simulates runoff due to excess saturation~~

~~In comparison,~~As explained in Section 4.2, when using the consistent dual node ~~displays more desirable behaviour. Namely, as explained in Section 4~~approach, ponding due to excess

705 infiltration occurs before reaching fully saturated conditions at the topmost subsurface node ~~which~~

706 ~~is arguably more correct if saturation occurs from the top-down, particularly if the vertical~~

707 ~~discretization is relatively coarse. When using the consistent dual node approach, the moment of~~

708 ~~ponding depends on the computation of the infiltrability.~~. More specifically, ponding occurs when

709 the infiltrability is exceeded. Compared to the condition for ponding in the common node approach

710 this is arguably more correct. Namely, if saturation occurs from the top-down then the saturation

711 at a certain depth occurs later than saturation at the land surface. Indeed, simulation results indicate

712 that when simulating excess infiltration the consistent dual node approach is less sensitive to the

713 vertical discretization in comparison to the common node approach. This is clearly indicated in

714 Figure 6b-d, 9a, 10b-d and 13a. To further explain this difference in accuracy, it is emphasized

715 that the spatial resolution only affects the accuracy of the flow computations when using the

716 consistent dual node approach and that the formulation of head continuity at the interface remains

717 correct. In contrast, when using the common node approach, if the spatial resolution is too coarse

718 then this does not only affect the accuracy of the flow computations but in addition the formulation

719 of head continuity becomes incorrect. It must be emphasized, however, that regardless of the

720 applied coupling approach, the vertical discretization must be relatively fine. As indicated by

721 Figure 6b-d, 9a, 10b-d and 13a the difference between the simulated results and the reference

722 solution increase for a coarser discretization. Eventually such differences will lead to unreasonable

723 results regardless of the coupling approach.

724 It is interesting to note that An and Yu [*An and Yu*, 2014] also found that their model was

725 less sensitive to the vertical discretization in comparison to ParFlow when simulating runoff due

726 to excess infiltration. Whereas An and Yu [*An and Yu*, 2014] hypothesized that this difference in

727 performance was related to using irregular grids instead of orthogonal grids as in ParFlow, it is

728 argued here that this difference can be explained by the fact that both models use a different

729 coupling approach.

730   Although the consistent dual node approach is less sensitive to the vertical discretization

731 in comparison to the common node approach, it is useful to explain in detail how the vertical

732 discretization affects the accuracy of the consistent dual node approach to the vertical

733 discretization. A relatively coarse vertical discretization may result in an underestimation of the

734 vertical pressure gradient at the land surface. This is because in a soil close to hydrostatic

735 conditions the pressure heads increase with depth. Therefore, the infiltrability during the early

736 stages of infiltration may be underestimated. If the applied flux rate is sufficiently large such that

737 the underestimated infiltrability is exceeded, then ~~this underestimation will result in an~~

738 ~~underestimation of the timing of~~ runoff~~. It may~~ during the early stages will be ~~observed from~~

739 ~~equation (8) that if the ratio q_R/K_z or the coupling length is sufficiently large, then ponding is~~

740 ~~initiated immediately.~~overestimated. Figure ~~10c and 14c illustrate~~6d illustrates that the ~~timing of~~

741 runoff ~~can indeed be underestimated due to~~as simulated with the cell-centered scheme, a relatively

742 coarse vertical discretization ~~when using the~~and a consistent dual node approach~~. However~~, is

743 indeed overestimated at early times. During the later stages of infiltration the pressure head at the

744 topmost subsurface node will be underestimated due to the combined effect of an underestimated

745 infiltration rate and the overly diffused saturation fronts. This results in an overestimation of the

746 infiltration rate in the later stages. Thus at some time after ponding has started, it is expected that

747 the amount of runoff is underestimated. ~~Contrary to the common node approach, however, there~~

748 ~~will be a time at which runoff is simulated correctly (Figure 10c and 14c).~~

749   If the ~~applied flux rate is not sufficiently large, then the~~ underestimated infiltrability ~~in the~~

750 ~~early stages of infiltration will not be~~ is not exceeded~~. In that case~~, then the overly diffused

751   saturation fronts resulting from a relatively coarse vertical discretization will eventually lead to an

752   underestimation of pressure head at the topmost subsurface node and as such the infiltrability may

753   be overestimated at later times. Consequently, when using the consistent dual node approach ~~the~~

754   ~~timing of runoff due to excess infiltration may also be underestimated. As discussed in section 4~~

755   ~~if the ratio q_R/K_z goes to unity, then the consistent dual node approach behaves practically similar~~

756   ~~to the common node approach. Indeed, Figure 2b which depicts a simulation with a relatively small~~

757   ~~ratio q_R/K_z clearly illustrates that the timing of runoff may be underestimated when using the~~

758   ~~consistent dual node approach.~~runoff due to excess infiltration may be delayed. However, the delay

759   in runoff as simulated by the consistent dual node approach will only equal the delay in runoff as

760   simulated by the common node approach in the limit when q_R/K_z goes to unity. Namely, as

761   explained in Section 4.2 if q_R/K_z goes to unity, then the consistent dual node approach behaves

762   similar as a common node approach. However, in general, if the consistent dual node approach

763   delays runoff, this delay will be smaller than the delay in runoff as simulated by the common node

764   approach ~~(Figure 2b). Overall, regardless if the consistent dual node approach underestimates of~~

765   ~~overestimates the timing of runoff, the simulation results indicate that the consistent dual node~~

766   ~~approach is generally less inaccurate than the common node approach for simulating excess~~

767   ~~infiltration when using a relatively coarse uniform vertical discretization.~~.

768   Comparing Figure 12a and 13a it can be observed that if the vertical discretization is

769   relatively coarse then a common node can act as an artificial barrier for a surface water wave

770   advancing across an initially unsaturated subsurface domain. Namely, as the wave travels

771   downstream the wave can only advance to the next common node once it is fully saturated. The

772   effect of this artificial barrier is that the front of the surface water wave is steepened. In contrast,

the consistent dual approach simulates a wave that becomes less steep as it advances downstream

for relatively fine as well as relatively coarse vertical discretizations as depicted in Figure 13a.

As illustrated in Figure 2b, 4b,6b-d, and 10b and 14b-d, if the coupling approach and the vertical discretization are identical and if the thin layer is absent, then the vertex-centered schemes are more accuratecloser to the reference solution with respect to the cell-centered schemes. This difference in accuracy results solely from the fact the primary mesh is the same for both schemes. As such the vertical extent of the topmost subsurface volumes is twice as small when using the vertex-centered scheme. This difference in vertical grid resolution near the land surface explains the differences in accuracy between the schemes.

When using a thin layer at the top of the model the common node approach and consistent dual node approach provide similar simulation results as shown in Figure 2c, 4c, 10d and 14d. This is expected, because the thin layer implies a small coupling length and as such the consistent dual node approach mimics the common node approach. In essence, in schemes using the consistent dual node approach the thin layer establishes a near head continuity between the dual nodes. If the simulation results are compared to the models based on the coarsest uniform discretization (Figure 2b, 4b, 10c and 14c), it is observed that adding a thin layer has only a positive effect on the cell-centered schemes based on the common node approach. This positive effect is explained by the fact that due to the thin layer the common node approach is in almost full agreement with the principle of head continuity at the land surface. Vis-à-vis the corresponding model without a thin layer, the thin layer has a negligible effect on the cell-centered scheme based on the consistent dual node approach. This is because the thin layer establishes a head continuity between the dual nodes and the topmost subsurface node and the adjacent subsurface node below act like the dual nodes in the model without the thin layer. The thin layer has also a negligible effect on the vertex-centered

796 ~~scheme based on the common approach. In this case the thin layer establishes a near head~~
797 ~~continuity between the topmost subsurface node and the adjacent node below and ponding due to~~
798 ~~excess infiltration will require almost fully saturated conditions in the two topmost subsurface~~
799 ~~volumes. The sum of these two volumes is equal to the topmost volume in the model without the~~
800 ~~thin layer and therefore the effect of the thin layer is minimal. In a vertex-centered scheme based~~
801 ~~on the consistent dual node approach, the thin layer has a clear negative effect. In essence the head~~
802 ~~continuity between the dual nodes removes the benefits of using the consistent dual node approach~~
803 ~~and contrary to the cell-centered scheme based on the consistent dual node approach the topmost~~
804 ~~subsurface node and the adjacent subsurface node below do not act like the dual nodes in the model~~
805 ~~without the thin layer. This is because the thin layer creates a non-uniform primary mesh in which~~
806 ~~the subsurface node directly below the topmost subsurface node is not located at the centroid of~~
807 ~~its associated dual cell.~~

808 ~~As indicated in figure 2d and 4d, when using an inconsistent dual node approach, the runoff~~
809 ~~is overestimated unless a very small coupling length is being used. As discussed in section 5, this~~
810 ~~is expected.~~

811 **6.2 Computational efficiency**

812 ~~During the early stages of ponding the rates at which the water depths are changing can be~~
813 ~~relatively fast as the applied flux rates on the land surface are possibly quite large. Typically, a~~
814 ~~numerical model with adaptive time-stepping will decrease the time step size at the moment of~~
815 ~~ponding to handle the non-linear flow terms and the high rates of change in water depth. Since a~~
816 ~~higher infiltration rate at the moment of ponding results in lower initial rates of change in water~~
817 ~~depth, it is expected that the most efficient coupling approach is characterized by a higher~~
818 ~~infiltration rate at the moment of ponding.~~

819   The computational efficiency of the schemes is measured in terms of the number of Newton steps.

820   The number of Newton steps equals the number of times that the linearized system of equations is

821   solved and this number depends on the time step sizes as well as the number of failed Newton

822   steps. It is emphasized that the measured efficiency depends crucially on the applied model code.

823   Nonetheless, as shown in the following, the measured differences in efficiencies can be explained

824   in terms of abrupt changes in how fast pressure heads near the surface-subsurface interface are

825   evolving with time. Regardless of the type of scheme used to solve the non-linear flow equations,

826   such abrupt changes are difficult to solve.

827   Once ponding occurs a surface-subsurface flow model will encounter significant numerical

828   difficulties as surface flow terms are activated. In essence, the activation of these terms represents

829   a discontinuity in flow behaviour which is challenging to resolve [*Osei-Kuffuor et al.*, 2014].

830   Indeed, the Newton steps as depicted in Figure 3 and 7 indicate that simulations encounter

831   difficulties at the moment of ponding. These figures also indicate that the consistent dual node

832   approach can be more efficient in comparison to the common node approach.

833   **6.2.1   Excess saturation**

834   ~~When simulating excess saturation the subsurface is fully pressurized at~~Just before the moment of

835   ponding due to excess saturation, the rate of change in pressure heads at the topmost subsurface

836   nodes is relatively high for both coupling approaches. This high rate is related to the shape of the

837   water retention curve. Typically, the derivative of the saturation with respect to the pressure head

838   goes to zero when approaching fully saturated conditions. Once ponding starts, the surface flow

839   terms are activated and ~~can only accommodate~~therefore the rate of changes in pressure heads at

840   the topmost subsurface nodes decreases drastically. Both approaches must handle this drastic

841 change. However, from Figure 4b and 5b it can be observed that the rate of change decreases more

842 abruptly when using the common node approach.

843 When using the common node approach the vertical hydraulic gradients in the subsurface

844 are close to zero at the moment of ponding, since additional water volumes can only be

845 accommodated by means of ~~the~~ specific storage. ~~As such the column will be close to hydrostatic~~

846 ~~conditions at the moment of ponding. When using the common node approach this~~ This implies

847 that the ~~hydraulic gradient between the common node and the adjacent subsurface node below is~~

848 ~~very close to zero. When using the consistent~~ infiltration rate drops instantaneously at the moment

849 of ponding. In contrast, in the dual node approach ponding ~~due to excess saturation occurs~~starts

850 when ~~$0 < p_{ss} < l$.~~the infiltrability is exceeded. Thus~~,~~ at the moment of ponding, the infiltration rate

851 is higher in comparison to the common node approach. After ponding this infiltration rate will

852 decrease quickly as the hydraulic ~~head~~heads at the dual nodes equilibrate. This difference in the

853 infiltration rate at the moment of ponding explains why the topmost subsurface hydraulic heads

854 change more smoothly when using the dual node approach. If the vertical discretization is

855 ~~generally still below the land surface. This means that~~coarser, then the infiltration rate at the

856 moment of ponding as computed ~~by the consistent dual node approach is higher in comparison to~~

857 ~~the rate as computed by the common node approach. It is thus expected that~~ with the consistent

858 dual node approach is ~~more efficient when simulating excess saturation. Indeed, Figure 8 and 13~~

859 ~~illustrate that, when simulating excess saturation, the consistent dual node approach is more~~

860 ~~efficient then the~~ even higher and this results in a lower initial rate initial rate of change in water

861 depth as depicted in Figure 5a.

862 The more abrupt changes in pressure heads at the common node in comparison to the

863 changes in pressure heads at the dual nodes mean that solving the activation of ponding with the

864 common node approach. ~~Figure 10 illustrates the pressure heads on the nodes near the land surface~~

865 ~~as~~ is more difficult. It is noted that the differences in the infiltration rates between the two coupling

866 approaches only occur at the moment of ponding and directly thereafter when water depths are

867 relatively small. Namely, quickly after ponding, the hydraulic heads at the dual nodes will

868 equilibrate and after that the two coupling approaches will behave similar. This explains why these

869 differences in infiltration rates do not significantly affect the accuracy of simulated ~~by the models~~

870 ~~based on the cell-centered scheme and the coarsest vertical discretization. As illustrated, the~~

871 ~~pressure head gradient governing the infiltration rate at the moment of ponding is larger when~~

872 ~~using the consistent dual node approach and consequently the rate of change in water depth is~~

873 ~~smaller.~~ runoff.

874 **6.2.2 Excess infiltration**

875 ~~As discussed in section 7.1.2, in comparison to the consistent dual node approach, the common~~

876 ~~node approach yields a later time of ponding due to excess infiltration. Since saturation fronts in a~~

877 ~~homogeneous medium become more diffused with time, it follows that the common node approach~~

878 ~~yields a smaller infiltration rate at the moment of ponding. Namely, if the saturation fronts are~~

879 ~~more diffused, then the pressure head gradient governing the infiltration rate is less sharp.~~

880 ~~Therefore, it is expected that the common node approach is computationally less efficient than the~~

881 ~~consistent dual node approach, particularly if ponding is significantly delayed. Figure 5 illustrates~~

882 ~~clearly, that the consistent dual node approach can be more computationally efficient. For the~~

883 ~~simulation scenario depicted in Figure 5, the consistent dual node approach is also more accurate.~~

884 ~~Figure 4b illustrates that, compared to the consistent dual node approach, the common node~~

885 ~~approach can result in a relatively high rate of change in runoff at the moment of ponding. This is~~

886 ~~indicative of a relatively high initial rate of change in water depth at the moment of ponding. Figure~~

887 ~~7 illustrates the pressure heads at the nodes near the land surface as simulated by the cell-centered~~

888 ~~schemes based on the coarsest vertical discretization. It can be observed that the pressure head~~

889 ~~gradient at the moment of ponding is larger when using the consistent dual node approach. This~~

890 ~~implies a higher infiltration rate and a lower rate of change in water depth. Figure 11 also illustrates~~

891 ~~that the consistent dual node approach is more efficient when handling the activation of ponding.~~

892 ~~However, considering the entire simulation period, the dual node approach is not always more~~

893 ~~efficient. As illustrated by Figure 11b and 11c, when the discretization is relatively coarse the~~

894 ~~common node approach is sometimes more efficient during the later stages of the simulation.~~

895 ~~However, in these cases the common node approach is only more efficient, because its inaccuracy~~

896 ~~leads to an easier flow problem to be solved. Namely, the underestimation of runoff results in more~~

897 ~~diffused saturation fronts in the subsurface.~~

898 ~~Figure 2, shows that if the ratio q_R/K_z is relatively small, then the differences in~~

899 ~~computational efficiency are relatively small. As discussed in section 4 this is because the~~

900 ~~consistent dual node approach behaves very similar to the common node approach if the ratio q_R/K_z~~

901 ~~is relatively small.~~

902 ~~Another factor that affects the efficiency of the common node approach is that the delay in~~

903 ~~ponding can act as an artificial barrier for a surface water wave advancing across an initially~~

904 ~~unsaturated subsurface domain. The effect of this artificial barrier is that the front of the surface~~

905 ~~water wave is steepened. This steepening of the surface wave front results in higher rates at which~~

906 ~~the water depth is changing and is undesirable because it decreases the computational efficiency.~~

907 ~~This is clearly illustrated in Figure 15. Figure 16 illustrates the evolution of water depth at the land~~

908 ~~surface for the cell-centered schemes using the coarsest vertical discretization. As shown, the~~

909 ~~common node approach delays and steepens the surface water front. This results in relatively high~~

910 ~~rates of change in water depth at the moment of ponding. Consequently, the common node~~

911 ~~approach is less efficient than the dual node approach. It is noted that for this scenario the~~

912 ~~consistent dual node approach is more efficient as well as more accurate.~~

913 Figure 8, 9, 12 and 13 illustrate the evolution of pressure heads at dual nodes and common nodes

914 when simulating excess infiltration. When applying the consistent dual approach, the net flux into

915 a topmost subsurface cell will decrease once ponding occurs, because the applied flux rate will be

916 partitioned between dual nodes (i.e. between the surface flow and subsurface flow domain). This

917 occurs while the topmost subsurface node is not yet fully saturated. After ponding the infiltration

918 rate decreases such that if the topmost subsurface node reaches fully saturated conditions the net

919 flux into the topmost subsurface node is relatively small. In contrast, partitioning of the applied

920 flux rate on a common node between the surface flow and subsurface domain starts when the

921 common node reaches fully saturated conditions at this node. This means that just before ponding

922 the rate of change in pressure head is relatively high as the common node is driven towards fully

923 saturated conditions while the infiltration rate is relatively high. This means that similar to the

924 excess saturation scenario the rate of change in pressure head at the common node is high just

925 before ponding. At the moment of ponding, this rate must drop abruptly as surface flow terms are

926 activated. This abrupt change explains why the common node approach is less efficient.

927   Figures 7 and 11 also indicate that a coarser vertical discretization only provides a

928 significant gain in efficiency in terms of Newton steps when using the consistent dual node

929 approach. When using the common node approach, a coarser discretization does not change the

930 fact that the topmost subsurface node must reach fully saturated conditions for ponding to occur

931 and that the infiltration rate is relatively high just before ponding. When using the consistent dual

932   node approach, a coarser vertical discretization means that the saturation fronts are more diffused

933   such that the flow problem becomes easier to solve.

934   Figure 8a and 9a illustrate that for the second simulation scenario, ponding occurs almost

935   simultaneously at all the surface nodes. Figure 12a and 13a show that this is different for the third

936   scenario where ponding occurs at different times as the flooding wave travels downstream. When

937   Figure 11a is compared with Figure 12a and when Figure 11d is compared with Figure 13a, it is

938   clear that the common node approach encounters difficulties around each time ponding starts at a

939   surface node. Figure 11 shows that these difficulties are encountered for all discretizations. In

940   contrast the consistent dual node approach has much less difficulties solving these difficulties. As

941   discussed in Section 6.1.2. the common node approach may result in steepening the advancing

942   wave. This implies that water depths will be changing more quickly. This presents an additional

943   difficulty for solving this flow problem with the common node approach.

944   **7   Conclusions**

945   In this study it is shown that ~~contrary to the common held view, the dual node approach if properly~~

946   ~~implemented is actually the more general, the more elegant as well as the more accurate coupling~~

947   ~~approach in comparison to the common node approach. This consistent dual node approach is~~

948   ~~implemented in cell-centered as well as vertex-centered finite difference schemes.~~

949   ~~The consistent dual node approach is derived from basic equations that govern infiltration and~~

950   ~~infiltrability at the land surface using~~the dual node approach should be conceptualized and

951   implemented as a one-sided finite differences approximation of the vertical hydraulic gradient at

952   the land surface. ~~In both cell-centered as vertex-centered schemes~~This provides an important new

953   insight into the coupling length. Namely, if the dual node approach is properly implemented then

954 the coupling length is related to the ~~grid geometry. As discussed, the dual node approach should~~

955 ~~not be conceptualized as a distinct interface between the surface and the subsurface. Moreover,~~

956 ~~this approach is in agreement with principle of head continuity along the land surface whereas the~~

957 ~~common node approach is not, unless the~~ vertical ~~discretization is~~grid resolution. Thus, the

958 coupling length does not represent an additional non-physical model parameter and therefore the

959 dual node approach is not automatically a less physically based approach in comparison to the

960 common node approach. Actually, this study shows if the vertical discretization is not sufficiently

961 fine then the head continuity at the surface-subsurface interface is formulated more correctly in

962 the consistent dual node scheme. This difference in formulation has consequences for how both

963 approaches compare in terms of accuracy and efficiency.

964 Numerical experiment indicate that ~~if the vertical discretization is relatively coarse, then~~

965 ~~the~~ the consistent dual node approach is ~~often less inaccurate as well as more computationally~~

966 ~~efficient~~ equally accurate or more accurate than the common node approach. It has been shown

967 that in comparison to the common node approach ~~for~~ the consistent dual node approach is less

968 sensitive to the vertical discretization when simulating excess infiltration. ~~For simulating excess~~

969 ~~saturation both coupling approaches are more or less equally accurate, but the consistent dual node~~

970 ~~approach was found to be more computationally efficient. Therefore, overall it can be argued that~~

971 ~~the consistent dual node approach is to be preferred to the common node approach unless~~However,

972 the practical advantage of the consistent dual node approach in terms of accuracy is limited.

973 Namely, if the vertical discretization is ~~sufficiently fine such that~~ refined, both approaches will

974 converge to more accurate and eventually similar results when simulating excess infiltration. When

975 simulating excess saturation both approaches yield similar results even if the vertical discretization

976 is relatively coarse.

Nonetheless, even though the advantage of the consistent dual node approach in terms of accuracy is limited, the fact that the consistent dual node approach is equally or more accurate than the common node approach is a significant finding. Namely, this finding is different from the commonly held view that a dual node approach is most accurate if it mimics the common node approach. Moreover, it also illustrates clearly that the consistent dual node approach is not similar to a common node approach.

Numerical experiment indicate that the consistent dual node approach can be more efficient than the common node approach while being equally or more accurate than the common node approach. It has been shown that this difference in efficiency is related to abrupt changes in the evolution of pressure heads around the moment that ponding is initiated.

Based on the findings in this study the models of An and Yu [*An and Yu*, 2014] and Kumar et al. [*Kumar et al.*, 2009] are expected to have some advantages with respect to models that are based on the common node approach. This is because these models are based on a consistent dual node approach. Moreover, given a model that uses an alternative dual node approach, it is relatively straightforward to implement the numerically more correct consistent dual node approach.

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abbreviation	meaning
cc	cell-centered
vc	vertex-centered
dn	dual node
cn	common node
~~TL~~	~~tiny layer~~

Table 1: Abbreviations as used in the figures.

a)

b)

c)

e)

g)

d)
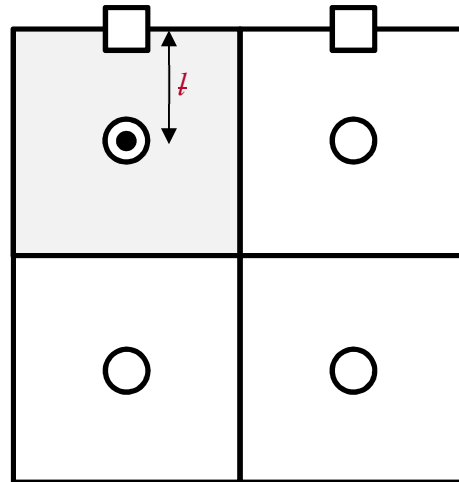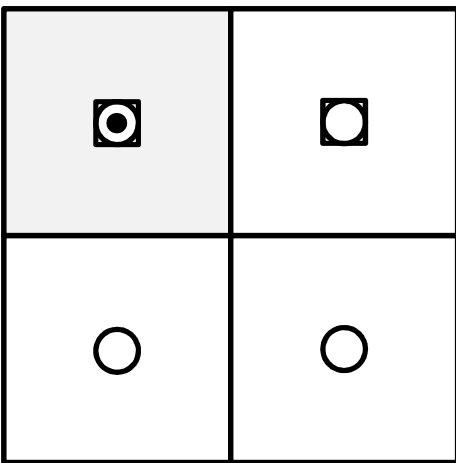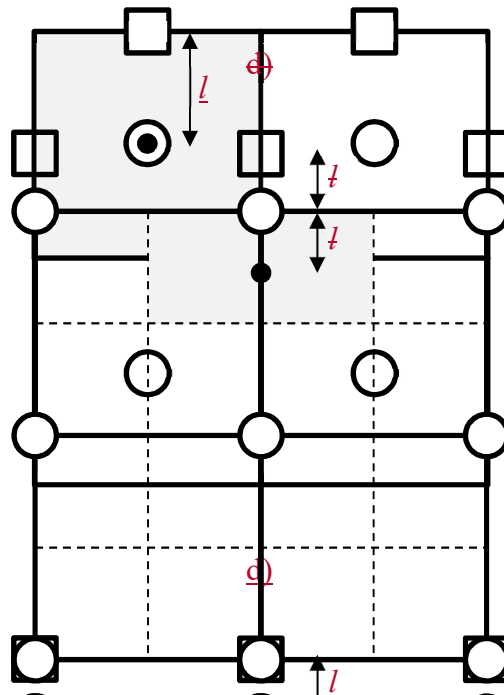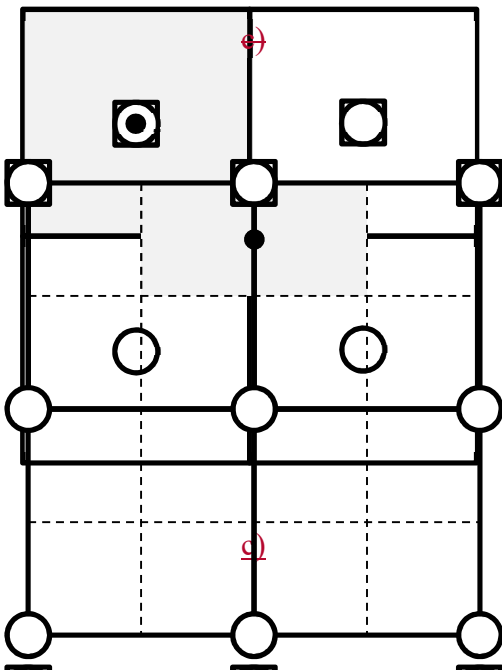
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1112 Figure 1: a) Common nodes ~~and co-located dual nodes~~ in cell-centered schemes. ~~b~~b) Dual nodes

1113 in cell-centered-centered schemes. c) Common nodes and co-located dual nodes in vertex-centered

1114 schemes. ~~c) Dual nodes in cell-centered-centered schemes (not col-located).~~ d) Dual nodes in

1115 vertex-centered schemes (not co-located). The white squares and white circles represent surface

1116 and subsurface nodes, respectively. The solid and dashed lines represent the primary mesh and the

1117 dual mesh, respectively. The grey-shaded area is a topmost discrete volume as associated with a

1118 topmost subsurface node. The black dot represents the centroid of this volume. The coupling length

1119 l as depicted in this figure applies to the consistent dual node approach.

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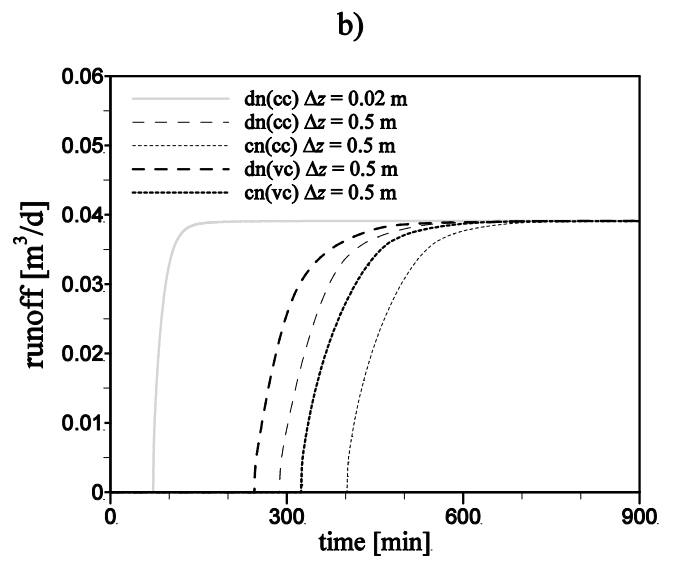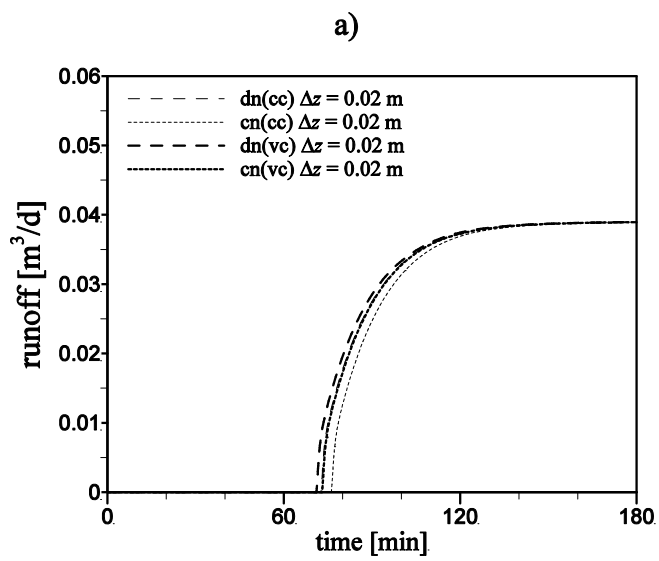
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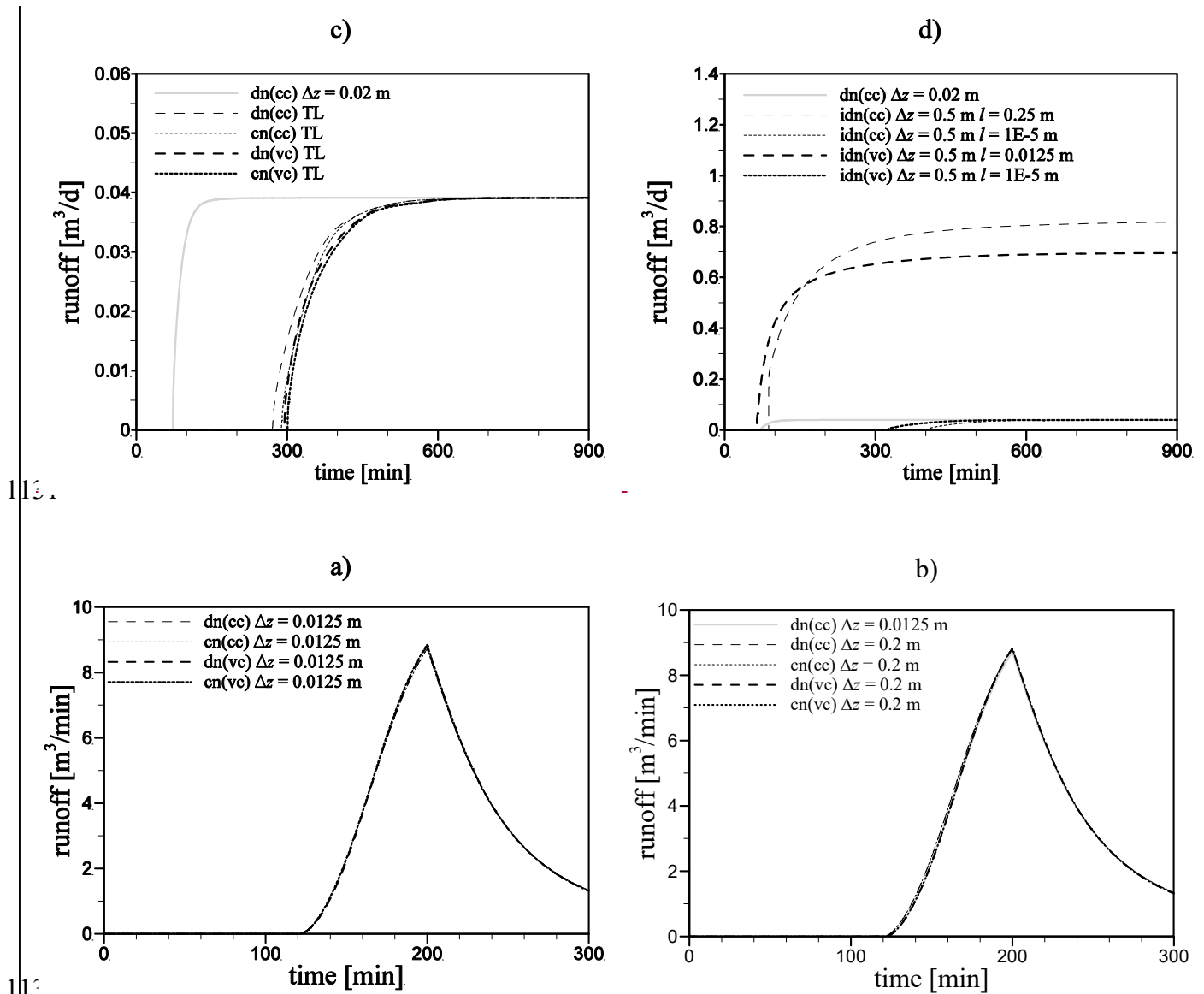
Figure 2: ~~Simulated runoff~~Outflow response for excess ~~infiltration in~~saturation on a ~~vertical soil columnhillslope~~hillslope (first scenario) using different vertical discretizations ~~($q_R = 1.1\ md^{-1}$)~~.

a)

Legend:
- dn(cc) $\Delta z = 0.02$ m
- cn(cc) $\Delta z = 0.02$ m
- dn(vc) $\Delta z = 0.02$ m
- cn(vc) $\Delta z = 0.02$ m

Axes: Newton steps (0–400) vs time [min] (0–180)

b)

Legend:
- dn(cc) $\Delta z = 0.5$ m
- cn(cc) $\Delta z = 0.5$ m
- dn(vc) $\Delta z = 0.5$ m
- cn(vc) $\Delta z = 0.5$ m

Axes: Newton steps (0–400) vs time [min] (0–900)

c)

Legend:
- dn(cc) TL
- cn(cc) TL
- dn(vc) TL
- cn(vc) TL

Axes: Newton steps (0–400) vs time [min] (0–900)

d)

Legend:
- idn(cc) $\Delta z = 0.5$ m $l = 0.25$ m
- idn(cc) $\Delta z = 0.5$ m $l = 1E-5$ m
- idn(vc) $\Delta z = 0.5$ m $l = 0.0125$ m
- idn(vc) $\Delta z = 0.5$ m $l = 1E-5$ m

Axes: Newton steps (0–500) vs time [min] (0–900)

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a)

b)
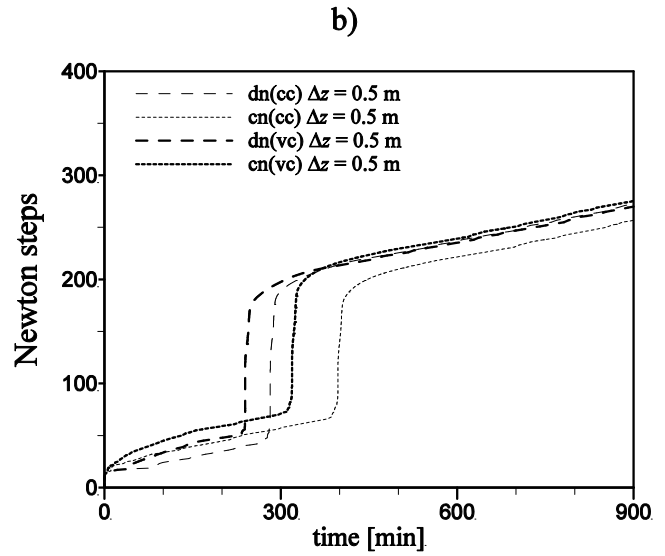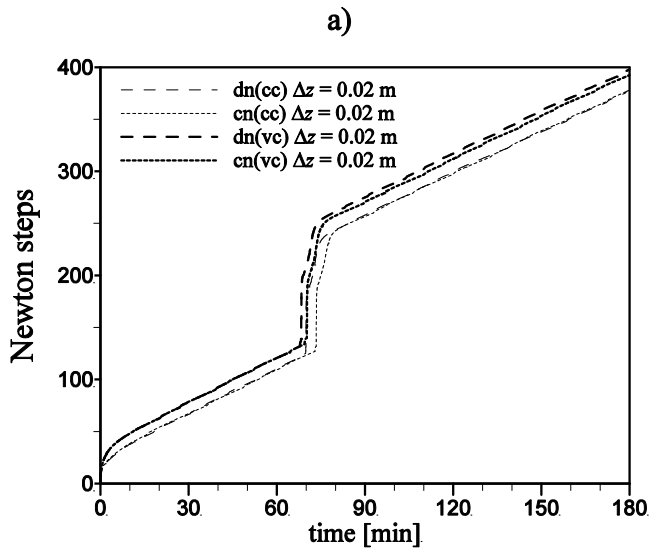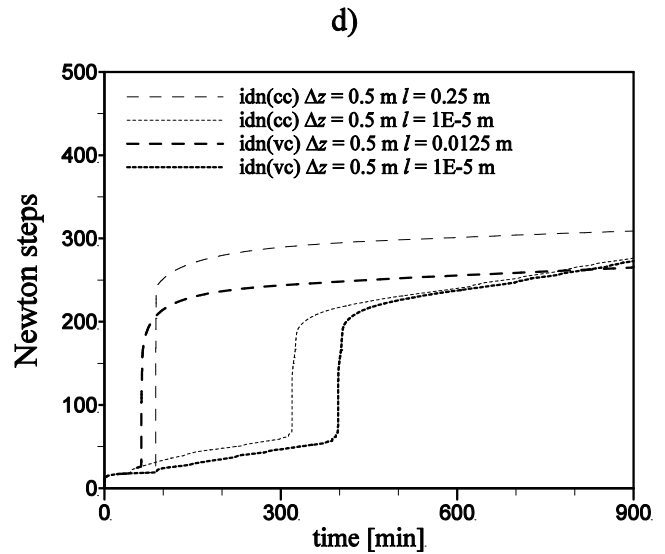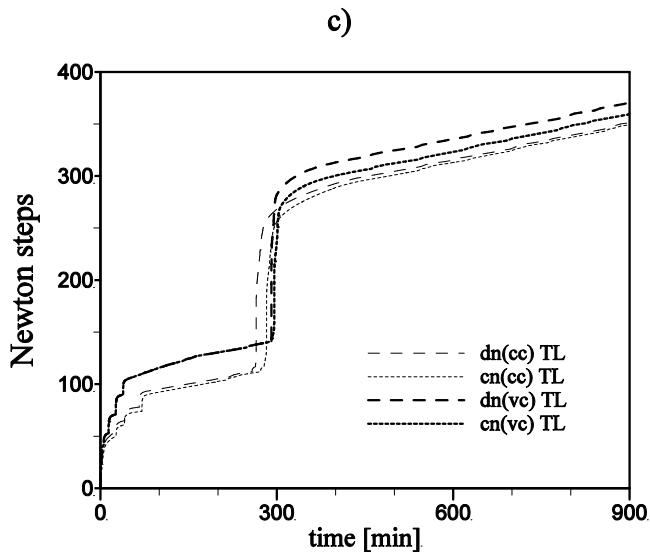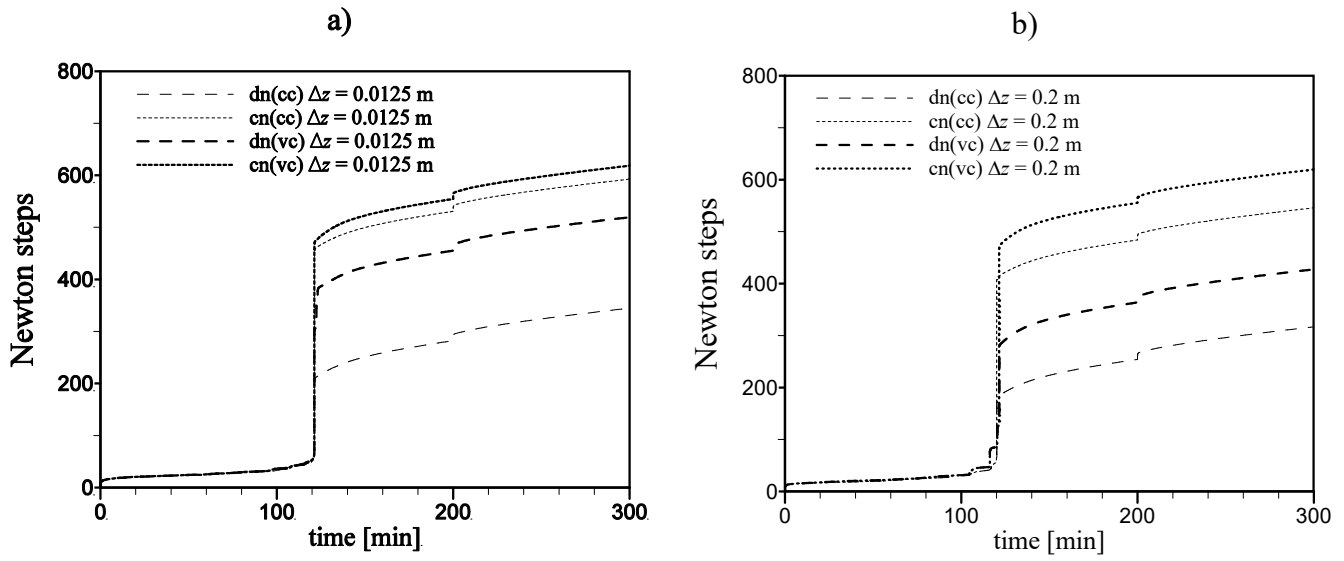
1141

1142   Figure 3: Number of Newton steps for excess ~~infiltration in a vertical soil column~~saturation on a

1143   hillslope (first scenario) using different vertical discretizations ~~($q_R = 1.1$ md^{-1})~~.

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1146

1147

a)

b)

1148

c)

d)

a)

b)
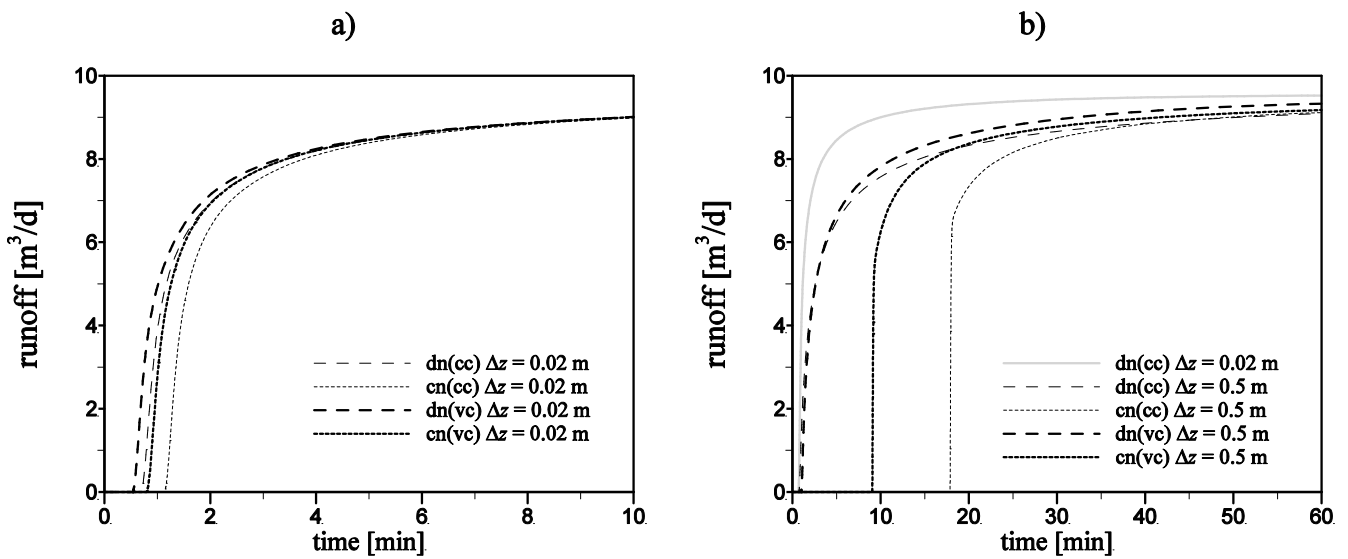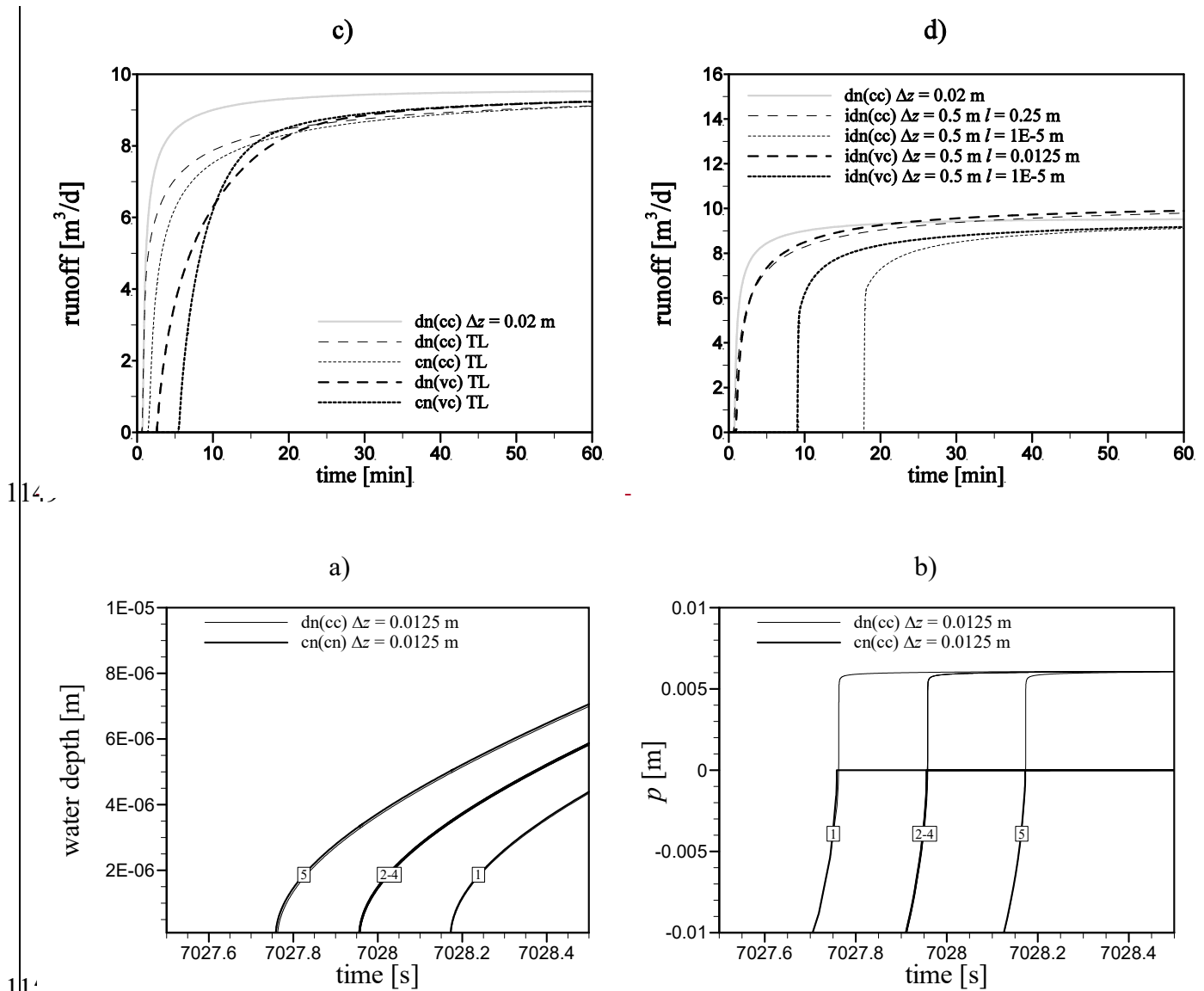
Figure 4: Simulated ~~runoff~~ values at the common nodes for excess saturation on a hillslope (first scenario) with a cell-centered scheme and $\Delta z = 0.0125$ m. a) Water depths. b) Pressure heads. Nodes are numbered 1-5 in the down-slope direction.

a)   b)
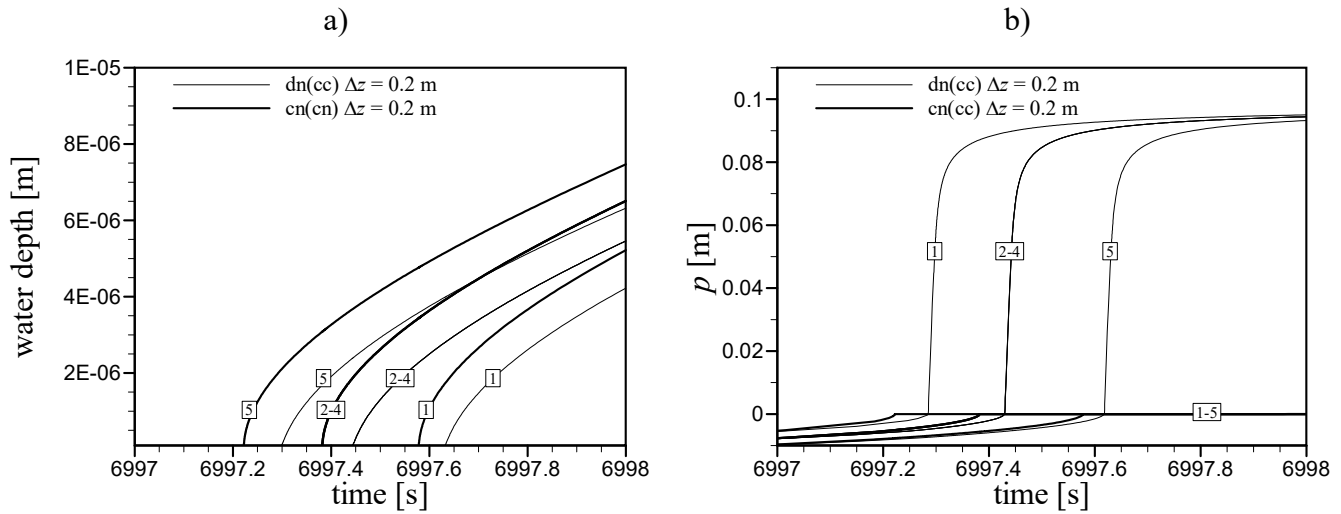
Figure 5: Simulated values for excess saturation on a hillslope (first scenario) with a cell-centered scheme and $\Delta z = 0.2$ m. a) Water depths at the surface nodes. b) Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the down-slope direction.

a)   b)
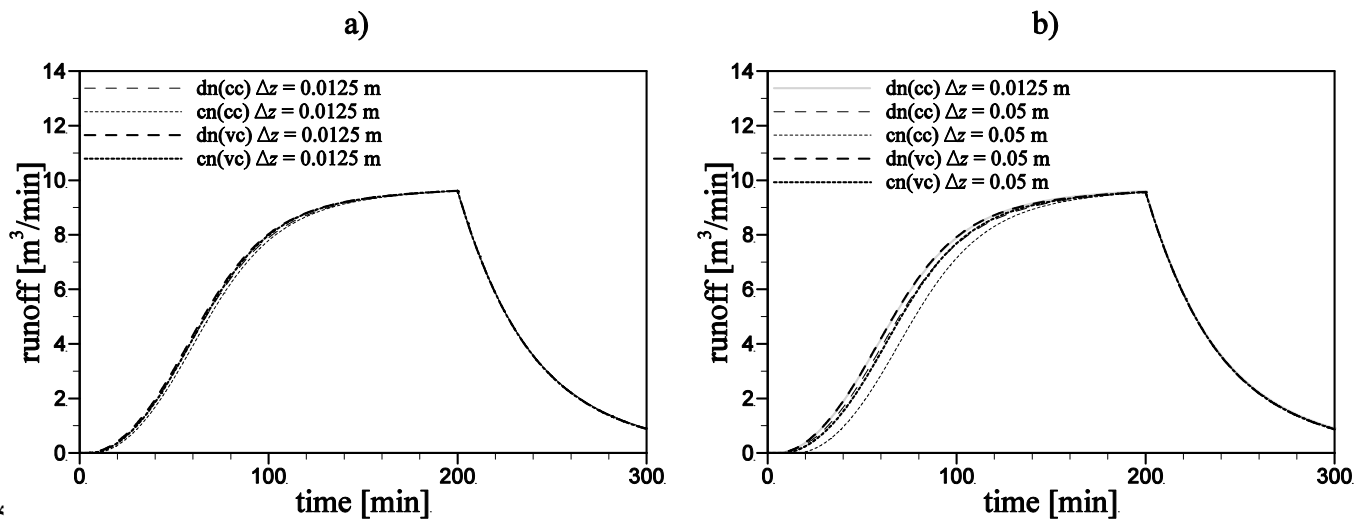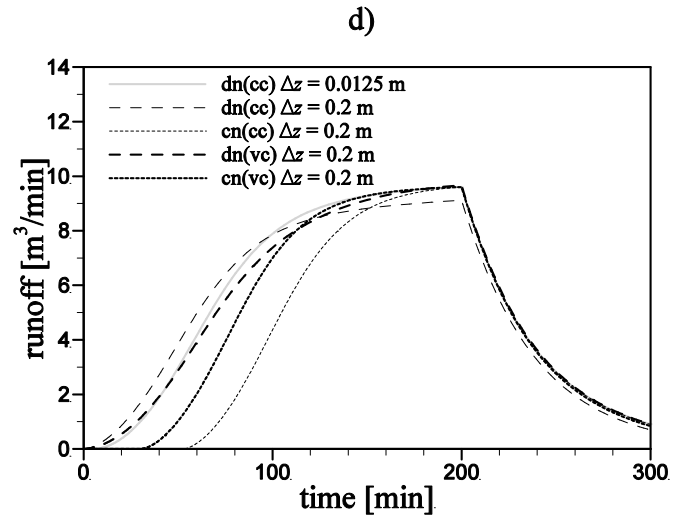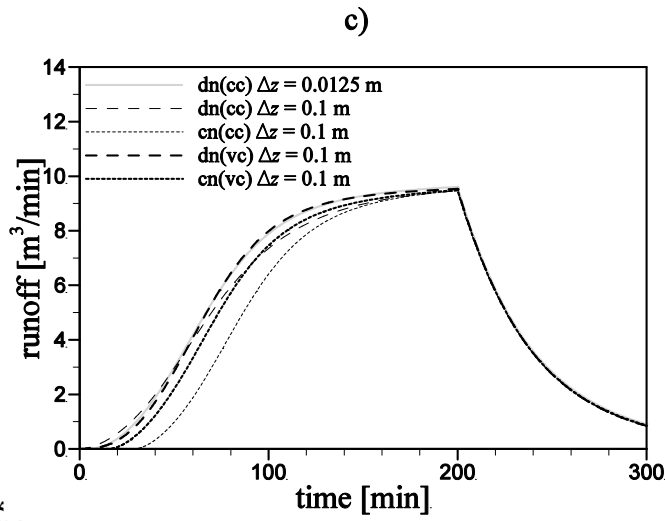
Figure 6: Outflow response for excess infiltration ~~in a vertical soil column~~on a hillslope (second scenario) using different vertical discretizations ~~($q_R = 10.608$ md⁻¹)~~.

c)

Legend (c):
- dn(cc) TL
- cn(cc) TL
- dn(vc) TL
- cn(vc) TL

X-axis: time [min], 0 to 60
Y-axis: Newton steps, 0 to 400

d)

Legend (d):
- idn(cc) $\Delta z = 0.5$ m $l = 0.25$ m
- idn(cc) $\Delta z = 0.5$ m $l = 1E\text{-}5$ m
- idn(vc) $\Delta z = 0.5$ m $l = 0.0125$ m
- idn(vc) $\Delta z = 0.5$ m $l = 1E\text{-}5$ m

X-axis: time [min], 0 to 60
Y-axis: Newton steps, 0 to 500

a)

Legend (a):
- dn(cc) $\Delta z = 0.0125$ m
- cn(cc) $\Delta z = 0.0125$ m
- dn(vc) $\Delta z = 0.0125$ m
- cn(vc) $\Delta z = 0.0125$ m

X-axis: time [min], 0 to 300
Y-axis: Newton steps, 0 to 600

b)

Legend (b):
- dn(cc) $\Delta z = 0.05$ m
- cn(cc) $\Delta z = 0.05$ m
- dn(vc) $\Delta z = 0.05$ m
- cn(vc) $\Delta z = 0.05$ m

X-axis: time [min], 0 to 300
Y-axis: Newton steps, 0 to 600

c)

d)
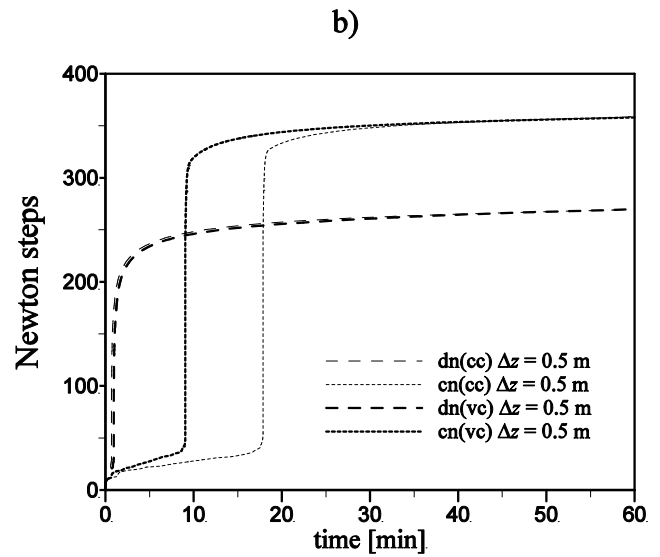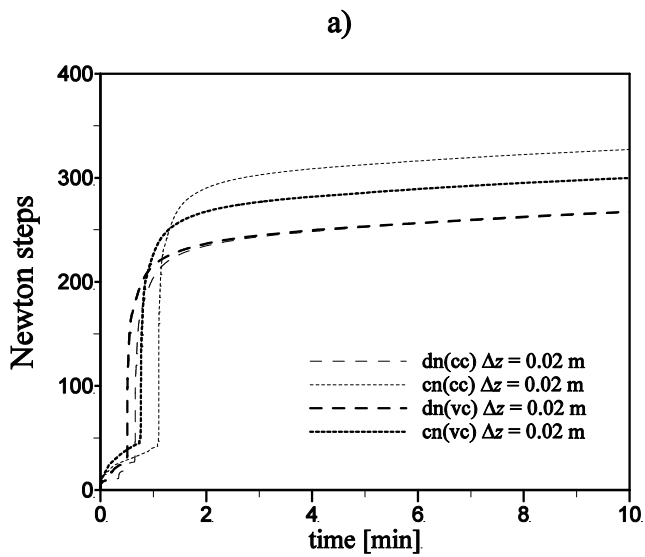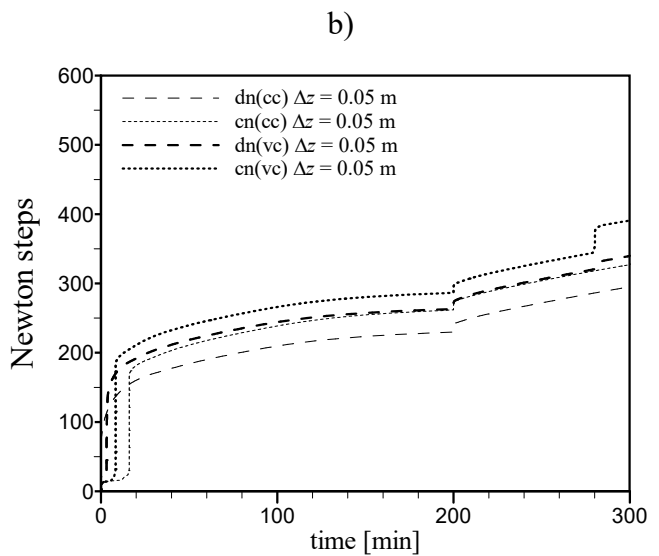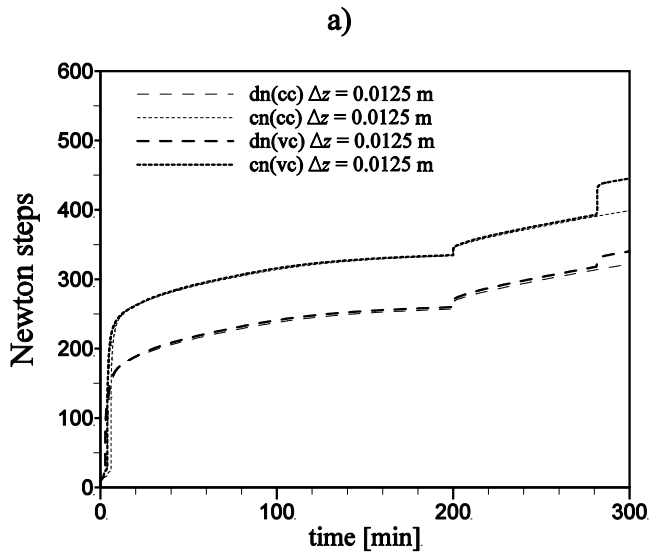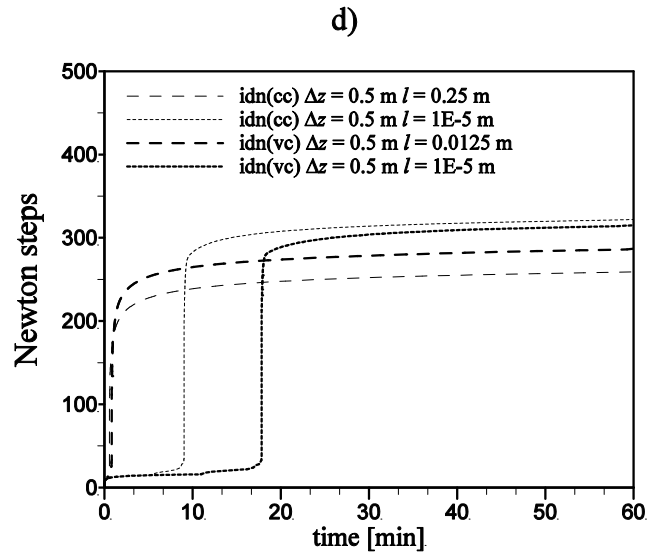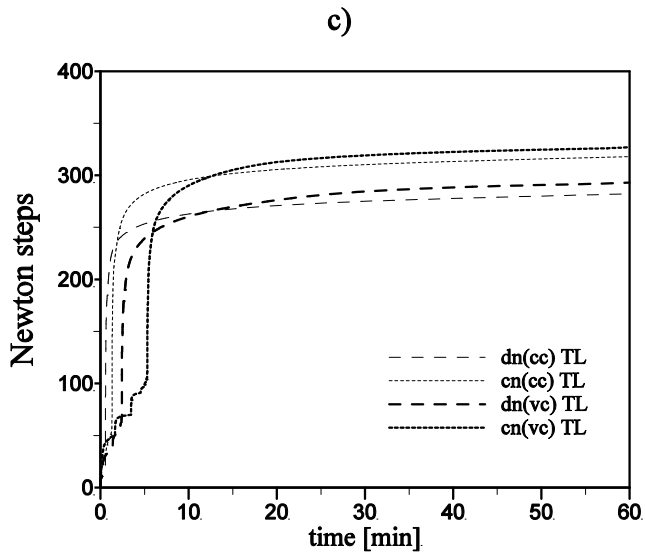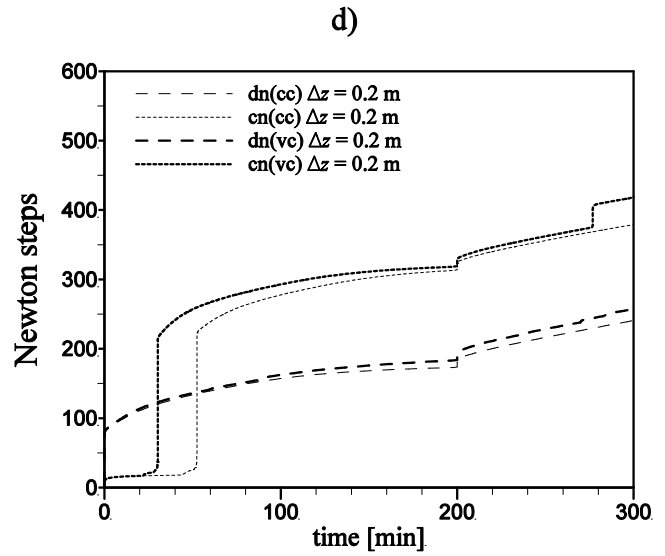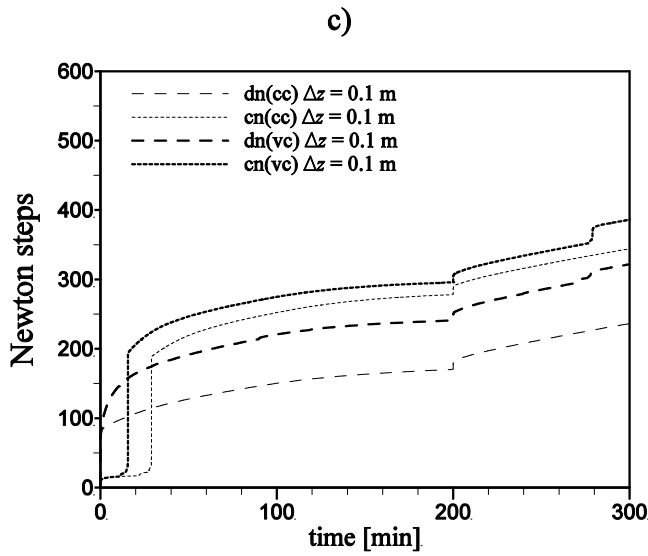
1170

1171 Figure ~~5:~~7: The total number of Newton steps for excess infiltration ~~in~~(second scenario) on a

1172 ~~vertical soil column~~hillslope using different vertical discretizations ~~(q_R = 10.608 md^{-1})~~.

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1174

1175

1176

a)

b)

1177

a)   b)
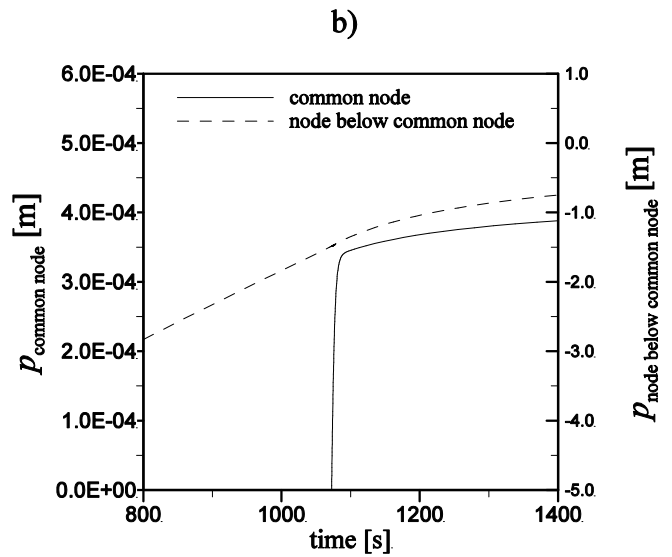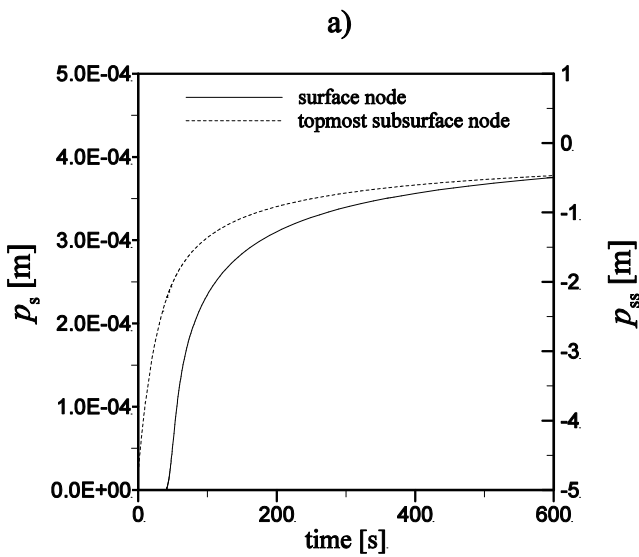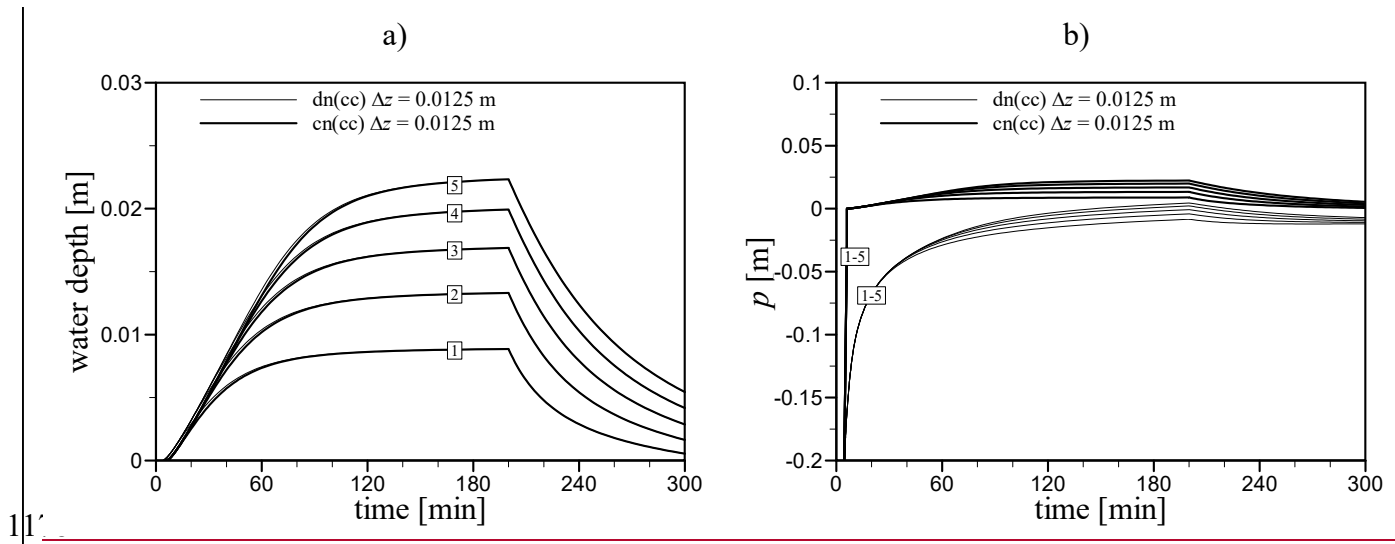
Figure 6: Changes in pressure heads near the surface-subsurface interface8: Simulated values at the common nodes for excess infiltration inon a vertical soil column (q_R = 10.608 md^{-1}). Left: dn(cc)hillslope (second scenario) with a cell-centered scheme and Δz = 0.50125 m. Right: cn(cc) Δz = 0.5 ma) Water depths. b) Pressure heads. Nodes are numbered 1-5 in the down-slope direction.

a)

b)

c)

d)

a)

b)
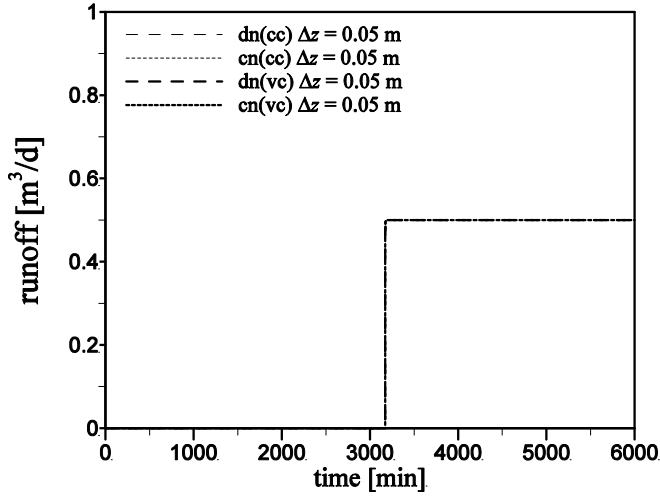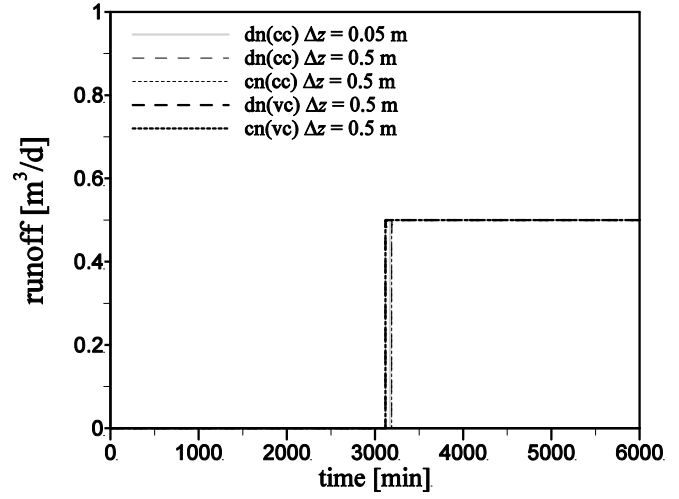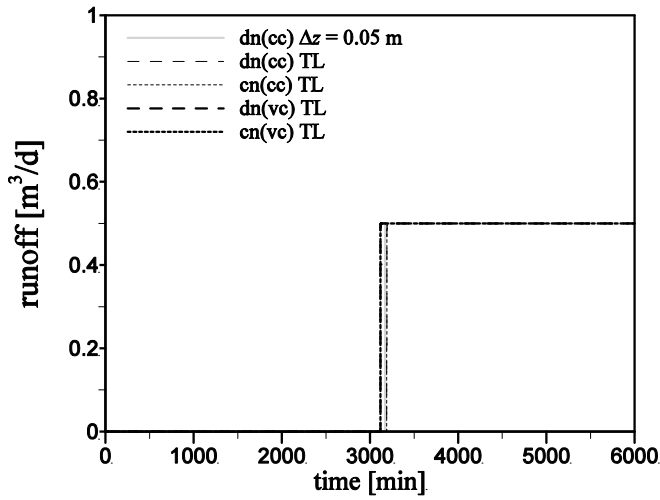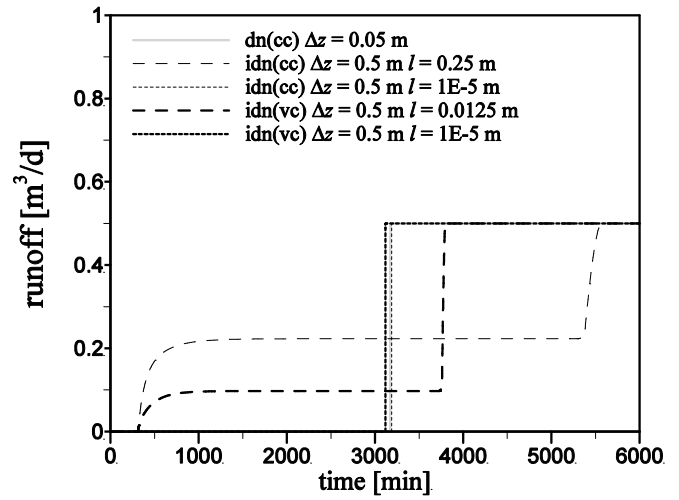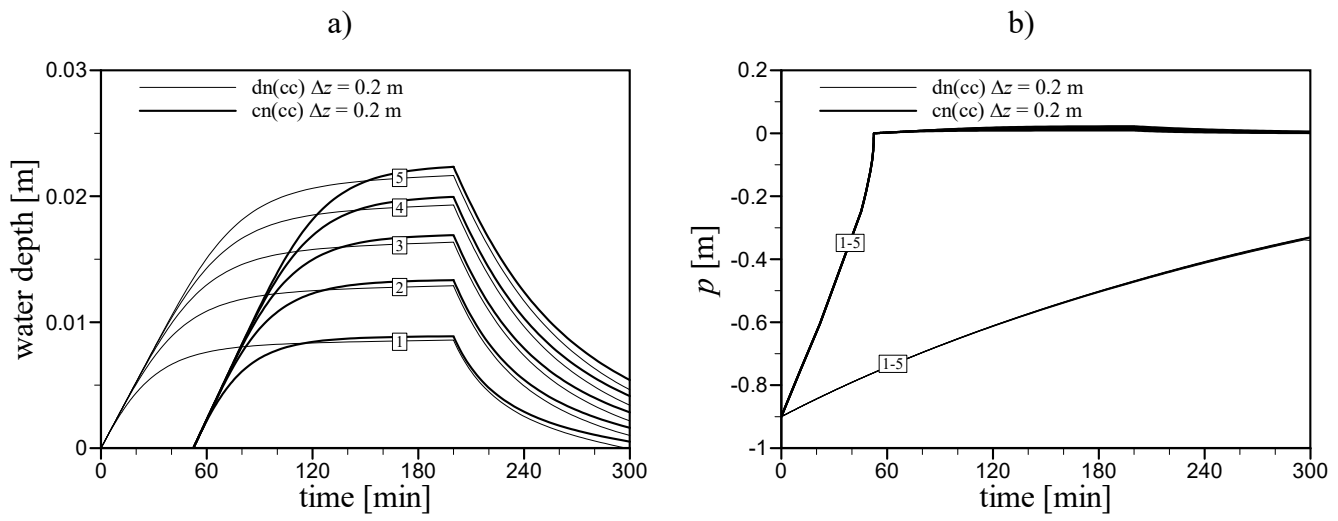
Figure 9: Simulated values for excess infiltration on a hillslope with a cell-centered scheme (second scenario) and $\Delta z = 0.2$ m. a) Water depths at the surface nodes. b) Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the down-slope direction.

a)

b)

1203
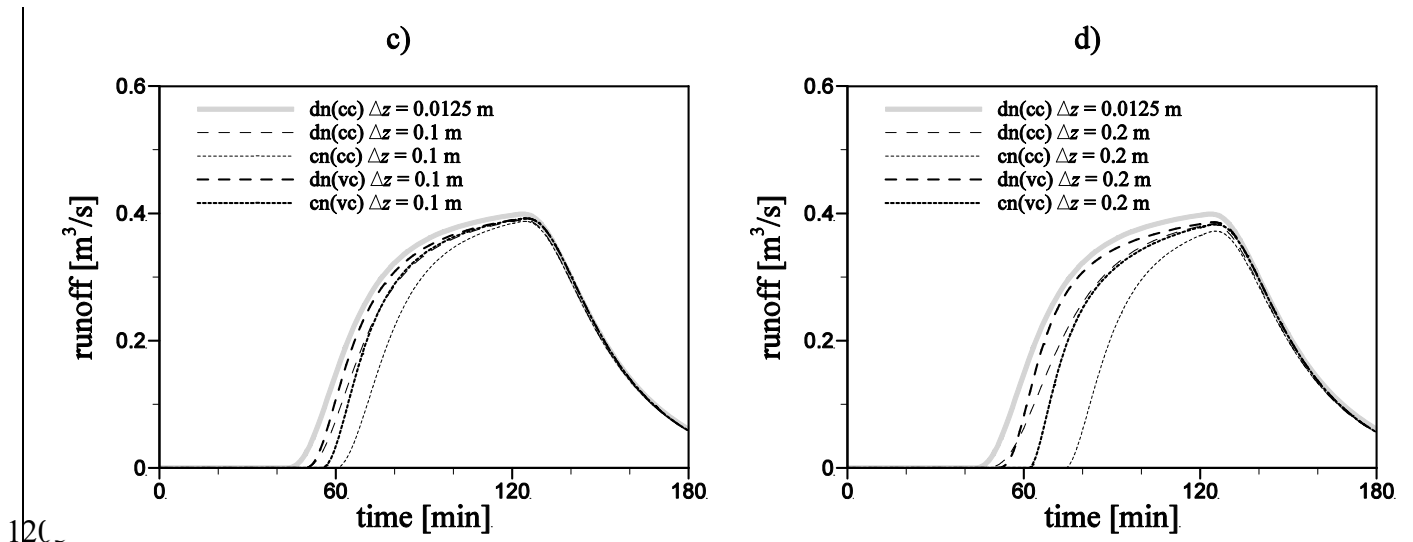
Figure 10: Outflow response for flooding an unsaturated hillslope using different vertical
discretizations.

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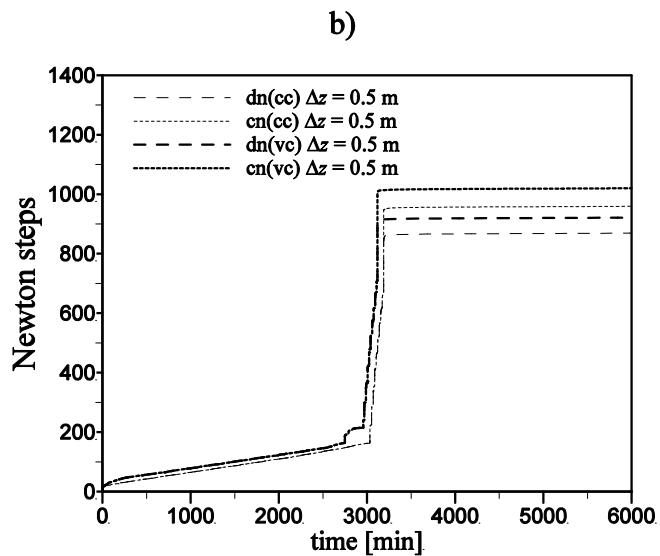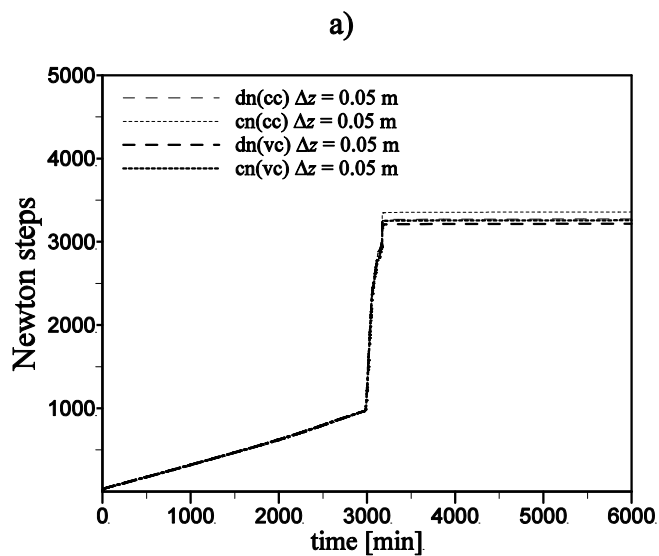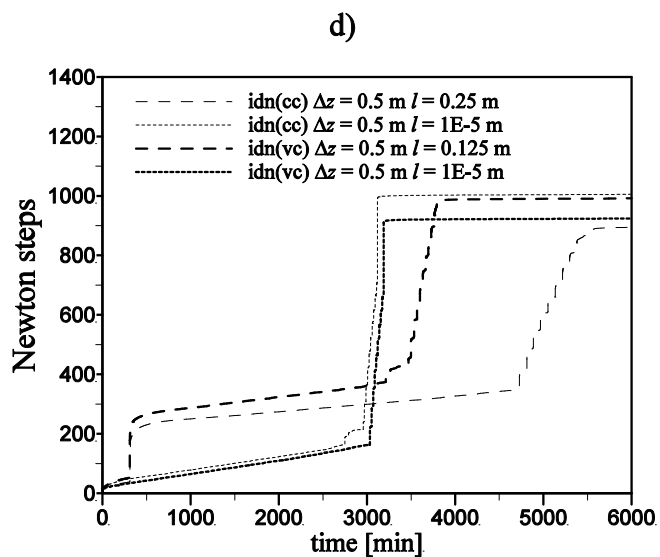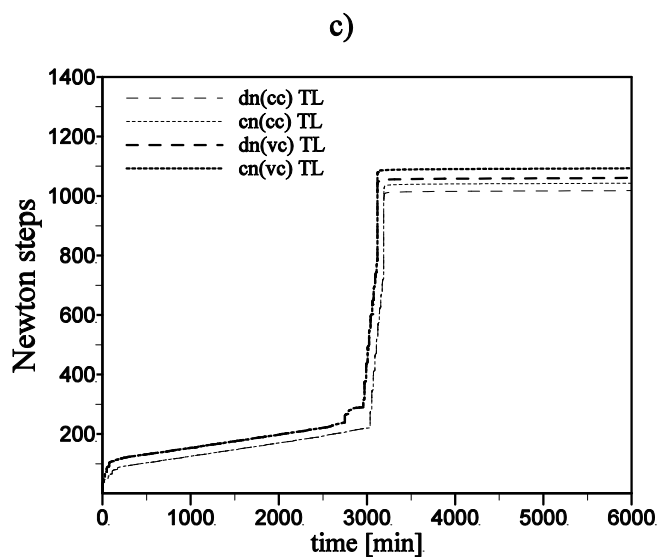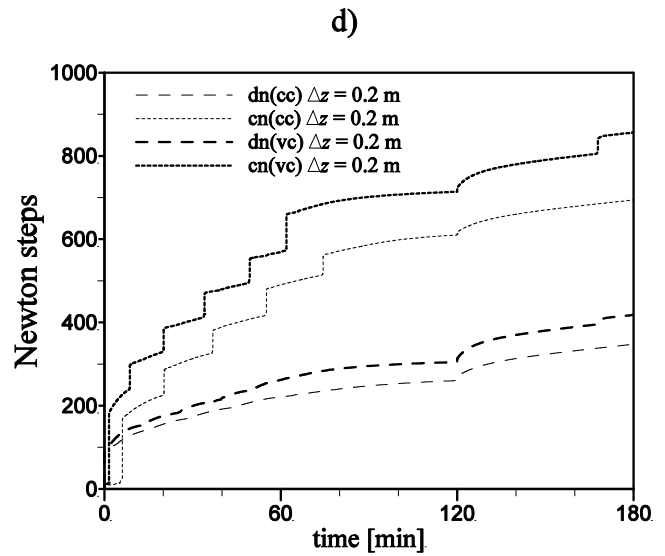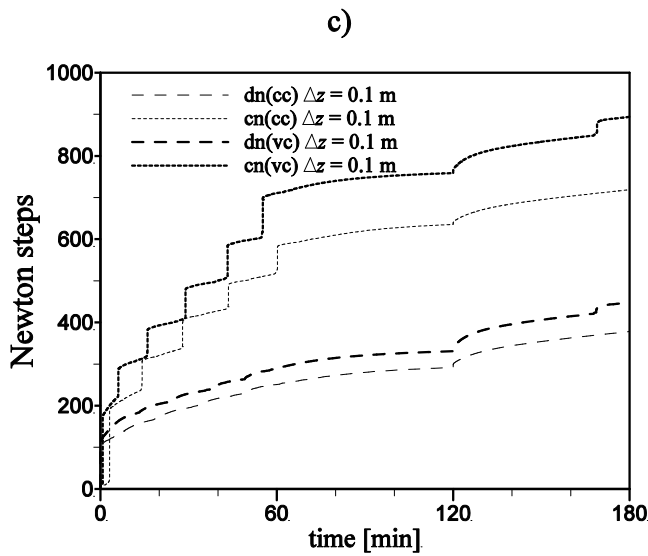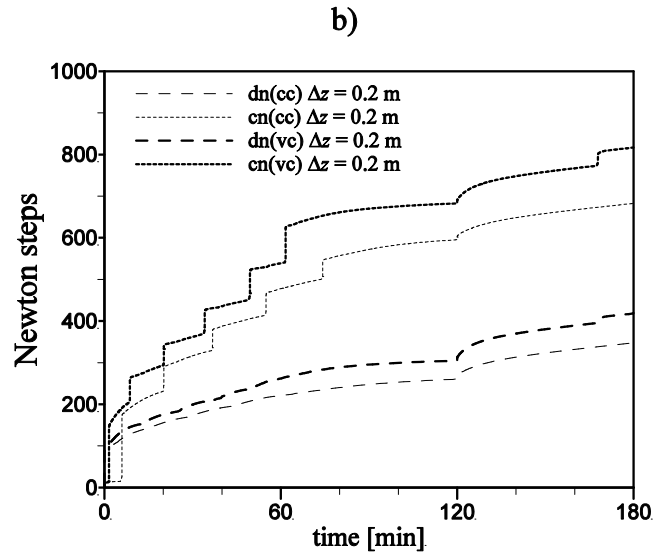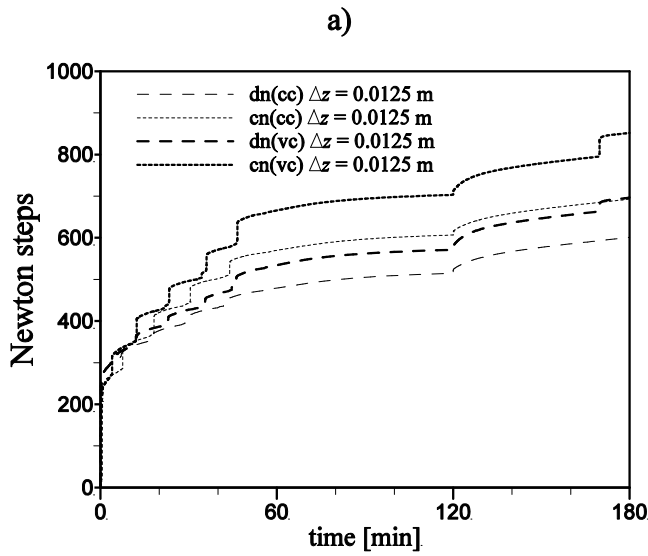
a)

Newton steps

- – – – dn(cc) Δz = 0.05 m
- ······· cn(cc) Δz = 0.05 m
- **– – –** dn(vc) Δz = 0.05 m
- **·······** cn(vc) Δz = 0.05 m

time [min]

b)

Newton steps

- – – – dn(cc) Δz = 0.5 m
- ······· cn(cc) Δz = 0.5 m
- **– – –** dn(vc) Δz = 0.5 m
- **·······** cn(vc) Δz = 0.5 m

time [min]

c)

Newton steps

- – – – dn(cc) TL
- ······· cn(cc) TL
- **– – –** dn(vc) TL
- **·······** cn(vc) TL

time [min]

d)

Newton steps

- – – – idn(cc) Δz = 0.5 m l = 0.25 m
- ······· idn(cc) Δz = 0.5 m l = 1E-5 m
- **– – –** idn(vc) Δz = 0.5 m l = 0.125 m
- **·······** idn(vc) Δz = 0.5 m l = 1E-5 m

time [min]

a)

b)

c)

d)

Figure 8: The total11: Number of Newton steps for excess saturation in a vertical soil columnflooding an unsaturated hillslope using different vertical discretizations.

a)

b)

1218

1219

1220

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a)

b)

1222
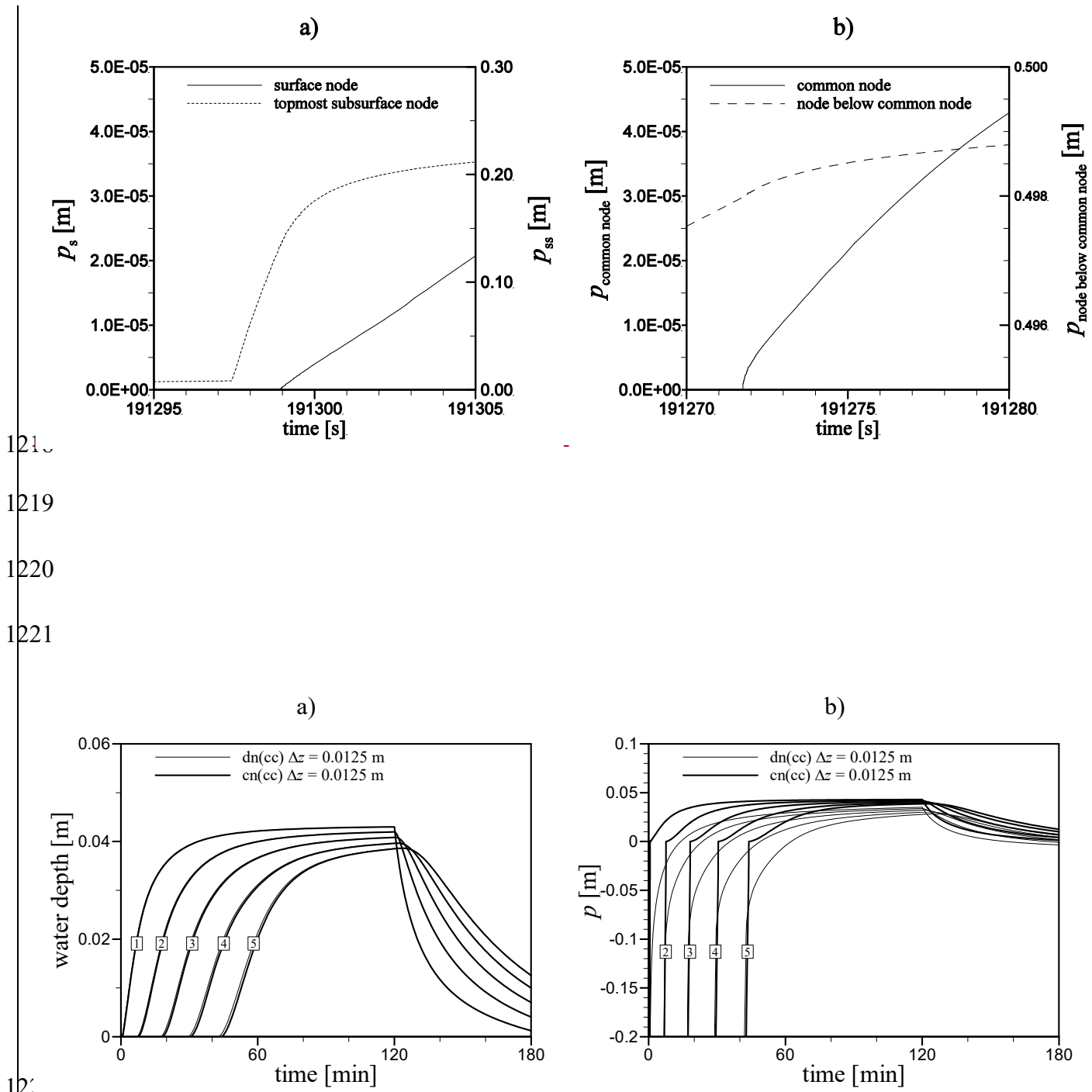
1223   Figure 9: Changes in pressure heads near the surface-subsurface interface for excess saturation in a vertical

1224   soil column. Left: dn(cc) Δz = 0.5 m. Right: cn(cc) Δz = 0.5 m.
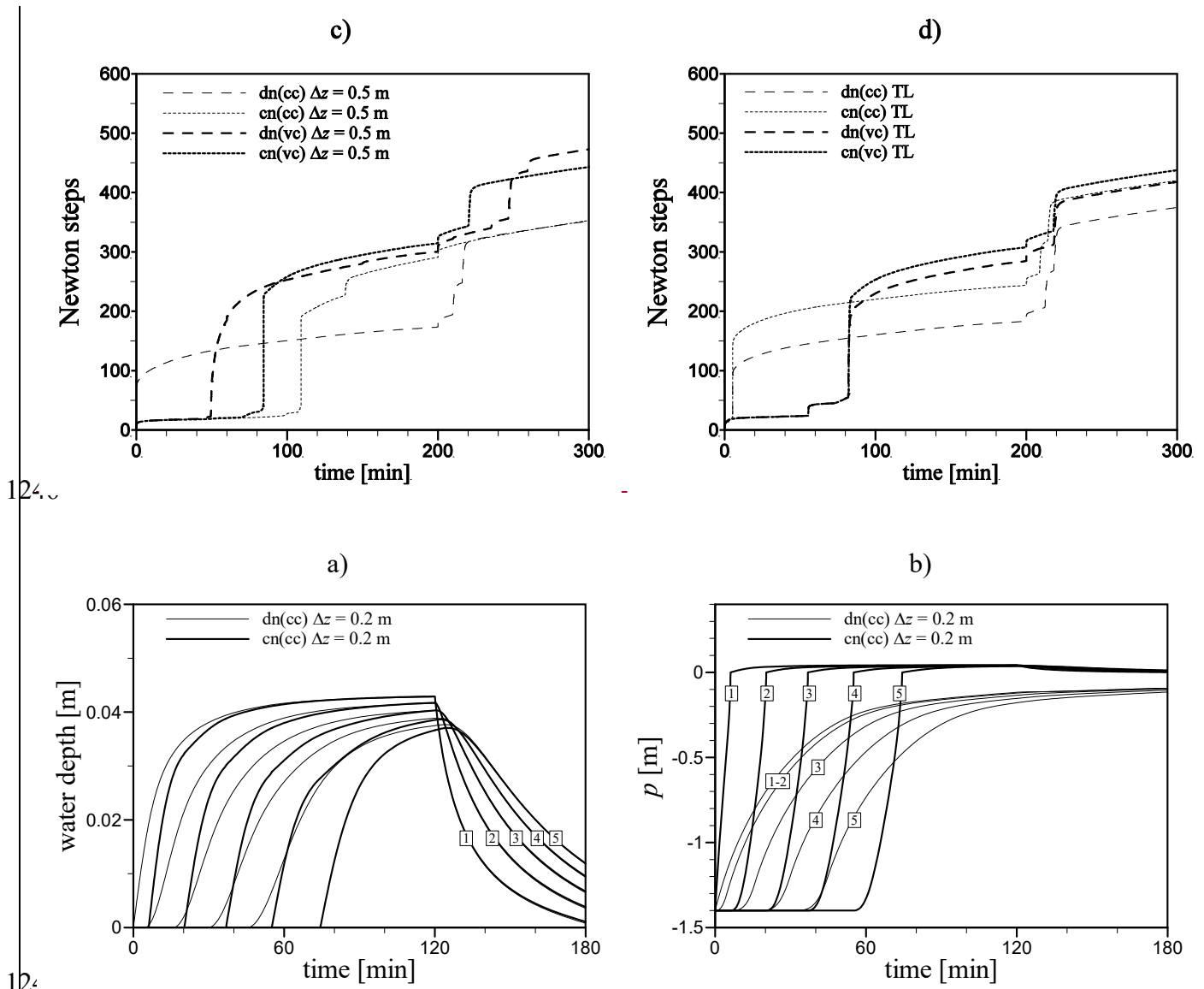
1225

a)

b)

c)

d)
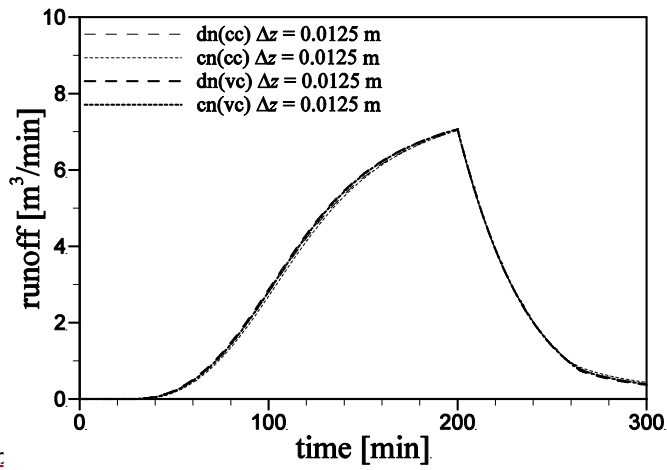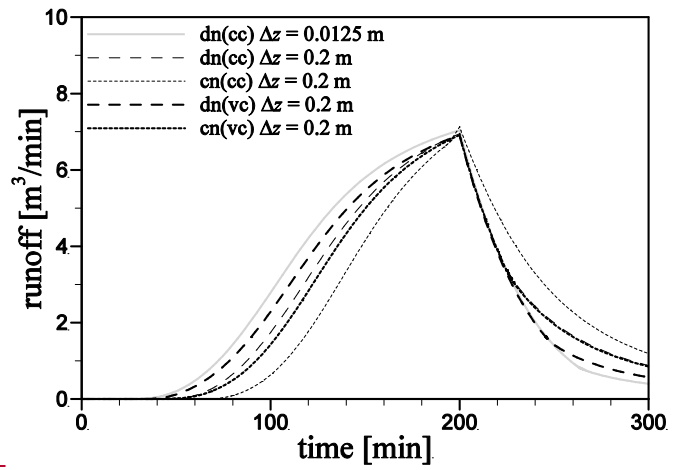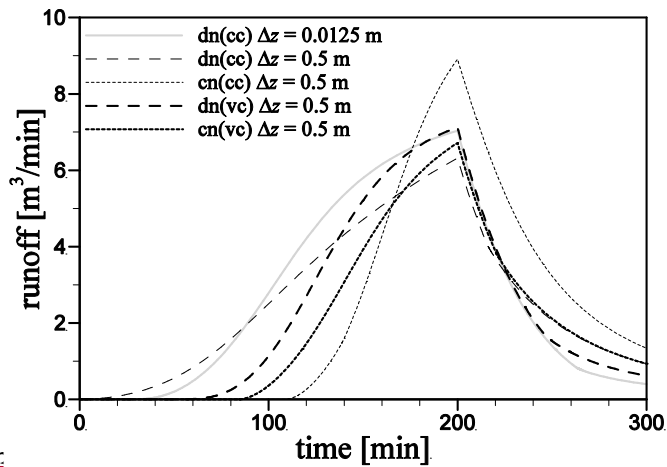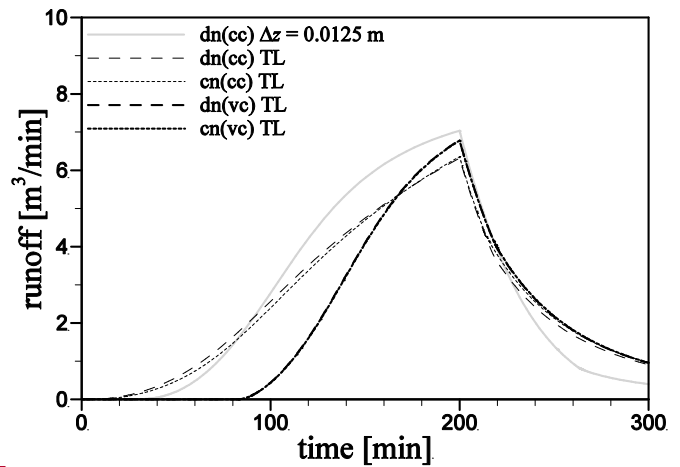
1235 ~~Figure 10: Outflow response~~12: Simulated values for excess infiltration (third scenario) on a ~~hill~~

1236 hillslope with a cell-centered scheme and $\Delta z = 0.0125$ m. a) Water depths at the surface nodes. b)

1237 Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the down-slope ~~using~~

1238 ~~different vertical discretizations.~~direction).
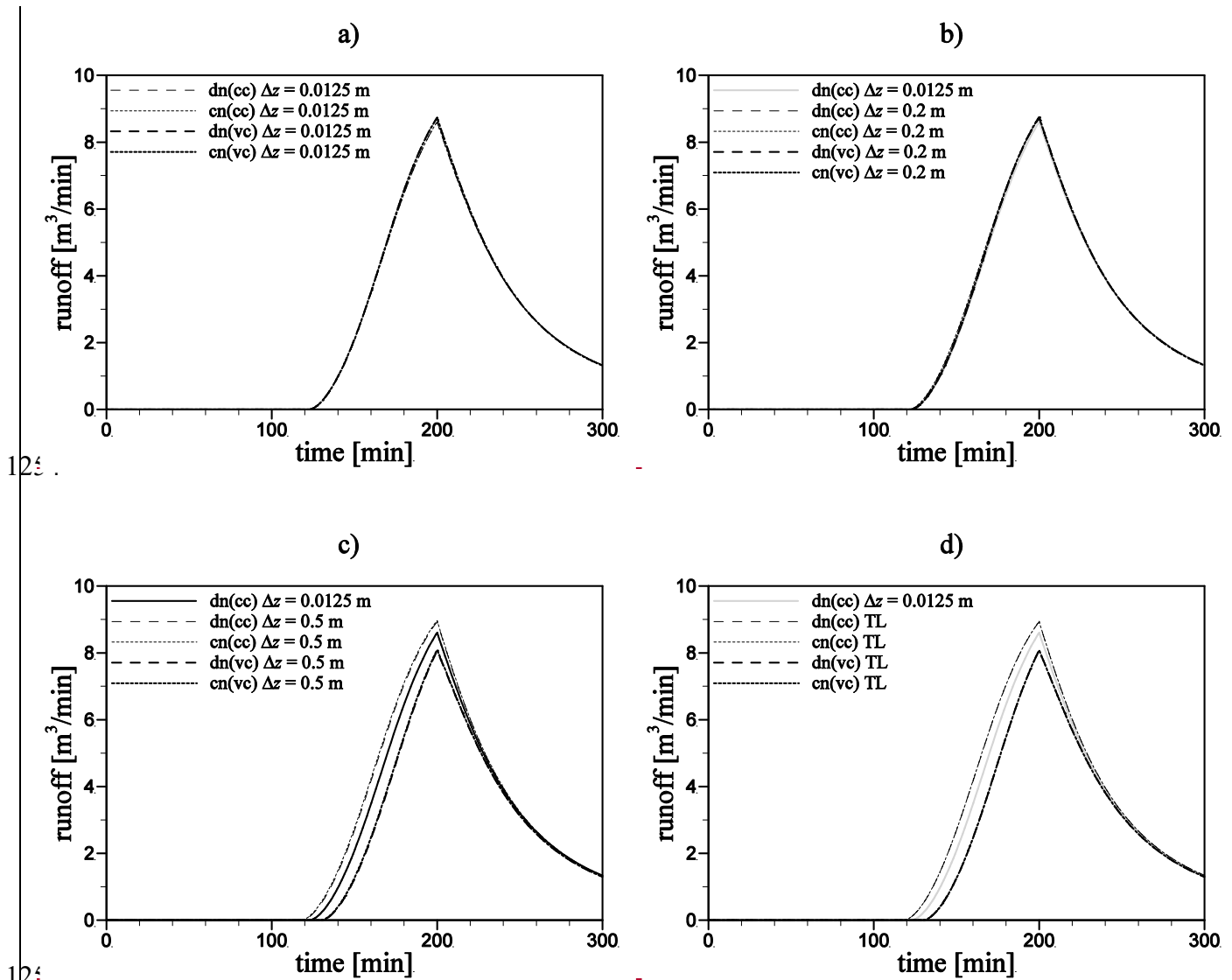
1239

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a)

b)

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c)

- – – dn(cc) $\Delta z = 0.5$ m
- · · · · cn(cc) $\Delta z = 0.5$ m
- **– –** dn(vc) $\Delta z = 0.5$ m
- **· · ·** cn(vc) $\Delta z = 0.5$ m

Newton steps

time [min]

d)

- – – dn(cc) TL
- · · · · cn(cc) TL
- **– –** dn(vc) TL
- **· · ·** cn(vc) TL

Newton steps

time [min]

a)

—— dn(cc) $\Delta z = 0.2$ m
—— cn(cc) $\Delta z = 0.2$ m

water depth [m]

1 2 3 4 5

time [min]

b)

—— dn(cc) $\Delta z = 0.2$ m
—— cn(cc) $\Delta z = 0.2$ m

p [m]

1 2 3 4 5

1-2 3

4 5

time [min]
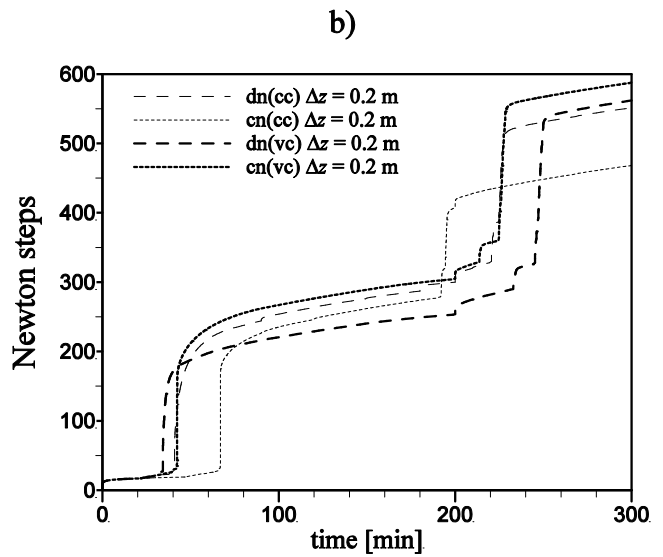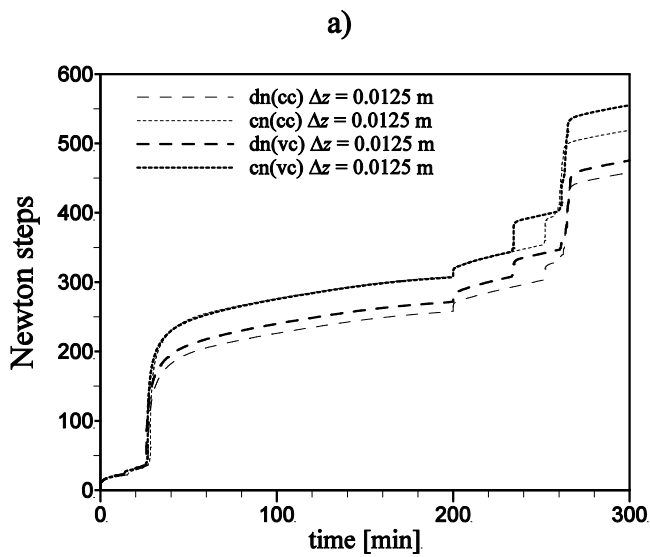
1248 Figure ~~11: The total number of Newton steps~~13: Simulated values for excess infiltration ~~on a hill slope~~

1249 ~~using different vertical discretizations.~~

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1253

a)

runoff [m³/min] vs time [min]
- dn(cc) Δz = 0.0125 m
- cn(cc) Δz = 0.0125 m
- dn(vc) Δz = 0.0125 m
- cn(vc) Δz = 0.0125 m

b)

runoff [m³/min] vs time [min]
- dn(cc) Δz = 0.0125 m
- dn(cc) Δz = 0.2 m
- cn(cc) Δz = 0.2 m
- dn(vc) Δz = 0.2 m
- cn(vc) Δz = 0.2 m

c)

runoff [m³/min] vs time [min]
- dn(cc) Δz = 0.0125 m
- dn(cc) Δz = 0.5 m
- cn(cc) Δz = 0.5 m
- dn(vc) Δz = 0.5 m
- cn(vc) Δz = 0.5 m

d)

runoff [m³/min] vs time [min]
- dn(cc) Δz = 0.0125 m
- dn(cc) TL
- cn(cc) TL
- dn(vc) TL
- cn(vc) TL

1256   Figure 12: Outflow response for excess saturation(third scenario) on a hill slope using different vertical

1257   discretizations.

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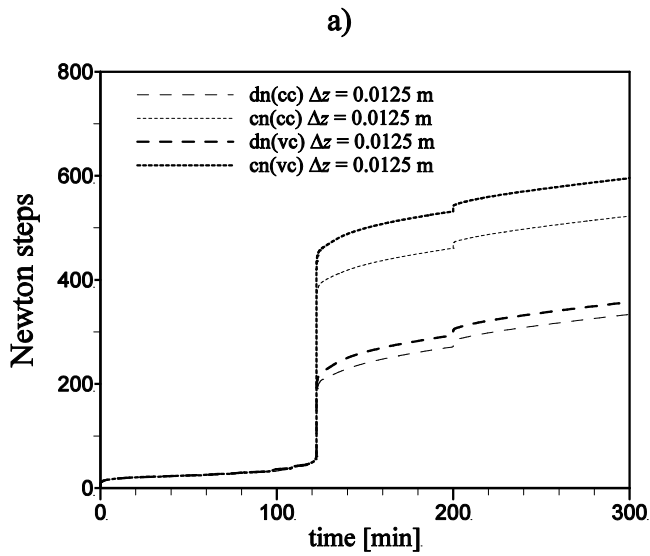1262

a)

b)

c)

d)
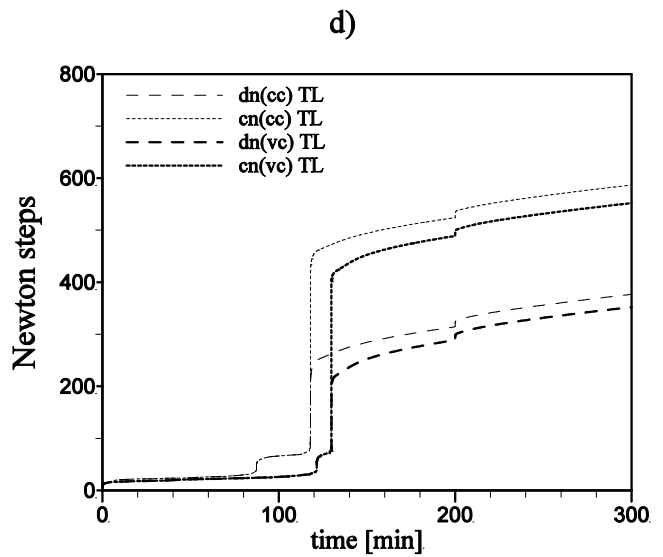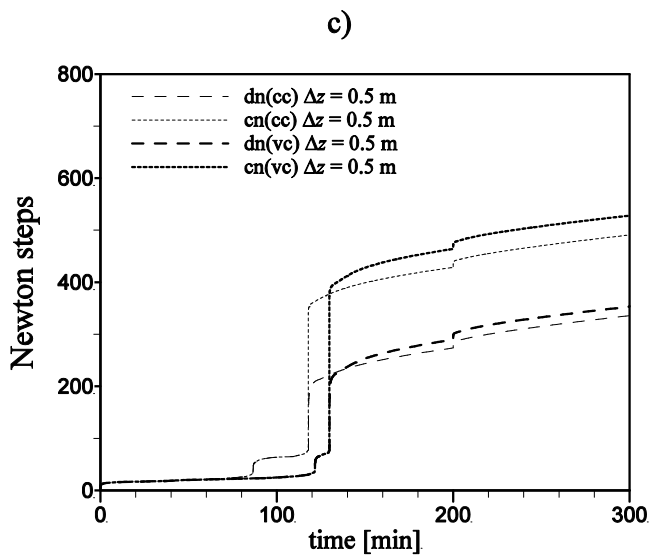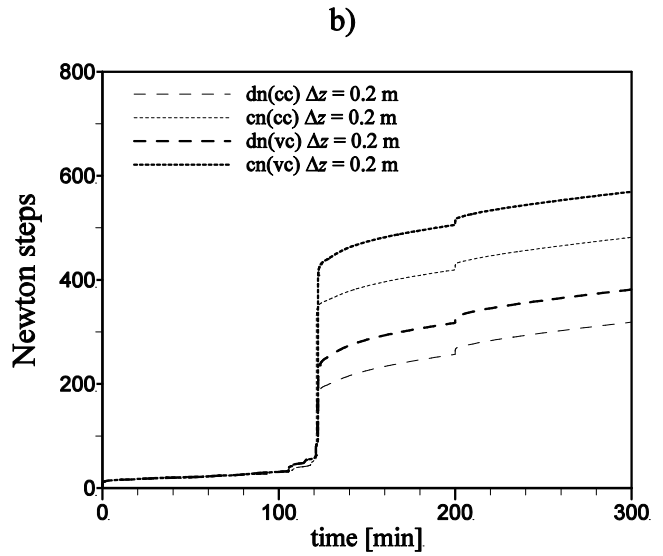
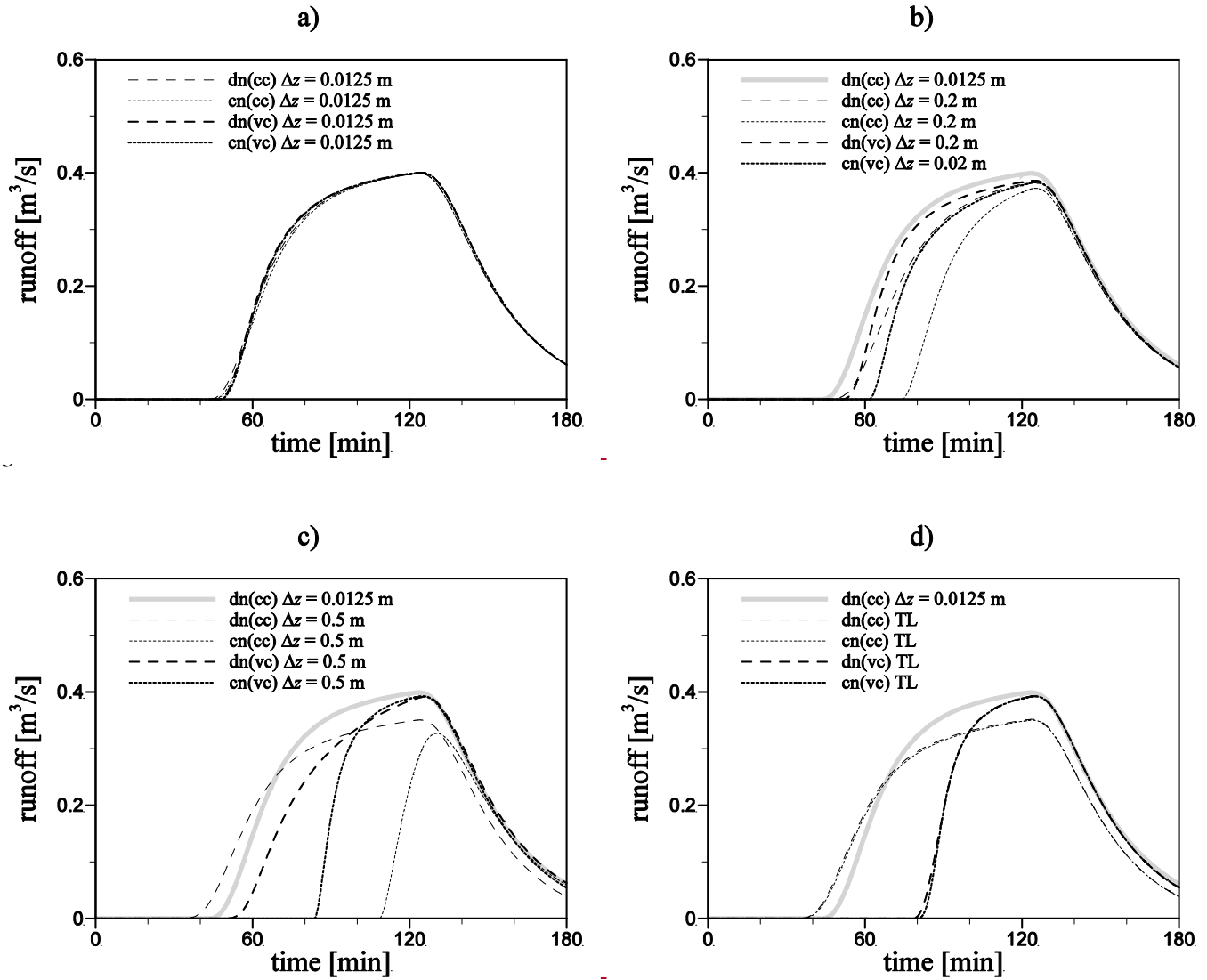Figure 13: Number of Newton steps for ~~excess saturation on~~hillslope with a ~~hill slope using different vertical discretizations.~~

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1272

a)

b)

c)

d)

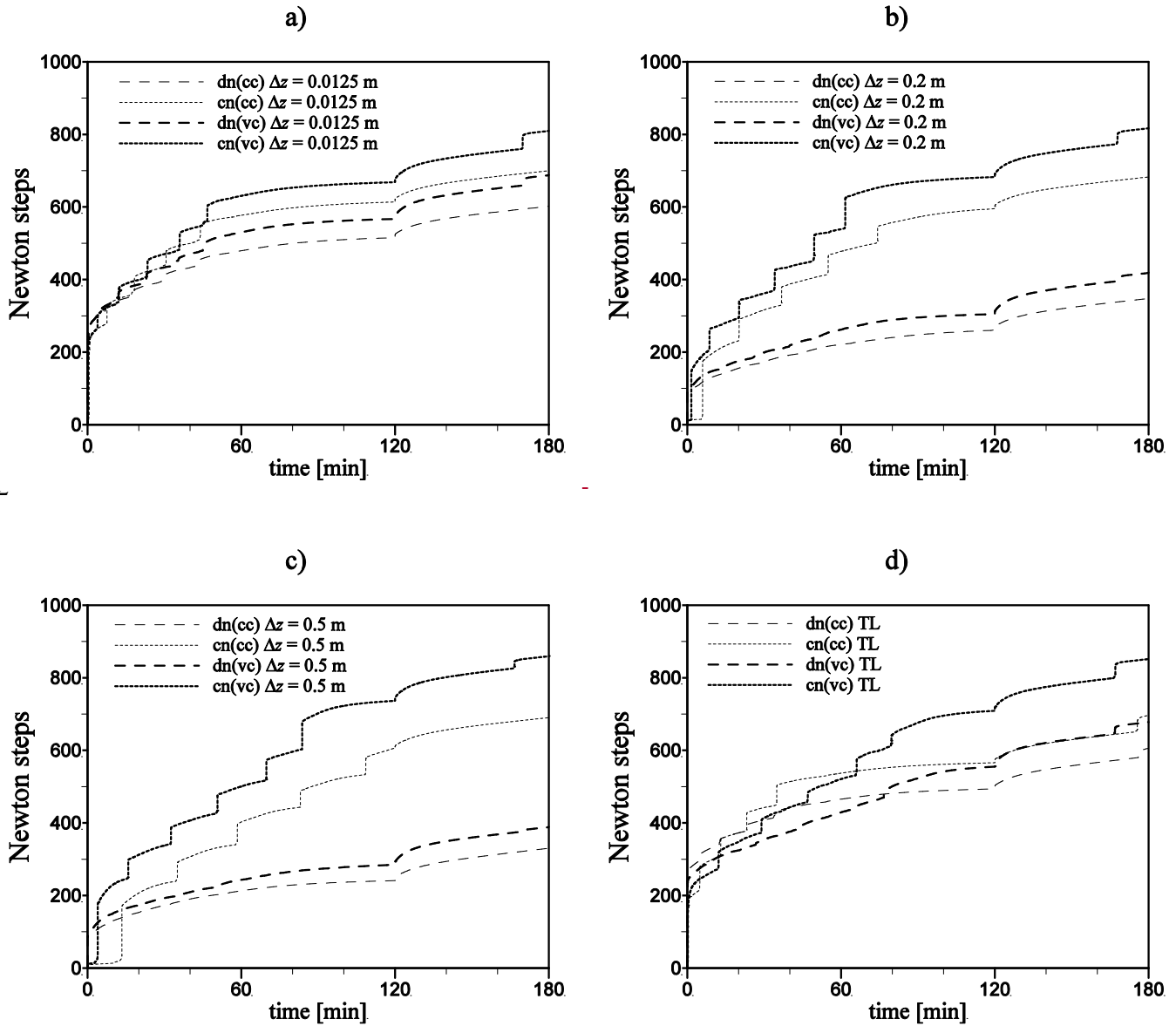Figure 14: Outflow response for flooding an unsaturated hill slope using different vertical discretizations.
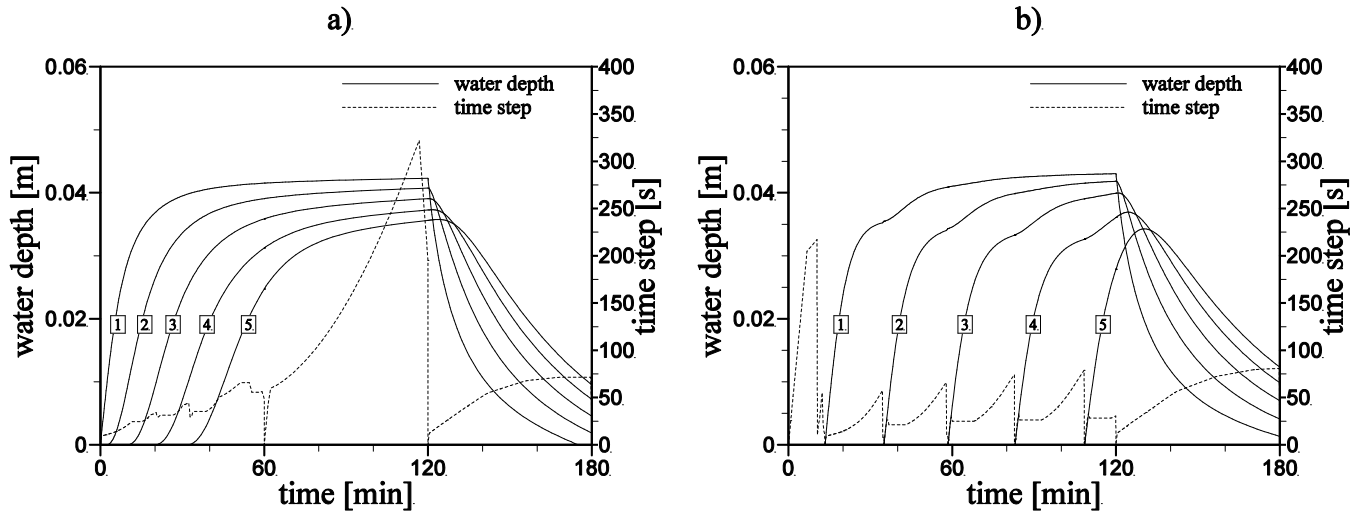
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a)

b)

c)

d)

1284 Figure 15: Number of Newton steps for flooding an unsaturated hill slope using different vertical

1285 discretizations.

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1289

a)

b)

Figure 16: ~~Response in water depth at the five~~cell-centered scheme and $\Delta z = 0.2$ m. a) Water depths at the surface nodes ~~(numbered from upstream to downstream) for flooding an unsaturated hill slope. Left: dn(cc) $\Delta z = 0.5$ m. Right: cn(cc) $\Delta z = 0.5$ m.~~ b) Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the down-slope direction).

1295