Technical Note: Design flood under hydrological uncertainty

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Abstract. Planning and verification of hydraulic infrastructures demands for a design estimate of hydrologic variables, usually provided by frequency analysis, neglecting hydrologic uncertainty. However, when hydrologic uncertainty is accounted for, the design flood value is no longer a deterministic value, but should be treated as a random variable itself. As a consequence, the design flood is no longer univocally defined, making the design process undetermined.

The Uncertainty Compliant Design Flood Estimation (UNCODE) procedure is a novel approach which allows one to fix the ambiguity in the selection of the design flood under uncertainty, by considering an additional constraint based on a cost-benefit criterion. This paper contributes to disseminate the UNCODE procedure without resorting to numerical computation, but using a correction coefficient that modifies the standard (i.e., uncertainty-free) design value on the basis of sample length and return period only. The procedure is robust and parsimonious, as it does not require additional parameters with respect to the traditional uncertainty-free analysis.

Simple equations to compute the correction term are provided for a number of probability distributions commonly used to represent the flood frequency curve. This new design tool provides a robust way to manage the hydrologic uncertainty and to go beyond the use of traditional safety factors. With all the other parameters being equal, an increase of the sample length reduces the correction factor, and thus the construction costs, still keeping the same safety level.

1 Introduction

The flood frequency curve is commonly used to derive the design flood as the quantile Q_T corresponding to a fixed return period T. For practical reasons, Q_T is commonly expressed only as a single value; however, Q_T can only be expressed in this way if its frequency distribution and its parameters are known perfectly. In practice, one can only estimate the frequency distribution and its parameters using a sample of observed data, thereby inflating the uncertainty in the estimate of Q_T. However, the design of an hydraulic infrastructure demands for a single design value to be selected. A gap therefore exists between theory and practice. Quantitative methods to measure the uncertainty associated to the quantiles of the flood frequency curve (e.g., through their variance or probability distribution) have been proposed (e.g., Cameron et al., 2000; De Michele and Rosso, 2001; Brath et al., 2006; Blazkova and Beven, 2009; Laio et al., 2011; Liang et al., 2012; Viglione et al., 2013), but very few suggestions are provided about how to extract a single design value from the probability distribution of possible design values.

Botto et al. (2014), with the development of the Uncertainty Compliant Design Flood Estimation (UNCODE) procedure, have shown that it is possible to select meaningful flood quantiles from their distribution by considering an additional constraint based on a cost-benefit criterion. Hence, the output is a unique design flood value Q_T^*. Before illustrating the UNCODE approach, it is worth recalling the working principles of the cost-benefit analysis, which is a core element of the procedure. Cost-benefit analysis can be used to estimate the design flood as the flow value which minimizes the total expected cost function, defined as the sum of the actual cost to build a flood protection infrastructure (cost function) and the expected damages caused by a flood event. An illustrative example of this approach is reported in Fig. 1a. The cost function is rather easy to understand, being an increasing function of the design flood. Instead, the expected damage function needs to be computed point-by-point: for any single tentative design flood value (see the inset in Fig. 1a) it equals the integral of the product of the probability density function (pdf) of the flood flow values and a specific damage function. The latter indicates the damage occurring when the flood exceeds the flow value used to design the infrastructure. The damage function depends on a number of parameters such as the exposure and vulnerability of the flooded goods, the flooding dynamics and the topography, to mention a few. For these reasons the damage function turns out to be very site-specific and often unavailable for the lack of information needed to compute it (Menoni et al., 2016).

The cost-benefit method thus appears as an attractive design approach; however, due to the cost and damage functions not easy to be determined and hardly generalizable, it often results inapplicable. To face this problem Botto et al. (2014) made the assumption that costs and damages can be represented by piecewise linear functions, with slope c and d respectively, as illustrated in Fig. 1b. Given this assumption, the total cost C_{TOT}, can be computed as

$$C_{TOT} = c \cdot Q^* + \int_{Q^*}^{\infty} d \cdot (Q - Q^*) \cdot p(Q|\mathbf{\Theta}) \, \mathrm{d}Q, \tag{1}$$

where Q^* is the generic design flood value and $p(Q|\mathbf{\Theta})$ is the probability density function of the flood flow with parameters $\mathbf{\Theta}$. The optimal design flood of the (uncertainty-free) cost-benefit framework can be then calculated as the value that minimizes Eq. (1). Examples of cost-benefit analysis in the hydrologic/hydraulic context can be found in the literature (Bao et al., 1987; Ganoulis, 2003; Jonkman et al., 2004; Tung, 2005), with only a few of them accounting for uncertainty (Al-Futaisi and Stedinger, 1999; Su and Tung, 2013).

Botto et al. (2014) further demonstrated that the optimal design flood obtained from the cost-benefit analysis with linear cost and damage functions is equivalent to the design flood Q_T obtained from the standard frequency analysis, provided that uncertainty is not accounted for and the ratio between d and c equals the return period T. This result can be shown by setting to 0 the derivative of C_{TOT} with respect to Q^*, in order to find the minimum of Eq. 1; this leads to the equivalence

$$\frac{d}{c} = \frac{1}{1 - P(Q^*|\mathbf{\Theta})} = T, \tag{2}$$

where $P(\cdot)$ is the cumulative distribution function of the flood values and T is the return period. This is valid provided that the probability distribution used in the cost-benefit framework is the same used in the standard frequency analysis.

The UNCODE approach is founded on these findings through the joint use of the cost-benefit approach of Eq. (1) and the constraint derived in Eq. (2). The rationale behind this approach is that it is possible to apply the cost-benefit framework with

standard, but meaningful, cost and damage functions. This is particularly convenient because the cost-benefit framework can be easily extended to include the hydrological uncertainty due to the limited data availability. This allows one to extend also the UNCODE framework to account for this kind of uncertainty. In uncertain conditions, the parameters of the flood frequency distribution, $\mathbf{\Theta}$, become a random vector; hence, the uncertainty can be included in the cost benefit analysis by compounding C_{TOT} over all the possible values of $\mathbf{\Theta}$. In mathematical terms, the cost-benefit framework with uncertainty is summarized by the equation

$$Q_T^* = \underset{Q^*}{\arg\min} \left[\int_{\mathbf{\Theta}} C_{TOT} \left(Q^* | c, d, p(\mathbf{\Theta}) \right) \cdot h(\mathbf{\Theta}) \, \mathrm{d}\mathbf{\Theta} \right], \tag{3}$$

where $h(\mathbf{\Theta})$ is the joint pdf of the parameters of the flood frequency curve. Equation (3) represents the full UNCODE model, which adopts linear cost and damage functions and accounts for uncertainty in a cost-benefit framework.

It is worth noting that, as a consequence of the inherent equivalence of Eq. (2), there are no additional parameters in the cost-benefit framework; in fact, c and d are related through the known value of the return period T. The remaining free parameter can be shown to affect only the magnitude of the integral in Equation (3), but not the position of its minimum thus avoiding the need for further parameters in the UNCODE framework with respect to the standard design flood procedure.

To simplify the UNCODE application, which requires the use of numerical computation of Q_T^*, we provide here an approximated, though reliable, method to estimate Q_T^* starting from Q_T. Other than a useful practical tool for design purposes, the analysis reported in this note also provides a method to quantify the "value" of newly available hydrological information or the effect of data scarcity on Q_T^* due to uncertainty.

2 Practical estimation of the UNCODE design flood

The UNCODE design flood, Q_T^*, results systematically larger than its corresponding standard value Q_T. The relative difference between the two values,

$$y = \frac{Q_T^* - Q_T}{Q_T}, \tag{4}$$

has been shown (Botto et al., 2014) to increase with the return period (as the quantile uncertainty increases) as well as, for fixed T, with the standard deviation of the probability distribution of Q_T (i.e., with the uncertainty of Q_T). We propose to calculate the approximated estimate of the UNCODE design flood, hereafter referred to as $\hat{Q}_T^*$, directly by inversion of Eq. (4), without resorting to the numerical solution of Eq. (3). This solution reads:

$$\hat{Q}_T^* = (1 + \hat{y}) \cdot Q_T, \tag{5}$$

where the correction factor $\hat{y}$ (i.e., the approximated estimator of y) needs to be computed separately. Given this background, we propose to model $\hat{y}$ according to the equation

$$\hat{y} = 10^{-2} \cdot \exp\left[a_0 + a_1\sqrt{n} + a_2 \ln T \right], \tag{6}$$

where T is the return period and n is the sample length which can be considered as a proxy of the standard deviation of Q_T; n can be computed from at-site records or as an equivalent sample length from the regional estimate of Q_T.

The coefficients a_0, a_1 and a_2 have been evaluated from an extensive simulation study in which the full UNCODE procedure has been systematically applied to many simulated records, created by combining the following criteria:

1. The parent distribution $\mathcal{P}$ selected from the list: log-Normal (LN3), Generalized Extreme Value (GEV), Generalized Logistic (GLO), Pearson type III (PE3) and log-Pearson type III (LP3). For details on the probability distribution equation and on the relationship between parameters and L-moments the reader is referred to Hosking and Wallis (1997). The LP3 corresponds to the PE3 with log-transformed variate.

2. The sample length n of annual maxima selected from the list: 30, 40, 50, 60, 70 80, 90, 100.

We generated 100 records for each combination of $\mathcal{P}$ and n. Looking at the L-moments space, 90% of the synthetic records fall within the ranges: $0.28 \leq$ L-CV ≤ 0.40, $0.14 \leq$ L-skewness ≤ 0.40 and $0.07 \leq$ L-kurtosis ≤ 0.32, which correspond well with values typically encountered in real-world applications. The standard design flood Q_T as well as the (exact) UNCODE estimator Q_T^* have been computed for each record of the simulated dataset. This step has been performed by adopting a suitable fitting distribution $\mathcal{F}$ to the whole synthetic dataset. To make the results more general, $\mathcal{F}$ has been selected from the list: LN3, GEV, GLO, PE3, LP3. Note that any $\mathcal{F}$ is used to fit records from any parent $\mathcal{P}$ as in real cases the exact parent distribution is not known a priori. In this way, the error due to the misspecification of the fitting distribution is included in the results. The correction factor y (Eq. 4) has been computed for all the available records in the simulated dataset and for different return periods T (respectively equal to 50, 100, 200, 500 and 1000 years). It depends on the fitting distribution $\mathcal{F}$ adopted in the frequency analysis. Finally, the exact y values have been regressed against n and T to obtain their estimate $\hat{y}$ (using an ordinary least squares linear regression on the log-transformed terms of Eq. 6). Different forms of Eq. (6) have also been tested, but are not reported as they provide less accurate results.

Coefficients a_0, a_1 and a_2 are reported in Table 1 for different fitting distributions commonly used in the hydrological practice to compute the design flood (in fact, the fitting distribution is always known, while the parent is not). It can be noticed that, when increasing the sample length n, the difference between Q_T^* and Q_T is reduced, due to the negative value of the coefficient a_1. Table 1 reports also some diagnostics of the regressions used to estimate the coefficients. The global performance of the regressions has been evaluated using the coefficient of determination and residuals analysis (through the mean absolute error, MAE, and root mean squared error, RMSE) for each fitting distribution. The value of the coefficient of determination ranges from 0.96 in case of the PE3 and 0.94 for the LN3, to 0.85 for the GEV and GLO. The MAE and the RMSE take values around 0.02, corresponding to a 2% variation in the design flood estimation, which is negligible in many situations. In general, the PE3 probability distribution is that with the best performances in terms of residuals analysis and R^2_{adj}.

The reliability of the approximated correction factor $\hat{y}$ estimated with the regression model has also been evaluated by comparing the $\hat{Q}_T^*$ value obtained through Eq. (5) and (6) with its exact counterpart calculated with the full UNCODE procedure (Eq. 3). As a reference, time series listed in Botto et al. (2014, Table 1) with at least 30 years of record length have been analyzed, assuming the LN3 and the GEV as possible fitting distributions and different return periods. Results show a very

good agreement between the exact (Q_T^*) and the approximated ($\hat{Q}_T^*$) UNCODE design flood values, as reported in Fig. 2, where each panel shows the estimates for all series and all the return periods.

A synthesis of the obtained results is shown in Fig. 3, where the values of $\hat{y}$ have been reported for the studied distributions, based on a set of typical sample length and return period values. As mentioned, a direct comparison of the results between different distributions is not possible, but it is relevant to observe that for all the distributions $\hat{y}$ evolves in the same way for varying n and T values. In general, the correction factor does not exceed 10% of the standard value Q_T for intermediate return periods (e.g., $T = 200$ years) even for small samples, although a significative variability is associated to the distribution type. It is around 10% for $T = 500$ years with sample length values ($n = 50$) commonly available at many gauged stations. On the other hand, the sample length plays an important role: for example, considering $T = 500$ years, the GEV distribution and varying the sample size, the reduction of the y value is about 0.075 between $n = 30$ and $n = 50$, and to 0.040 between $n = 50$ and $n = 70$.

3 Discussion of the application conditions

The UNCODE approach to flood frequency analysis provides an answer to the ambiguity due to the uncertainty in the quantile estimation. Application of the full UNCODE procedure may be cumbersome and computationally demanding. For a quick estimation of the design value an approximated but reliable framework has been proposed here to easily compute the UNCODE flood starting from the standard design value.

The extensive simulation analysis at the base of this study shows that the coefficients relating the UNCODE value $\hat{Q}_T^*$ to the standard value Q_T are distribution-dependent. For the most used distributions they have been computed and provided. The choice of the distribution and the management of its associated uncertainty is a problem of model selection; hence it cannot be solved by the UNCODE procedure, but depends on the preliminary standard flood frequency analysis.

The obtained results demonstrate that an increase in the length of relatively short samples has a noticeable impact in terms of reduction of $\hat{y}$ and of the UNCODE estimate $\hat{Q}_T^*$. This implies that, while the infrastructure keeps the same safety level, additional data reduce construction costs as the actual design value is reduced. The mentioned results agree with findings recently obtained by Ganora and Laio (2016) in a study on the relative role of regional and at-site flood frequency modeling approaches, where the value of at-site data has been highlighted and regarded as a reliable way to improve regional predictions, even with short records. Under this perspective, the correction factor can be used as a metric for uncertainty comparison and quantification, thus providing a further tool to combine different modeling approaches, similarly to the applications of Kjeldsen and Jones (2007) and Ganora et al. (2013) who, with different methodologies, have exploited measures of hydrologic uncertainty to merge regional and at-site information. The coefficient $\hat{y}$ can be considered a measure of the value of data. In fact, with all other parameters being equal, increasing n leads to a reduced $\hat{y}$ value and, consequently, to a reduced UNCODE design flood $\hat{Q}_T^*$. As a consequence, while the design value is still based on the same return period, costs will reduce.

Finally, the correction factor is a new and easy-to-implement design tool which provides a quantitative way to determine the design flood value accounting for hydrologic uncertainty, while keeping the same design hazard level considered in standard

uncertainty-free analyses. This is a novel approach when compared to the common engineering practice, which accounts for hydrologic uncertainty by considering, for instance, the hydraulic freeboard. The use of the freeboard is equivalent to increasing the design flood value, but without accounting for the size of the system (e.g., the basin area), nor for the hydrologic information available at the section (i.e., observed of equivalent record length used to compute the standard design flood); this approach is thus not tailored to the specific case study. The correction factor represents an advance with respect to the use of "all-encompassing" safety factors and towards a clearer way to manage the different sources of uncertainty in hydrological and hydraulic design.

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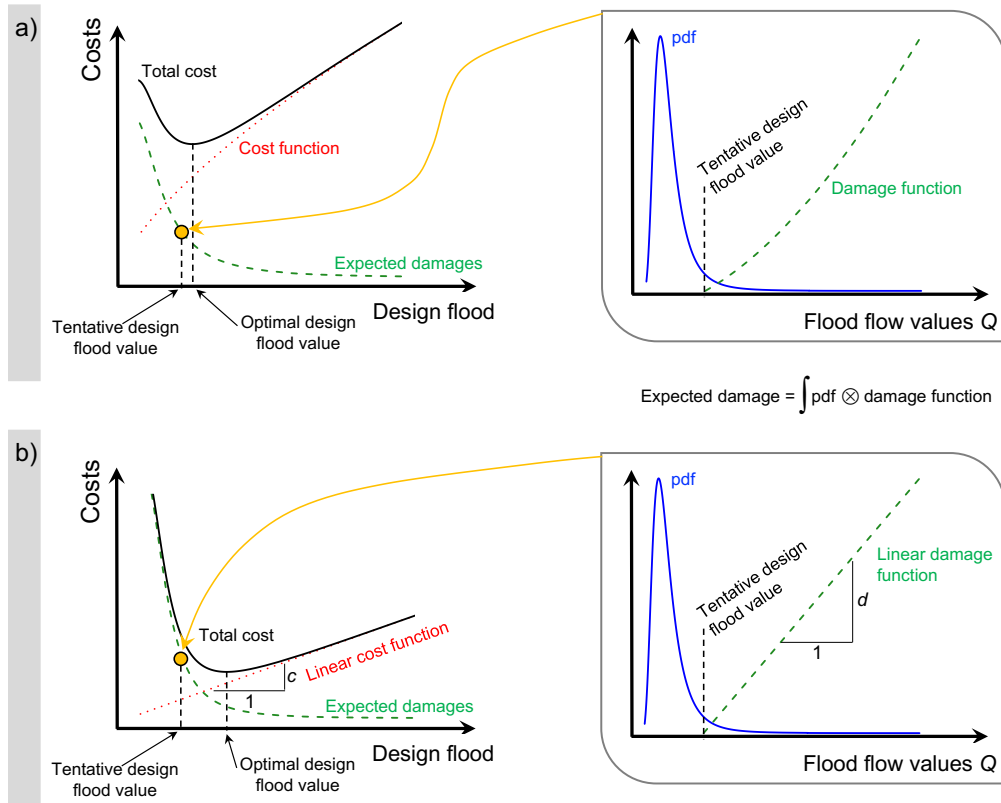
Figure 1. Illustrative example (without uncertainty) of the application of the cost-benefit framework to compute the design flood. Two generic cost and damage functions are reported in panel a, while panel b shows the linear functions adopted in the UNCODE framework.
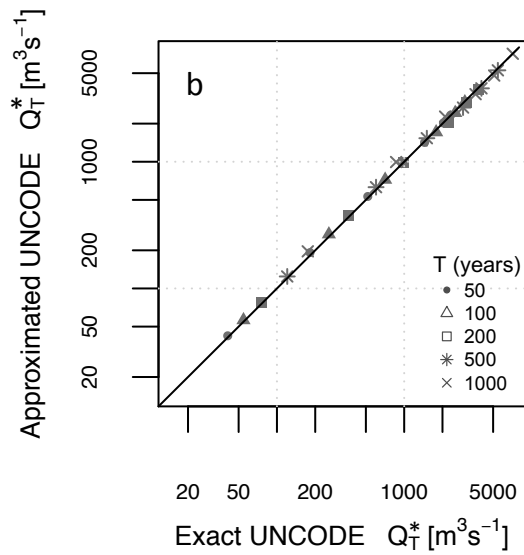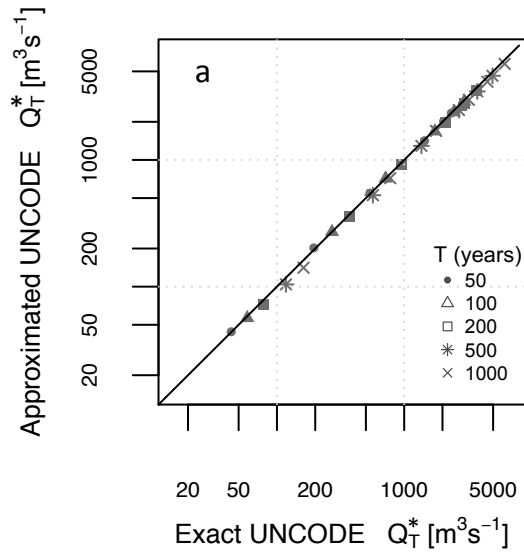
Figure 2. Comparison between the exact, Q_T^*, and the approximated, $\hat{Q}_T^*$, UNCODE estimators of the design flood for a pool of 6 flood records considered in Botto et al. (2014, Table 1) with at least 30 years of data. Different return periods are listed in the legend. The reference distribution used for this flood frequency analysis is the 3-parameter log-Normal (LN3) in panel "a" and the generalized extreme value (GEV) in panel "b".
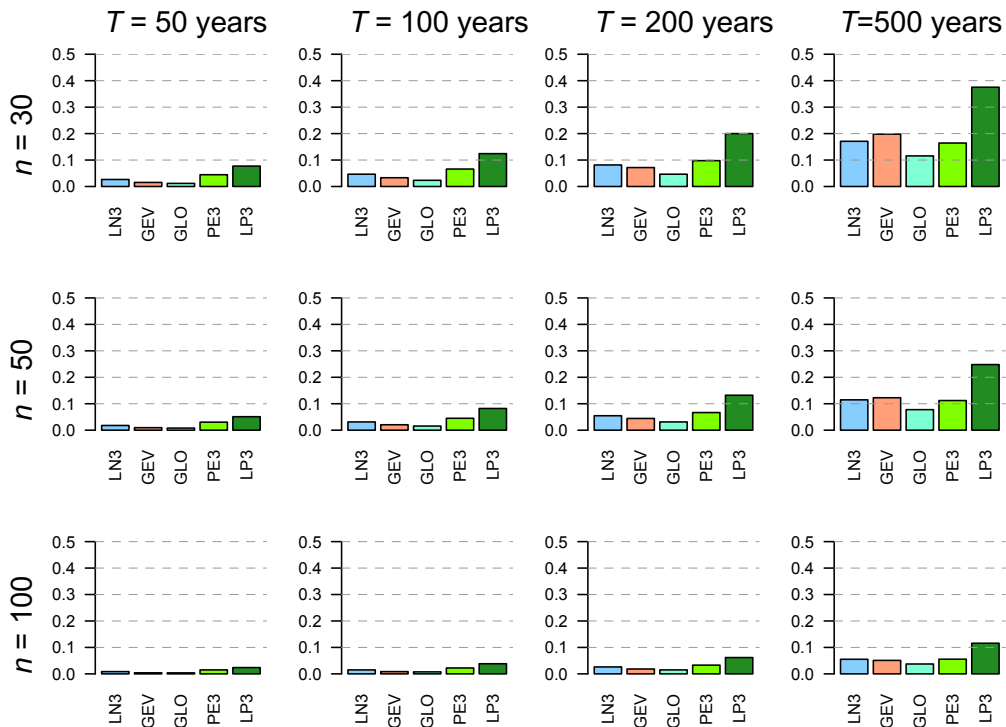
Figure 3. Values of the correction factor $\hat{y}$ from Eq. (6) for some values of the sample length n and return period T and for different three-parameter fitting distributions (LN3 = log-Normal; GEV= generalized extreme value; GLO = generalized logistic; PE3 = Pearson type III; LP3 = log Pearson type III). The LP3 corresponds to the PE3 with log-transformed variate.

Table 1. Coefficients to be used to estimate $\hat{y}$ based on the sample length n and the return period T (eq. 6) and corresponding regression diagnostics, for different 3-parameter fitting distributions (LN3 = log-Normal; GEV= generalized extreme value; GLO = generalized logistic; PE3 = Pearson type III; LP3 = log Pearson type III). The LP3 corresponds to the PE3 with log-transformed variate.

	a_0	a_1	a_2	R^2_{adj}	MAE	$RMSE$
LN3	-0.82	-0.25	0.809	0.94	0.0107	0.0160
GEV	-2.27	-0.3	1.110	0.85	0.0190	0.0321
GLO	-2.36	-0.25	0.994	0.85	0.0096	0.0145
PE3	0.59	-0.24	0.567	0.96	0.0080	0.0115
LP3	0.78	-0.26	0.687	0.89	0.0235	0.0363