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Cascade of submerged reservoirs as a rainfall-runoff model

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Abstract. The rainfall-runoff conceptual model as a cascade of submerged linear reservoirs with particular outflows depending on storages of adjoining reservoirs is developed. The model output contains different exponential functions with roots of Chebyshev polynomials of the first kind as exponents. The model is applied to IUH and recession curves problems and compared with the analogous results of the Nash cascade. Case study is performed on a basis of 46 recession periods. Obtained results show the usefulness of the model as an alternative concept to the Nash cascade.

Keywords: rainfall-runoff models, submerged reservoirs, Chebyshev polynomials, IUH, recession

1 Introduction

The significance of the conceptual model of the rainfall-runoff relation introduced by Nash as a linear cascade of reservoirs (Nash, 1957) and developed later as parallel cascades (Wittenberg, 1975; Oben-Nyarko, 1976) known nowadays as the Diskin model (Diskin *et al*., 1978; Diskin, 1980) cannot be overestimated. These models have been widely applied in the mathematical modeling of catchments for many years and still are in use. Undoubtedly, one of the advantages of these models is the simplicity related to the linearity, what corresponds *inter alia* to the real baseflow features (Fenicia *et al*., 2006). However, the Nash and Diskin models do not represent many real hydrographs correctly enough, including peak flows (Singh, 1976). Bárdossy (2007) noticed the great uncertainty of the identified cascade parameters and related difficulties with the determination of the optimum parameters set for a particular catchment. These problems considered together with the high diversity of real hydrographs shapes including recession curves (Stoelzle *et al*., 2003) imply searches for new solutions. One of the modern tendencies are nonlinear models (e.g. Liu and Todini, 2002; Ding, 2011; Kim and Georgakakos, 2015). This direction of researches may be perceived as an expression of disappointment due to unsatisfactory results of linear models applications. On the other hand it seems, however, that the possibilities of linear models have not been exploited enough. The linear model of cascaded reservoirs generating outputs different from the classical Nash hydrographs, which may be an alternative solution to standard ones is presented below.

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2 Submerged cascade model

2.1 Theoretical considerations

The peculiarity of the model is replacing classical reservoirs of the Nash cascade by submerged ones (Fig. 1), where outflows depend on storages of adjoining reservoirs (except the last reservoir in a chain). Assuming the linearity of the system it is described by the set of constitutive equations:

$$\begin{aligned} &Q_1 = k_1 \cdot (S_1 - S_2) \\ &\ldots \\ &Q_{n-1} = k_{n-1} \cdot (S_{n-1} - S_n) \\ &Q_n = k_n \cdot S_n \end{aligned} \tag{1}$$

and continuity equations:

$$\begin{aligned} &\frac{dS_1}{dt} = P - Q_1 \\ &\frac{dS_2}{dt} = Q_1 - Q_2 \\ &\ldots \\ &\frac{dS_n}{dt} = Q_{n-1} - Q_n \end{aligned} \tag{2}$$

Substituting (1) to (2) and introducing a commonly used simplification:

$$k_1 = k_2 = \ldots = k_n = k \tag{3}$$

yields the following set of equations:

$$\begin{aligned} &\frac{dQ_1}{dt} = k \cdot (P - 2Q_1 + Q_2) \\ &\frac{dQ_2}{dt} = k \cdot (Q_1 - 2Q_2 + Q_3) \\ &\ldots \\ &\frac{dQ_{n-1}}{dt} = k \cdot (Q_{n-2} - 2Q_{n-1} + Q_n) \\ &\frac{dQ_n}{dt} = k \cdot (Q_{n-1} - Q_n) \end{aligned} \tag{4}$$

To solve the nonhomogeneous set of equations (4), the solution to a homogeneous set is necessary. At $P = 0$ the set of equations (4) generates a tridiagonal matrix:

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$$\mathbf{A} = k \cdot \begin{bmatrix} -2 & 1 & 0 & \dots & \dots & \dots & 0 \\ 1 & -2 & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ \dots & & & & & & \\ 0 & \dots & \dots & 0 & 1 & -2 & 1 \\ 0 & \dots & \dots & \dots & 0 & 1 & -1 \end{bmatrix} \tag{5}$$

If all eigenvalues of the matrix **A** are different, the global solution to the set (4) with the condition $P = 0$ is:

$$Q_i = \sum_{j=1}^{n} c_j \gamma_{ij} e^{\lambda_j t} \quad \text{for } i = 1,2,\dots n \tag{6}$$

where $\boldsymbol{\lambda}$ is a vector of the matrix **A** eigenvalues, $\boldsymbol{\gamma}$ – matrix of coefficients creating fundamental set of solutions and $\boldsymbol{c}$ – vector of coefficients depending on initial conditions.

After substituting $\boldsymbol{\lambda} = k \cdot \boldsymbol{\delta}$ the determination of the eigenvalues vector requires the solution to the equation:

$$W_n(\delta) = \begin{vmatrix} -2-\delta & 1 & 0 & \dots & \dots & \dots & 0 \\ 1 & -2-\delta & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -2-\delta & 1 & 0 & \dots & 0 \\ \dots & & & & & & \\ 0 & \dots & \dots & 0 & 1 & -2-\delta & 1 \\ 0 & \dots & \dots & \dots & 0 & 1 & -1-\delta \end{vmatrix} = 0 \tag{7}$$

Values $W_n(\delta)$ may be determined by the recurrence formula:

$$\begin{aligned} W_0(\delta) &= 1 \\ W_1(\delta) &= -(\delta + 1) \\ W_i(\delta) &= -(\delta + 2) \cdot W_{i-1} - W_{i-2} \end{aligned} \tag{8}$$

Figure 2 shows the $W_n(\delta)$ values for different numbers of reservoirs n. Due to the Favard's theorem (Favard, 1935) the values W_i produce a sequence of orthogonal polynomials, what results from the 3-term recurrence relation. However, the roots of these polynomials of higher degrees are difficult to calculate. Therefore, the above concept of submerged cascade requires the modification facilitating calculations of the consecutive eigenvalues (as a consequence, also $\boldsymbol{\gamma}$ coefficients). This can be done by increasing the storage coefficient k for the last reservoir in a chain twice (model SC2):

$$k_1 = k_2 = \dots = k_{n-1} = k, \quad k_n = 2k \tag{9}$$

It is worth noting that the concept of differentiating k value of the last reservoir in relation to the rest of the chain is not new; in 2006 was introduced by Szilagyi to a model with fractional number of reservoirs (Szilagyi, 2006).

After (9) the matrix **A** has the form:

$$\mathbf{A} = k \cdot \begin{bmatrix} -2 & 1 & 0 & \dots & \dots & \dots & 0 \\ 1 & -2 & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ \dots \\ 0 & \dots & \dots & 0 & 1 & -2 & 1 \\ 0 & \dots & \dots & \dots & 0 & 2 & -2 \end{bmatrix} \tag{10}$$

and the determinant $W_n(\delta)$ may be calculated recursively:

$$\begin{aligned} &W_0(\delta) = 2 \\ &W_1(\delta) = -(\delta + 2) \\ &W_n(\delta) = -(\delta + 2) \cdot W_{n-1} - W_{n-2} \end{aligned} \tag{11}$$

Thus,

$$W_n = 2T_n\left(-\frac{\delta + 2}{2}\right) \tag{12}$$

where T_n is a Chebyshev polynomial of the first kind and n-th degree. Functions $W_n(\delta)$ are shown in Fig. 3.

Roots of the Chebyshev polynomials of any degree satisfy the relation:

$$T_n(\delta) = 0 \ \text{for} \ \delta_j = \cos\left(\frac{2j-1}{2n} \cdot \Pi\right), \quad j = 1,2,\dots n \tag{13}$$

so the eigenvalues of the matrix (10) yield:

$$\lambda_j = (-2 + 2 \cdot \beta_{j,n}) \cdot k \text{, where } \beta_{j,n} = -\cos\left(\frac{2j-1}{2n} \cdot \Pi\right), \ j = 1,2,\dots n \tag{14}$$

Determination of the matrix $\boldsymbol{\gamma}$ consists in the solution to the following set of equations at successive values of j ($j = 1,2,\dots n$):

$$\begin{bmatrix} -2\beta_{j,n} & 1 & 0 & \dots & \dots & \dots & 0 \\ 1 & -2\beta_{j,n} & 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -2\beta_{j,n} & 1 & 0 & \dots & 0 \\ \dots \\ 0 & \dots & \dots & 0 & 1 & -2\beta_{j,n} & 1 \\ 0 & \dots & \dots & \dots & 0 & 2 & -2\beta_{j,n} \end{bmatrix} \cdot \begin{bmatrix} \gamma_{1,j} \\ \gamma_{2,j} \\ \dots \\ \\ \gamma_{n,j} \end{bmatrix} = 0 \tag{15}$$

Theory of differential equations proves that the order of the matrix from (15) equals $n-1$; hence, one of the equations depends on the others, what allows to assume any value of one coefficient. Thus, putting:

$$\gamma_{n,j} = 1 \tag{16}$$

remaining coefficients may be calculated from the relations:

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$$\gamma_{n-1,j} = \beta_{j,n}$$
$$\gamma_{n-2,j} = 2\beta_{j,n} \cdot \gamma_{n-1,j} - \gamma_{n-2,j} \tag{17}$$

Hence:

$$\gamma_{n-m,j} = T_m\left(\beta_{j,n}\right)$$

or (18)

$$\gamma_{i,j} = T_{n-i}\left(\beta_{j,n}\right)$$

Since for Chebyshev polynomials at any values p, x the following identity is satisfied:

$$T_p(\cos x) = \cos px \tag{19}$$

the coefficients $\gamma_{i,j}$ may be calculated as:

$$\gamma_{i,j} = \cos\left[(n-i)\frac{2j-1}{2n}\cdot\Pi\right] \tag{20}$$

Finally, the general solution (6) for SC2 yields:

$$Q_i = \sum_{j=1}^{n} C_j \cos\left[(n-i)\frac{2j-1}{2n}\cdot\Pi\right] e^{-\left[2+2\cos\left(\frac{2j-1}{2n}\cdot\Pi\right)\right]kt} \tag{21}$$

In particular, for the last reservoir of the cascade:

$$Q_n = \sum_{j=1}^{n} C_j \cdot e^{-\left[2+2\cos\left(\frac{2j-1}{2n}\right)\cdot\Pi\right]kt} \tag{22}$$

Determination of the constants of integration to the SC2 model requires solution to the equation set:

$$\boldsymbol{\gamma C} = \boldsymbol{Q}(\mathbf{0}) \tag{23}$$

where $\boldsymbol{Q}(\mathbf{0})$ is a vector of initial conditions, depending on the analyzed problem. For any number of reservoirs n the set (23) can be solved numerically by the calculation of the inverse matrix $\boldsymbol{\gamma}^{\mathbf{1}}$.

2.2 Solution to the IUH problem

Considering the IUH problem the following initial conditions are introduced:

$$S_1 = 1, \quad S_2 = \ldots = S_n = 0 \tag{24}$$

Hence,

$$Q_1(0) = k, \quad Q_2(0) = \ldots = Q_n(0) = 0 \tag{25}$$

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Exact values of the constants of integration C_j for IUH in the SC2 model obtained by analytical solution to the equation (23) with conditions (25) from $n = 2$ to $n = 6$ at $k = 1$ are given in Table 1.

Figure 4 shows the IUHs for consecutive reservoirs of the SC2 cascade for number of reservoirs varying from $n = 2$ to $n = 6$ ($k = 1$). The relatively small difference between IUH values for Q_5 and Q_6 at $n = 6$ is apparent, what suggests irrationality of increasing n above these numbers in practical applications.

2.3 Solution to recession curves

Initial conditions for recession curves in the Nash model may be determined considering the equal storage for each reservoir with no rainfall supply. Such assumption is rational and justified in particular at long-lasting rainfall before the recession period. However, in the SC2 model such a rainfall does not lead to the situation of equal storage of reservoirs since no flows between adjoining reservoirs exist then. Therefore, the initial conditions for SC2 may be formulated as:

$$Q_1(0) = Q_2(0) = \ldots = Q_n(0) = Q_0 \tag{26}$$

This corresponds to the situation of permanent decrease of storage for successive reservoirs. Table 2 shows numerically calculated constants of integration C_j for recession curves with initial conditions (26) at $Q_0 = 1$ and $k = 1$ from $n = 2$ to $n = 6$. Analytical forms of these constants are not given due to its high complexity at higher n values.

Figure 5 shows recession curves for successive reservoirs of the SC2 cascade from $n = 2$ to $n = 6$ ($k = 1$). Similarly to IUH problem, the difference between ordinates of curves for Q_5 i Q_6 at $n = 6$ is inconsiderable.

3 Comparison of SC2 and Nash model hydrographs

IUHs and recession curves yielded by SC2 were compared with analogous Nash model results. In order to ensure similarity of both cascades, the storage coefficient k for the last reservoir in the Nash model was doubled. Additionally, the following conditions were assumed:

$$\begin{gathered} Q_N(0) = Q_{SC2}(0) = 1 \\ \int_0^\infty Q_N dt = \int_0^\infty Q_{SC2} dt \end{gathered} \tag{27}$$

Lower indices in (27) represent values in Nash and SC2 models, respectively. To fulfill (27), the parameter k in the Nash model should be assumed as:

$$k_N = 2k_{SC2} \cdot \frac{n - \frac{1}{2}}{n^2} \tag{28}$$

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Figures (6) and (7) show IUHs and recession curves for both models with different number of reservoirs n. It should be noticed that IUH of SC2 model attenuates at higher n values much more than IUH of Nash cascade, what may suggest better condition for this number identification for SC2. However, the same feature can be a disadvantage of SC2, since this model, opposite to the Nash one, does not have the possibility of non-integer number of reservoirs application and may create too large discretization of the solutions space. Next, the recession curves generated by SC2 having initially stronger tendency to decrease in relation to Nash ones and apparent differences of both hydrographs shapes lead to the suggestion that SC2 can be a good alternative to the Nash cascade at rapid transitions of curvatures from concave to convex.

4 Case study – recession curves for real catchments

To examine the usefulness of the SC2 model for practical purposes 12 catchments of Vistula and Oder rivers basins with areas of 500-1000 km^2 were selected. Next, for the set of 46 rainless periods lasting from 7 to 32 days the recession curves were distinguished. For each catchment the condition of minimum number of recession curves equal to three was applied. Flow values for these catchments were taken from published records of the Polish Institute of Meteorology and Water Management – National Research Institute and were determined by the Institute due to the stage-discharge relations with the accuracy of three significant digits.

Since each of the selected periods was preceded by rains of different height and intensity, application of initial conditions neither relating to the equal storage of all reservoirs in the Nash cascade nor to the condition (26) in the SC2 model was possible. Therefore, the initial conditions defined by the vector $\boldsymbol{C}$ were optimized for each recession curve together with the storage coefficient k, assuming the Nash-Sutcliffe efficiency index (Nash and Sutcliffe, 1970) as an objective function. Calculations were carried out separately for both models according to the following formulas:

– in the SC2 model – equation (22);

– in the Nash model:

$$Q_n = e^{-kt} \sum_{j=1}^{n} C_j \cdot \frac{(kt)^{j-1}}{(j-1)!} \tag{29}$$

Figures 8 and 9 show the optimization results. Despite the fact that the SC2 model does not allow to apply the non-integer number of reservoirs and the Nash model was not analyzed from this point of view, graphs are presented as continuous lines, what facilitates the analysis of the variability of the optimized values. Figure 8 shows exemplary results of the optimization for one of the catchments (Ścinawka river, Gorzuchów gauge station) and Fig. 9 shows the averaged values of storage coefficients k and Nash-Sutcliffe indices E_f for particular catchments.

Comparison of graphs for both models leads to the following regularities:

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- E_f values exceed 0,95 for both models as a rule, in particular at high numbers of reservoirs, what testifies the quality of both models quite well;
- at low number n the value E_f in the SC2 model is generally higher than in the Nash one, although at higher n the SC2 model does not show any significant growth of this value, opposite to the Nash model achieving the highest E_f at high n values. This may testify the better elasticity of the Nash model, i.e. better ability to fit the modeled hydrographs shapes to the various recession curves;
- optimized values of storage coefficient k in the SC2 depend on the assumed n value insignificantly (except transition from $n = 2$ to $n = 3$). In the Nash model these values successively increase due to n. This regularity may suggest possibility of the SC2 model application to determine the characteristic value of k for given catchment and, consequently, facilitate the model calibration process by independent optimization of the parameters k and n.

5 Conclusions

In this study the rainfall-runoff conceptual model as a cascade of submerged linear reservoirs is proposed. The supply of each reservoir (except the first one in a chain) depends on the storage of as the upper reservoir as the considered one. Additionally, to obtain the recurrence solution to the set of equations describing water flow throughout the cascade, the value of the storage coefficient k for the last reservoir in the chain is doubled in relation to the previous reservoirs (model SC2), what allows to determine the eigenvalues of the equations set as roots of successive Chebyshev polynomials of the first kind. Obtained output hydrographs contain exponential functions with different exponents in contradistinction to the Nash model, which generates hydrographs with the one and only exponent.

Comparison of features of IUHs and theoretical recession curves generated by SC2 and Nash models suggests possibility and even advisability of next attempts to replace the Nash model by the SC2 one, in particular at baseflow modeling. This is confirmed by the analysis of the measured recession curves. Results of the analysis show that the optimized values of storage coefficients k in the SC2 model are practically constant for each curve and independent of number of reservoirs n, what can be useful considering as the identification process carried on separately for both calibrated parameters (n, k) as the possible correlation between values of identified storage coefficients and catchment parameters. However, the lack of solutions at non-integer number of reservoirs can be a serious disadvantage of the SC2 model. Thus, the applicability of SC2 requires further researches with greater number of catchments. Application of the SC2 model to one of the cascades representing baseflow in the Diskin model may be an interesting experience as well.

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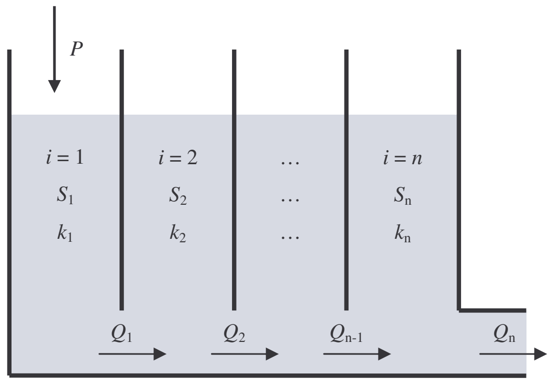

Fig. 1. Conceptual model of submerged reservoirs

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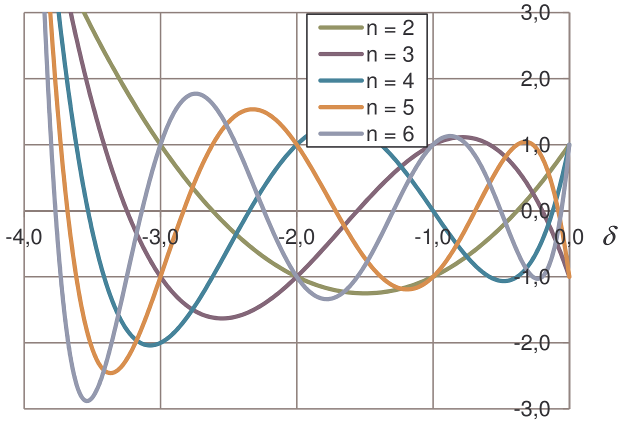

Fig. 2. Graphs $W_n(\delta)$ at different numbers of reservoirs n; values of k the same for all reservoirs (model SC)

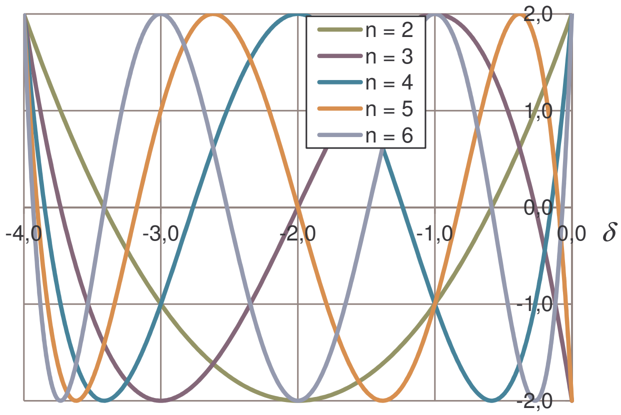

Fig. 3. Graphs $W_n(\delta)$ at different numbers of reservoirs n; value of k for last reservoir in a chain doubled (model SC2)

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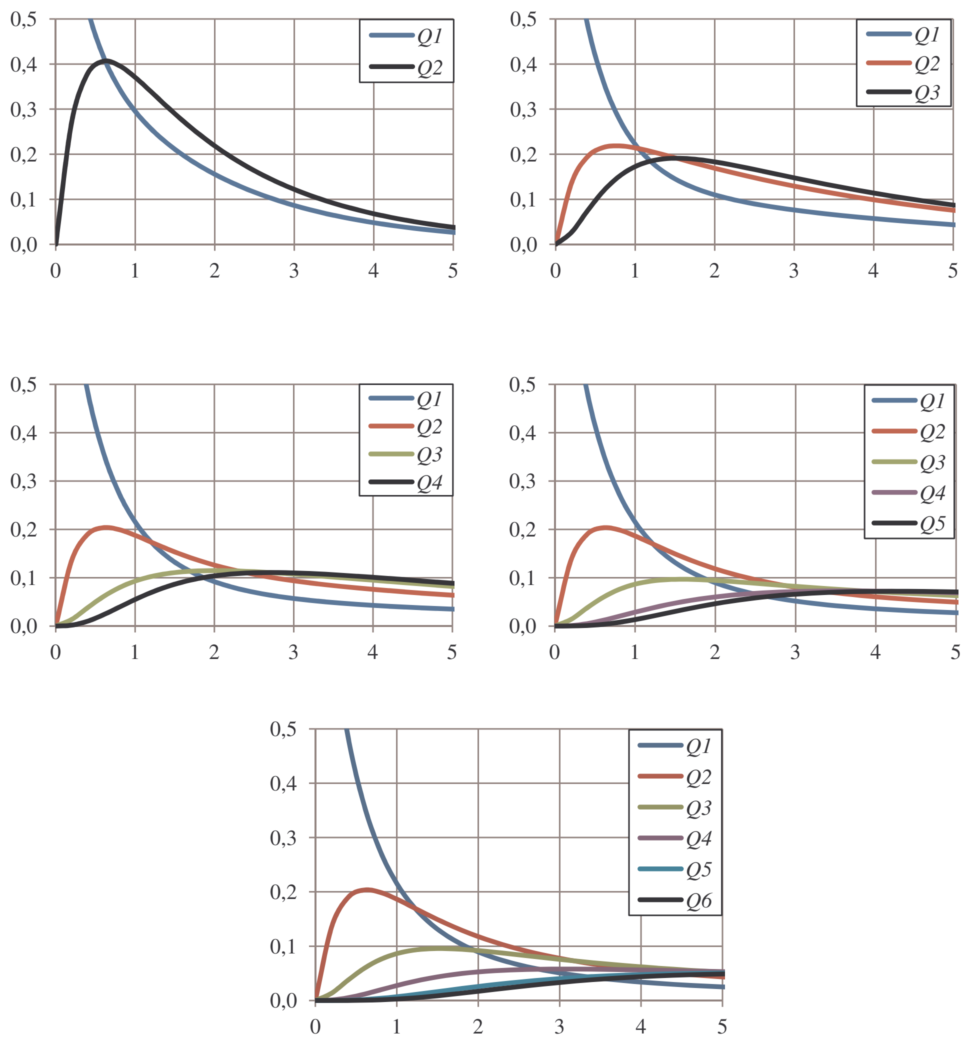

Fig. 4. IUH at different numbers of reservoirs in SC2 model, k_{SC2} = 1

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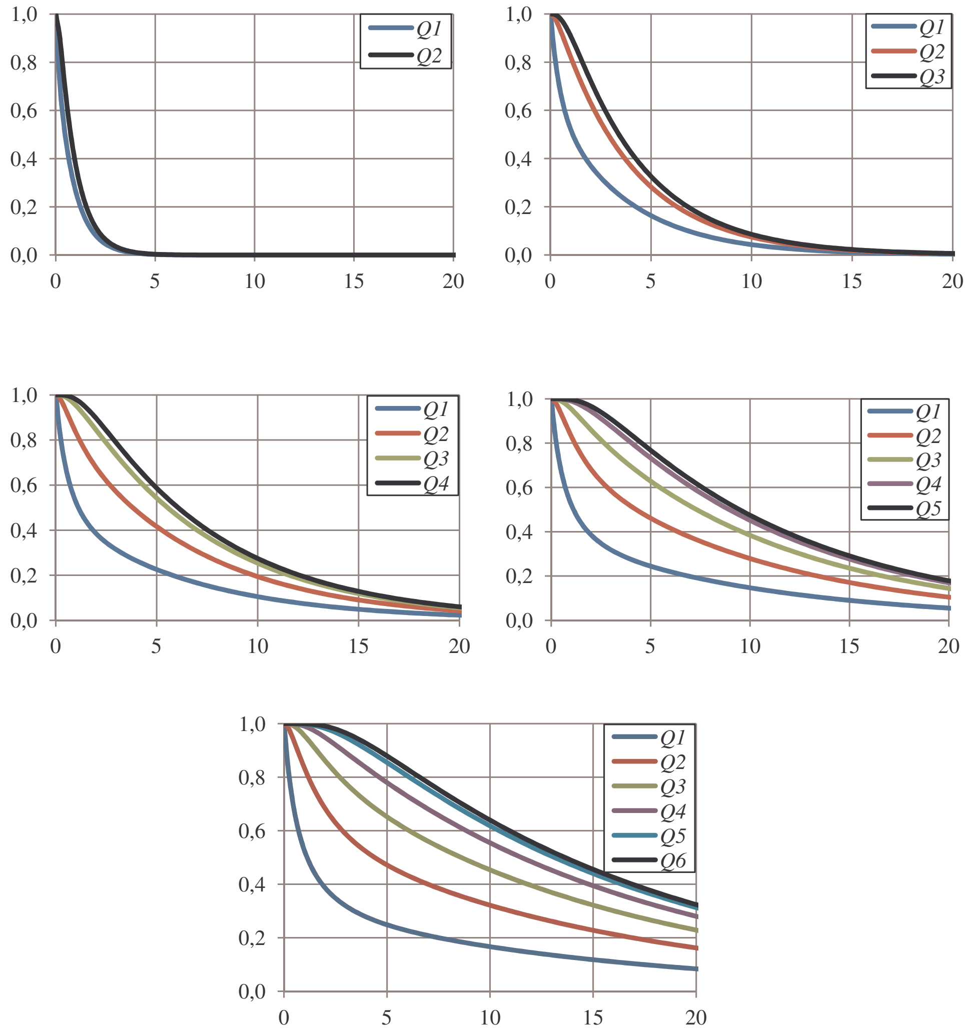

Fig. 5. Recession curves at different numbers of reservoirs in SC2 model, k_{SC2} = 1

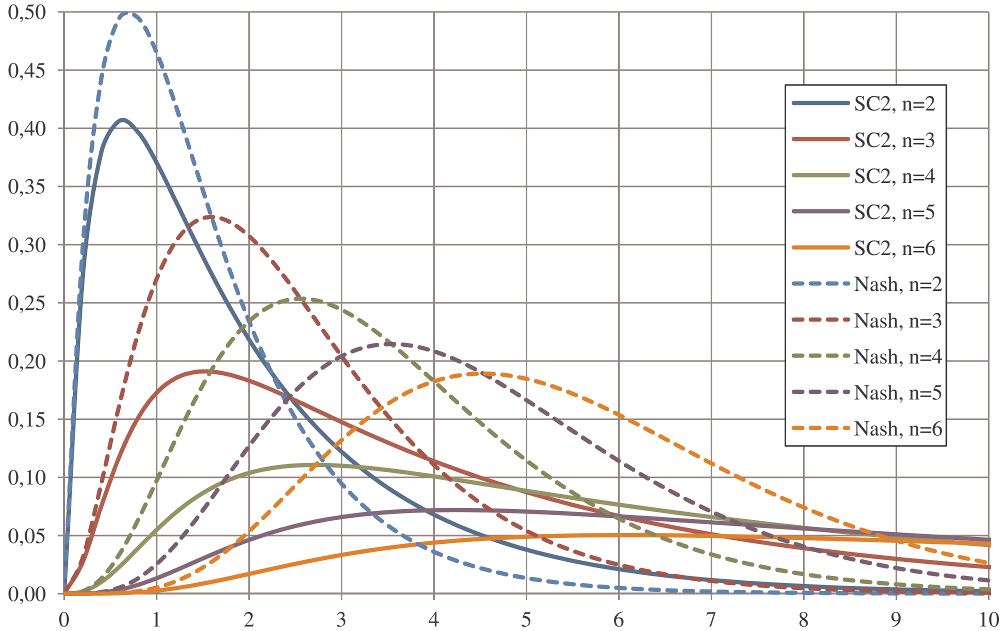

Fig. 6. Comparison of IUH in SC2 and Nash model, k_{SC2} = 1

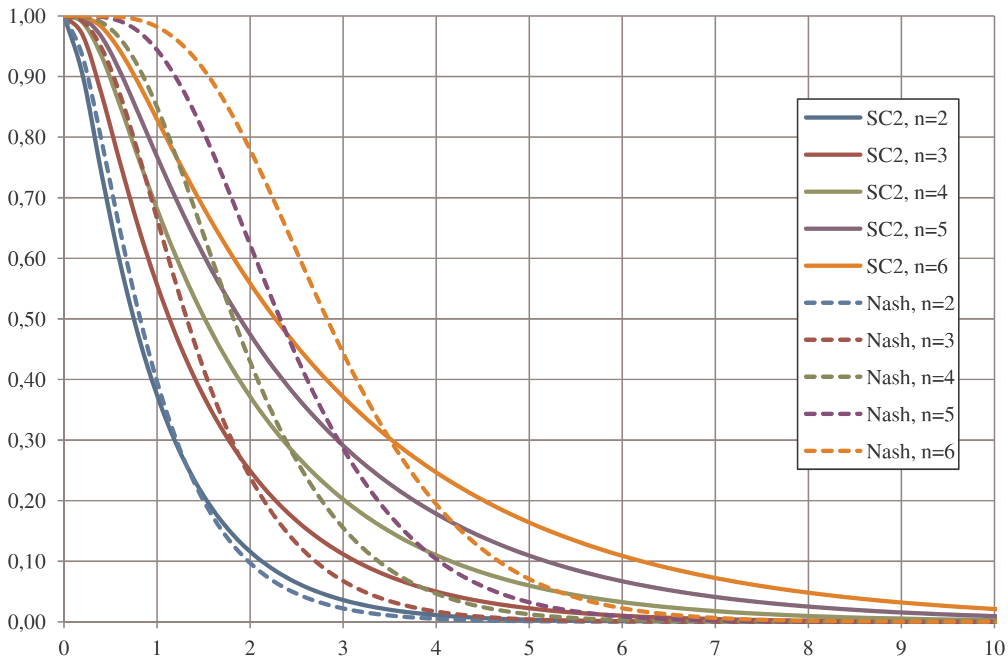

Fig. 7. Comparison of recession curves in SC2 and Nash model, k_{SC2} = 1

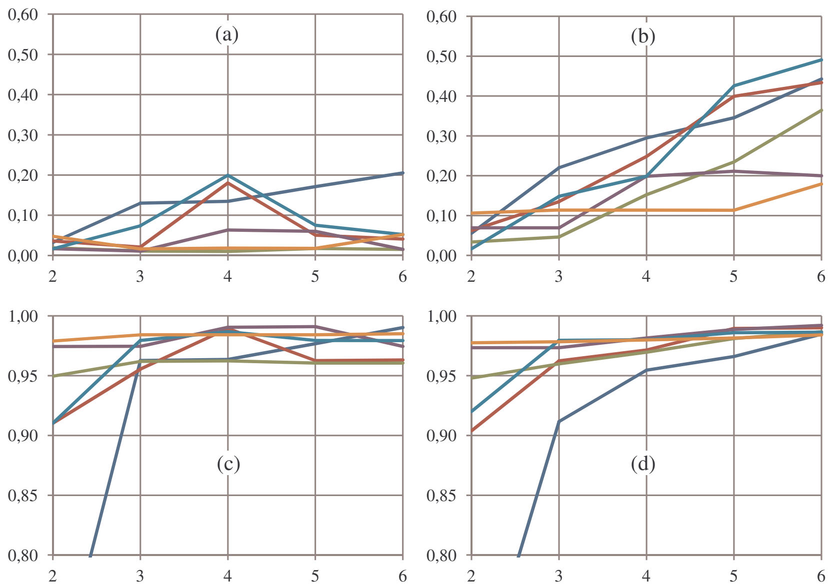

Fig. 8. Exemplary values of storage coefficient *k* [d^{-1}] in the SC2 model (a) and Nash one (b) and Nash-Sutcliffe efficiency index E_f for the SC2 model (c) and the Nash one (d) versus number of reservoirs; the Ścinawka river catchment, water gauge Gorzuchów, 6 independent recession curves

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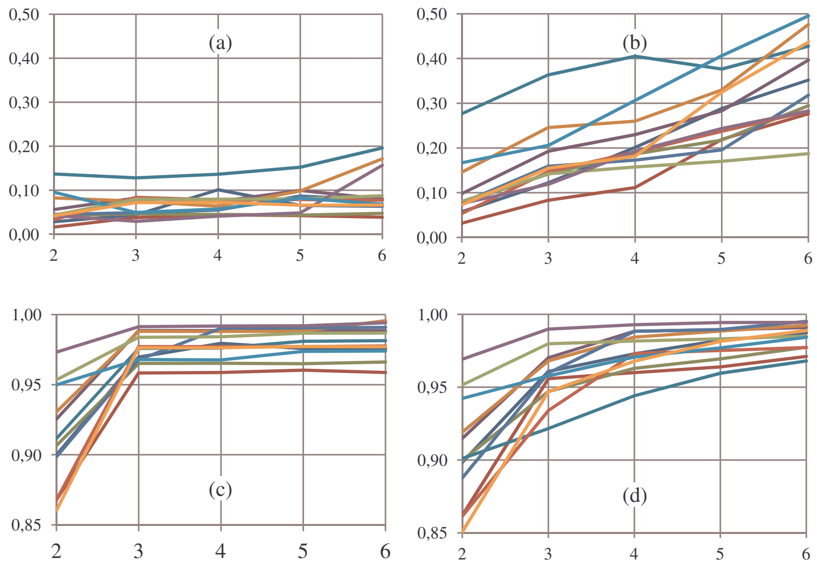

Fig. 9. Mean values of storage coefficient *k* [d^{-1}] in the SC2 model (a) and Nash one (b) and Nash-Sutcliffe efficiency index E_f for the SC2 model (c) and the Nash one (d) versus number of reservoirs for particular catchments

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Table 1. Analytically determined constants of integration to IUH in the SC2 model, *k* = 1

Constant	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
C_1	$\frac{\sqrt{2}}{2}$	$\frac{1}{3}$	$-\frac{\sqrt{2-\sqrt{2}}}{4}$	$\frac{\sqrt{5}-1}{10}$	$-\frac{\sqrt{6}-\sqrt{2}}{12}$
C_2	$-\frac{\sqrt{2}}{2}$	$-\frac{2}{3}$	$\frac{\sqrt{2+\sqrt{2}}}{4}$	$-\frac{\sqrt{5}+1}{10}$	$-\frac{\sqrt{6}+\sqrt{2}}{12}$
C_3		$\frac{1}{3}$	$-\frac{\sqrt{2+\sqrt{2}}}{4}$	$\frac{2}{5}$	$\frac{\sqrt{2}}{6}$
C_4			$\frac{\sqrt{2-\sqrt{2}}}{4}$	$-\frac{\sqrt{5}+1}{10}$	$-\frac{\sqrt{2}}{6}$
C_5				$\frac{\sqrt{5}-1}{10}$	$\frac{\sqrt{6}+\sqrt{2}}{12}$
C_6					$\frac{\sqrt{6}-\sqrt{2}}{12}$

Table 2. Numerically determined constants of integration to recession curves in the SC2 model, $Q_0 = 1$

Constant	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
C_1	–0,20711	0,08932	–0,04973	0,03168	–0,02194
C_2	1,20711	–0,33333	0,16704	–0,10191	0,06904
C_3		1,24402	–0,37415	0,20000	–0,12789
C_4			1,25684	–0,39252	0,21720
C_5				1,26275	–0,40237
C_6					1,26596