

Dear Editor,

Based on the comments we received from the 2 referees who reviewed the first version of the manuscript, we have revisited several parts of our analysis and thoroughly revised the manuscript. In the “track-changes” version of the manuscript, we have indicated the sections where major changes were made (text removed and new texts added). We have included additional results on computation of R-B flashiness indices and compared these to the IAT flashiness index we have defined. Below we briefly explain how each of the referees’ comments was dealt with in the revision. We thank the two reviewers for their critical and constructive comments which have helped us to deepen our analysis and bring out more clearly some interesting findings based on adaptive sampling using inter-amount times.

Review of “Characterising hydrological response in urban watersheds based on inter-amount time distributions” by ten Veldhuis and Schleiss
Referee #1

The study propose a methodology for sampling flow data sets from water courses in urban watersheds based on samples of equal volume.

Data

The stream gauge data used in this study has a temporal resolution of maximum 15 minutes (p4117) but all results reported are for longer aggregation periods: in Figure 8 minimum of 12 hours and in Figure 9 and 10 a minimum of 3 hours. With data available at 15 minutes resolution it should be possible to detect rapid changes in the flow and the manuscript seriously lack comparisons at the sub-hourly scales.

Reply: The minimum scale at which analyses have been conducted was determined by the definition of minimum inter-amount scale, as explained in section 2.4, equations (8) and (9). To limit estimation errors of inter-amount times, we set the mean of absolute relative errors to be smaller than 50%. This requirement determines the minimum sampling scale for IAT analysis; flow analysis and IAT were conducted over the same range of scales. We would like to emphasise that because IAT sampling is an adaptive sampling strategy, much shorter IATs than the mean sampling scale are observed during periods of high flows. We revised sentences explaining the equations (8) and (9) to make this point more clear (p.8, 117-22). Additionally, we added a paragraph at the start of section 3 to explain that analyses were first conducted at 24 hour scale, then across a range of scales (p12, 120-23).

One exception is in Table 3 where the flashiness index is reported for 15 min observational resolution; but here it is unclear whether the 99%-tile of the flow measurements at 15 minutes resolution would give the same answer. Please investigate this and add the results to the discussion.

Reply: We added a paragraph analysing 10-90% and 1-99% ranges for both flows and IATs (section 3.3, p17-18, 120-23, 10-14). We also expanded the section on flashiness index by adding results for computation of Richards-Baker flashiness indices, a flashiness index commonly used in the hydrology literature. We compared results for R-B flashiness indices with those for IAT flashiness based on the 99% of IATs (sections 2.7 and 3.4).

As illustrated by Figure 3 the inter-amount time methodology result in much less data points than the original data set, but it is very unclear why it provides a better data basis for discussing the hydrological properties than the original 15 minute resolution data.

Reply: This is a misunderstanding ; we have rephrased the captions for figures 3a and 3b to explain that figure 3a shows original observed time series and figure 3b compares flow data and inter-amount data at a same resolution to illustrate how sampling strategies differ.

“Missing data were treated as zeros” (p4I23): how does this influence the results and the estimation error?

Reply: We have added several lines to discuss the effect of missing data on IATs (p7 I3-9)

The catchments used in the study are to some degree sub catchments of each other. This could mean that some stream gauges are correlated (e.g. 507 and 530 (Figure 1)) but this is not discussed in the manuscript. This should be discussed in general and specifically in relation to the results where all catchments are discussed (e.g. Tables 2 and 3 and Figures 5, 7, 20 and 13).

Reply: We have added a discussion on relations between results for catchments that are subcatchments of each other in section 3.2 (p15, I9-15) and section 3.3 (p16, 20-21) and added a table (Table 4) summarising results for the 3 sets of related subcatchments in our study.

Figure 1 is very hard to read and could benefit from being enlarged. Also, some will not know where in the world Charlotte, NC, is and it would be beneficial to add a panel of the North American East Coast with a marking of where the study area is.

Reply: We have added a panel showing the location of Charlotte and increased font size.

Figure 2 is not providing any insight and should be removed.

Reply: Figure 2 visualises times series of flows and associated inter-amount times and shows how the inter-amount times have lower variability and fewer outliers as a result of the different sampling strategies. These aspects are discussed later in relation to the respective statistical distributions. Since many readers will never have seen IAT time series we prefer to keep this figure.

Inter-amount times

The definition of inter-amounts (p4I24-p5I2) is brief and to the point. The section on normalization of inter-amounts (p5I13-12) is also brief and the arguments for the methodology are good. The section on sample estimate (p5I13-p6I18) is somewhat harder to follow. The section thoroughly explain how to convert a time series of flow measurements to a series of inter-amount times and the possible error introduced by the approach but in the results section a measure of the error associated with the present 15 minute resolution data and the present catchments is not reported. This is really needed as it should be really limited how important this is at this fine temporal resolution of the flow data.

Reply: The error in flow measurement associated with temporal observational resolution, in relation to the real flow variability, is not known for these datasets. For this, a set of observations at higher temporal resolution would be required, for comparison. As reported in the paper, flow data are derived from water depth measurements using flow-rating curves that regularly recalibrated by USGS. Documentation reporting quantitative error estimates is not available to the authors.

CV, skewness and medcouple are used to compare inter-amount time and flow distributions. In general the discussion of the results (p9I5-p11I9) are for daily values. This section could be much more interesting by adding results for higher resolution since the native data resolution is so much higher than the daily scale.

Reply: As explained above, the minimum scale for IAT analysis is limited by our definition of acceptable error in IAT estimation (equations (8) and (9)). Minimum scales vary between 3 and 13.75 hours (see table 5). We first analysed and compared IAT analysis at daily scale, sections 3.1-3.3, then then over the range of available scales, in sections 3.4-3.6. A brief introduction has been added at the start of section 3, to explain this (p12, I20-23).

At p10I28-29 a bi-modal histogram for catchments with low flow regulation is discussed but not shown, please add these in a supplement.

Reply: A supplement has been added, presenting histograms of flows and IATs for all gauges as well as histograms of differences in flows and IATs for all gauges and quantile plots for all gauges.

From p10l30 to the end of the section is repetition that could and should be left out of the manuscript.

Reply: We have rephrased and shortened this paragraph (p14, l9-13)

The distribution of changes in inter-amount times is used to identify rapidly increasing and decreasing trends (p11l10-30). Figure 6 summarizes the results (again at daily scale) between flow based and inter-amount time based investigations but I cannot see how there can be both inter-amount times and flows in the figure. And it is not clear how to quantitatively get more knowledge from the inter-amount times since the qualitative conclusions will be the same between flows and inter-amount times (even though the skew will be in opposite directions). Please elaborate on this and correct the figure.

Reply: Figure 6 has been changed, the new version presents histograms for differences in flows and in IATs for 2 gauges (histograms for other gauges available in the supplement). We have revisited the analysis of differences in IATs and substantially revised the text section 3.2 to present clear conclusions of this analysis.

Inter-amount times are further compared to flows in Figure 8. Figure 8 is a really good example of all the problems you get from having box-plots on a log scale. For both values span several orders of magnitude and is vastly skewed (as indicated by the large difference between the mean and the median). The associated discussion (p11l31-13l15) is very hard to follow and whether a given percentile is following a power law (p12l10-12) or not is effectively impossible to see from the figure. I would suggest a form of normalization of the results to avoid the logarithmic axes in Figure 8 and make the scaling discussion much more accessible.

Reply: The reviewer is correct in that the values span 4 orders of magnitude in scale, hence our choice for visualising the Q-Q plots on a log-scale. Normalisation as suggested by the reviewer has been applied, the current figures are based on values normalised by basin area. The distributions are indeed vastly skewed; understanding the origins of the skewness and how it differs between flows and IATs, between gauges and how it changes with scale is what we are analysis in this subsection (3.3). The text in section 3.3 has been thoroughly revised to improve the explanations and discussion.

Flashiness

From the very first sentences of the abstract (p1l1-5) flashiness is highlighted as a key parameter where inter-amount time distributions can really make a difference. In the introduction it is concluded from literature that it is difficult to predict the flashiness of urban watersheds, but no methodologies or results are directly presented. Please add a more thorough introduction on how flashiness is normally calculated from traditional flow data.

Reply: We have added a brief literature review on flashiness indicators to the introduction (section 1.2, p3, l4-18). The most commonly used is the R-B flashiness index, introduced by Baker (2004):

$$R-B \text{ Index} = R - B \text{ Index} = \frac{\sum_{i=1}^n |q_i - q_{i-1}|}{\sum_{i=1}^n q_i}$$

We have computed this index for our datasets and compared results with those for the flashiness indicator derived from IAT analysis. Results are presented in a largely new section, section 3.5.

The flashiness indicator formulated in the study (p6l23-24) is very briefly described and a discussion of why this choice was made and why this is a good indicator for flashiness is completely lacking. Please add these.

Reply: We have added a new section 2.7, explaining the definition of IAT flashiness and that of R-B flashiness (p.11)

The results and discussion for flashiness and minimum observable scale (p13l32 +p14) is not easy to follow. In the first section (p13l33-p14l10) It is discussed that very high peak flows cannot be measured correctly every time with 15 minutes resolution data; but is this interesting at all and is it important to know flow variation at this high resolution? Please add a discussion of this.

Reply: We have thoroughly revised this section and added discussion and comparison with R-B flashiness index (section 3.5, p20-21).

In the next section ('Table 2' should really be 'Table 3') a clear correlation between minimum observable scale and flashiness is reported (p14l18-20) as well as a clear correlation between flashiness and basin area (p14l20-22 + Figure 10); these make perfect sense, but would they be different if flow-based flashiness indicators had been used? Please add a comparison to other flashiness indicators.

Reply: We have added values for R-B flashiness in the table 5 and corrected table references in the text. Results are discussed in a new section on flashiness, section 3.5.

In the very end of the section (p14l30-34) a discussion of results not shown is given indicating that the manuscript would benefit from addition of a supplement containing results from all catchments and also the further analysis that has apparently been carried out.

Reply: We have looked into this more thoroughly and found that investigating variation of flashiness with scale is not straightforward. As discussed in section 3.5, the well-established R-B flashiness index does not allow comparisons between values computed at different scales. IAT flashiness can be computed and compared at different scales, but this adds a lot of complexity to the analysis due to the interaction between data resolution and IAT sampling scale. Results require thorough interpretation and discussion, which we plan to work on in the future, but is beyond the scope of the current paper.

Multifractal analysis

The whole section on scaling (p15l1-29) could really be shortened to one sentence simply stating that scaling is great for both flow and inter amounts accompanied by the left side of Figure 11 unless you can show that there is a statistical significant better fit of one of them. Also the identified departures from linearity (p15l6-88 and the right side of Figure 11) should be statistically significant to be relevant for discussion. Please provide relevant statistics to support the conclusions drawn or shorten the section.

Reply: We have revised parts of this section to better explain the two main aspects of multifractal analysis of IATs and flows: (1) IATs scale better than flows (we have provide R2 values in the log-log plots and in table 5) and (2) UM parameters C1 and alpha are different between the two approaches. Moreover, the C1 and alpha values for IATs are less sensitive to the selected range of scales.

Conclusions

In the conclusions it is stated that: "Flows sampled over fixed time intervals did not clearly exhibit this transition. This is result of peak flow variability being poorly sampled by fixed time window sampling." (p16l28-29) but until you add results where you utilize the 15 minutes resolution this cannot be concluded.

Reply: We have revised the text in the related section (section 3.4, p17, l15-18) as well as in the conclusions (p25, l10-15) to make this point more clear.

Another sentence: "Based on inter-amount times distribution we were able to define a flashiness indicator that incorporates both the rising and falling components of the hydrological response" (p17l28-29) seems to be unsupported as the flashiness indicator, as I have understood it, really only tell how many hours of mean flow one can expect as peak flow within a given much shorter time frame (e.g. an indicator of 100 hours for 15 minutes inter-amount times mean that the 99%-tile peak flow is 400 times the mean flow) and how the rise and fall of the peaks are incorporated is not clear.

Reply: We have replaced this text by a new conclusion based on our more extensive analysis of IAT flashiness and new results for R-B indices (p25-26, l29-30, l1-9)

It is also concluded from the multifractal analysis that: “This showed that inter-amount times can help better predict peak flow characteristics at small unobservable scales based on coarse resolution data. Additionally, they provide new interesting alternatives for the stochastic modelling and downscaling of flow data.” (p17l18-20) and “Scaling analysis showed that inter-amount times provide a promising way to better predict peak flow characteristics at small unobservable scales from coarse resolution data” (p17l31- 32) but this was not discussed at all before in the manuscript and if it is true you should really add results to support this.

Reply: We have replaced this text by a new conclusion (p26).

Figures

In general the figures need some work before publication.

The fonts used are generally very small (e.g. the legend for Figure 1 which is unreadable when printed).

Reply: we have increased the font size of some of the figures

The use of sub-figure numbering is inconsistent between text and figures (e.g. Figure 4c and d are not mentioned in the caption and for Figures 6, 8 and 9 the sub-plot labels are missing).

Reply: we have corrected figure references

In Figure 9 there is no marking of which color corresponds to which data set.

Reply: we have added an explanation to the figure caption

Inconsistent use of ‘IATs’ (Figure 11) and ‘inter-amount time’ (Figure 8) as well as ‘Flow’ (Figure 2) and ‘Amounts’ (Figure 11) and ‘medcouple’ (Figure 7) and ‘MC’ (Figure 5).

Reply: we have corrected the terms in captions and in figures

What are the units of the x-axes of Figure 6?

Reply: we have added the units in the caption

Also put the unit directly on the x-axes of Figure 4 and not only in the text of the figure.

Reply: we have added unit to the x-axes

For Figure 8 the x-axis seem confusing. For the inter-amount times the volumes are based on time and should be reported something like “0.51 mm (12h)” but for the flow plot the axis should only be time. Similarly for Figure 11 where the x-axes for flow and inter-amount times should be different.

Reply: we have changed the of units of the axes in the plots

Concluding remark

Indeed, this approach is very interesting as it generate data sets with higher sampling frequency when high flow occur and lower sampling frequency for low flow periods.

This is nicely pointed out by the authors. However, from the reported results I am not convinced that the methodology adds so much to the field.

Reply: We have thoroughly revised large parts of the results section and extended our analysis of flashiness. We believe that the results that are presented in the revised manuscript bring out much more clearly the additional insights that were obtained from IAT analysis.

It is also unclear how exactly the authors see that this knowledge can be utilized in future research as 1) very high resolution flow data is used in this study and 2) it is unclear how the results can be used to better describe watersheds with much coarser data available as it is already pointed out that considerable uncertainty is associated with estimation of the peak flow from 15 minutes resolution data; how will that then look if only daily data is available?. Also downscaling of coarse flow data is mentioned, but for the same reasons as just mentioned it is very unpredictable how this will work.

Reply: For opportunities with respect to downscaling of flow data, we refer to previous replies related to the scaling analysis. Regarding the comment on observational resolution, the authors think it safe to say that nowadays automated gauges are replacing manual gauges in many places and with automated gauges observational resolutions of 15 minutes are quite common and likely to go down to 5 min and 1 min resolution. Especially in urban areas, given the high flow variability at small scales.

Review of "Characterising hydrological response in urban watersheds based on inter-amount time distributions" by ten Veldhuis and Schleiss

Referee #2

Received and published: 29 November 2016

This paper applies an existing framework (Schleiss and Smith, 2016) for analysing inter-amount-times (IAT) to time series of daily streamflow from 17 urbanised watershed monitored by the USGS. The stated aim of the paper is to explore how IAT can be used to characterize the hydrological response and compare results to analysis based on traditional analysis techniques.

In general I found the manuscript difficult to read for a number of reasons; The introduction is very unstructured. It starts by discussing effects of urbanisation, then gives a general A-Z of general challenges in applied hydrology, incl. flood frequency analysis, flow duration curves, multivariate statistics, unit hydrographs, baseflow separation, before ending-up with a very detailed summary of scaling issues with a high level of assumed knowledge on behalf of the reader. I suggest a much more focussed introduction is necessary to better justify the scientific gaps being addressed by the study.

Reply: We have thoroughly revised the introduction section, following the reviewer's suggestion for a more focused literature review. We have also added a brief literature review on flashiness indices

The aim is presented very loosely as 'to explore how inter-amounts can be used to characterise hydrologic response for a range of (semi)urban watersheds in North Carolina, US.' I think it would be more convincing if the aim could be linked more strongly to a distinct problem/gap when using existing methods for analysing the hydrological response from urban watersheds. What is the actual scientific and/or applied problem being addressed here?

Reply: We have rephrased the scope and objective of the paper in section 1.4 in the introduction section (p 5).

I was struggling with some of the technical description in Section 2. This is partly down to a number of key places where the notation is hard to follow (see more detailed comments below), but also because I could not follow how this method was going to add new insight that was not available from a direct analysis of runoff time series. Again, I think a more focussed and readable introduction might have been helpful here.

Reply: Parts of the text in section 2 have been revised, equations in section 2 have been checked and corrected in case they were not consistent and, as explained above, we revised the introduction section to make the scope and objective of the paper more clear.

The results discussion (Section 3) is hard to follow. Maybe consider introducing more subsection and better explain how the different analysis comes together to answer the scientific aim, rather than at present where I get the impression it is a series of independent and individual analysis undertaken because it is possible.

Reply: The results section has been thoroughly revised, as indicated in the revised, tracked-changes manuscript. We have added paragraph breaks to better separate topics discussed in the text.

The main conclusion seems to be that flashiness is related to watershed area, but that no link to urbanisation could be identified. While this is, of course, not the fault of the researchers it does perhaps suggest that the title of the paper (urban watersheds) is not really appropriate as no new insight into the effects of urbanisation seems to have been discovered.

Reply: In the revised text of chapter 3 and in the revised conclusions presented in chapter 4, the effect of urbanisation on results we obtained in IAT analysis are discussed more clearly and explicitly.

I don't think the strong conclusion on page 17, line 21-22 is justified as it is not clear what characteristics is being referred to that could not have been established using conventional time series?

Reply: We have revised this section to summarise the arguments more distinctly, with more explicit links to the presented results.

Detailed comments:

I think HESS uses British rather than US spelling, so catchment rather than watershed

Reply: We have replaced watershed by catchment throughout the text.

Page 2, line 25: Define 'scale-break'

Reply: this text has been revised

Page 3, line 5: Not sure what 'moments $q[0.1-4]$ ' signifies?

Reply: the text has been revised and the definition of moments is explained better.

Eq. (4): this equation sums over q_i , but in the lines above the sample flow time series is defined in terms of r (line 14). Should it be ' r ' in Eq(4) or else ' q_i ' need to be defined somewhere

Reply: the equation has been corrected

Eq (5): I don't understand the notation used in this equation. What does ' $Q_t, nq-nq$ ' mean?

Reply: the equation has been corrected

Eq (6): I don't know how to link this equation to Eq. (5) – I think some more explanation is required here.

Reply : the equations have been corrected

Eq(7): ' T ' is not defined anywhere?

Reply : the equations has been changed, T is now defined already in equation (3)

Page 6, lines 23-24: Given that the flashiness is one of the main conclusions of the study, I think a more comprehensive description of the concept is required, for example include a conceptual figure.

reply: a new section on flashiness indices has been added in the methods section (2.7) and in the results section (3.5)

Page 7, line 16: What is 'Scott's rule'?

reply: Scott's rule defines how bin widths are determined for histograms. A reference was provided

Page 19=0, line 14: what does ' cq ' refers to?

reply: abbreviation ' cq ' has been removed.

Statistical analysis of hydrological response in urbanising catchments based on adaptive sampling using inter-amount times

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Abstract. Urban catchments are typically characterised by a more flashy nature of the hydrological response compared to natural catchments. Predicting flow changes associated with urbanisation is not straightforward, as they are influenced by interactions between impervious cover, basin size, drainage connectivity and stormwater management infrastructure. In this study, we present an alternative approach to statistical analysis of hydrological response variability and basin flashiness, based on the distribution of inter-amount times. We analyse inter-amount time distributions of high-resolution streamflow time series for 17 (semi)urbanised basins in North Carolina, US, ranging from 13 km² to 238 km² in size. We show that in the inter-amount times framework, sampling frequency is tuned to the local variability of the flow pattern, resulting in a different representation and weighting of high and low flow periods. ^{c1}This leads to important differences in the way the quantiles, mean, coefficient of variation and skewness of the distributions vary across scales and results in lower mean intermittency and improved scaling. Moreover, we show that inter-amount times distributions can be used to detect regulation effects on flow patterns, identify critical sampling scales and characterise flashiness of hydrological response. The possibility to use both the classical approach and the inter-amount time framework to identify minimum observable scales and analyse flow data opens up interesting areas for future research.

1 Introduction

Hydrological response in urban ^{c2} ^{c3}catchments tends to be more flashy compared to natural ones as a result of their higher degree of imperviousness. Increase in flashiness is typically characterised by shorter response times to rainfall, higher runoff ratios and higher peak flows (Berne et al., 2004; Smith et al., 2005). On the other hand, high impervious degrees may ^{c4}reduce base flows and ^{c5} lead to intermittent flow during dry periods. At the same time, urbanisation is usually tied to development of urban drainage infrastructure, ^{c6}associated with artificial flow control as well as higher peak flows due to increased drainage

^{c1} Text added.

^{c2} watersheds

^{c3} Text added.

^{c4} Text added.

^{c5} ultimately

^{c6} Text added.

connectivity. Predicting the degree of flashiness or base flow reduction associated with urbanisation is not straightforward, as it depends on the interplay of impervious cover, basin size and shape, soil properties, basin slope, drainage connectivity and control structures such as detention ponds, weirs and pumps (Emmanuel et al., 2012; Fletcher et al., 2013; Smith et al., 2013).^{c7}Traditional analyses of flow time series tend to focus on specific aspects and flow characteristics, aiming for example at predicting low flow durations or peak flow magnitudes. For analysis of change in hydrological response, it may be beneficial to combine both peak flow and low flow statistics into a single framework. This applies in particular to the context of urban hydrology where urbanisation and human intervention alter both high flow and low flow characteristics of the hydrological response. Combining both aspects in a single analysis is difficult, as flow distributions are highly skewed and frequencies of low and high flow values are very different. In this paper, we show how alternative sampling of flow time series based on inter-amount times leads to more balanced statistical distributions, better representation of both high and low flows in a single framework and more robust behaviour of statistical distributions across scales.

1.1 Statistical analysis of hydrological response

Many authors have investigated methods for characterising hydrological response and changes therein, including ^{c1}univariate analysis and multivariate statistics, combining several hydrograph properties such as flood peak, flood volume and flood duration (e.g., Salvadori and De Michele, 2004; Favre et al., 2004; Grimaldi and Serinaldi, 2006; Vittal et al., 2015). ^{c2}Traditional statistical analysis techniques tend to focus on either left or right tail properties of statistical distributions, but not necessarily using the same statistical framework. Low flow analyses for example are primarily concerned with the total time the flow stays below a critical threshold (see e.g. Smakhtin (2001) for an extensive review). ^{c3}By contrast, peak flow analysis puts more weight on total accumulated flows at a given time scale using annual flow maxima or peak-over-threshold values to derive extreme value statistics and establish flood frequency curves (e.g., Stedinger, 1983; Lang et al., 1999; Villarini et al., 2009; Smith and Smith, 2015). ^{c4}Both approaches are valid and solidly rooted in the context of extreme event analysis with numerous applications in drought and flood risk analysis. However, the statistical frameworks they rely on are not necessarily the same. Low flow analysis favours 'time' as a random variable. Peak flow analysis on the other hand treats the 'flow amount' over a fixed time interval as the main random quantity. This might seem more intuitive to many but there is no strong compelling reason to prefer one approach over the other a priori. For example, one might as well adopt an alternative framework in which the unknown random variable is the 'time' necessary to cumulate a fixed, critical amount of flow. This approach is known as the inter-amount time (IAT) method (Schleiss and Smith, 2016) and has been previously proposed to analyse the properties of intermittent rainfall time series.^{c5}One of the main goals of this paper is to apply the IAT formalism to flow time series to derive

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^{c3} Text added.

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properties of statistical distributions and compare the results to the ones obtained using the classical fixed-time framework. ^{c6} ^{c7}

1.2 Change in hydrological response, basin flashiness

^{c1} An important characteristic that has been used to analyse change in hydrological response is basin flashiness, qualitatively
5 described by Poff (2002) ^{c2} as one of the indicators characterising change in natural flow regimes and how this affects the ecological integrity of river ecosystems. Richter (1996) ^{c3} developed a set of 33 indices, the Indicators of Hydrological Alteration (IHA), including indicators for conditions associated with flashiness, such as frequency and duration of high and low pulses, and rate and frequency of change in flow conditions. Smith and Smith (2015) ^{c4} quantified flashiness of 5436 catchments in the contiguous United States based on peak flows exceeding $1 \text{ m}^3\text{s}^{-1}\text{km}^{-2}$ normalised flows (i.e., flows normalised by basin
10 area). A frequently used index in the literature is the Richards-Baker (R-B) Flashiness Index (Baker et al., 2004), ^{c5} based on the Richards pathlength (Gustafson et al., 2004). ^{c6} The R-B index is defined as the sum of absolute values of changes in flow values divided by the total cumulative flow, and is usually computed at the daily time scale. Similar to the coefficient of variation, it measures the relative dispersion of the flow at a given scale. A downside of the R-B index is that it highly sensitive to the scale of analysis. Baker et al. (2004) ^{c7} argued that for smaller basins ($< 50 \text{ km}^2$) the use of hourly instead of daily flow
15 data should be considered to compute R-B flashiness index, but also found that R-B flashiness values computed at hourly scale are highly sensitive to diurnal or other sub-daily low flow fluctuations. An important still unanswered question remains how to overcome scale sensitivity of flashiness indicators in different hydrological basins. This is crucial for establishing how urbanisation impacts flashiness and how changes relate to basin characteristics such as size, slope, imperviousness degree and whether

^{c6} ~~statistical approaches as well as hydrograph analysis and multivariate regression. Most of these studies have used daily flow time series, while more recently hourly and sub-hourly flow data series have increasingly been used. Traditionally, flow duration curves, representing the frequency distribution of flows, have been used to characterise hydrological response of a given basin. Wood and Hebson (1986) developed dimensionless flood frequency curves and investigated relations with basin and rainfall characteristics, length ratio, a geoclimatic scaling factor and dimensionless mean storm duration. Others have investigated the use of regional flood frequency curves to predict flood response in ungauged basins. Similarly, low flow frequency curves have been used to characterise the influence of prolonged periods of drought, see for instance Smakhtin (2001) for an extensive review. More recently, authors have concluded that univariate flood frequency curves are insufficient to describe flow response and multivariate approaches have been developed, combining several hydrograph properties such as flood peak, flood volume and flood duration~~

^{c7} ~~Other approaches for characterising basin flow response and flood probability include establishment of Instantaneous Unit Hydrographs and design flood hydrographs. At the other end of the spectrum, base flow indices and subflow separation techniques have been developed to characterise low flow conditions. Others have investigated whether statistical properties derived from flow time series are still valid under changing climate conditions. One of the problems in analysing hydrological response across different events and basins is that hydrological response variables need to be normalised for comparison. This is usually done by dividing instantaneous flows and cumulative flow volumes by basin area. The downside of this approach is that definition of basin boundaries is prone to errors and that, especially for basins with inhomogeneous urbanisation coverage, basin area may imperfectly represent flow generation.~~

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urbanisation thresholds can be identified above which basin response is characteristically urban (Praskievicz and Chang, 2009).

c1

1.3 Scaling analysis of hydrological flows

- 5 ^{c2}Scaling behaviour of river flows has been investigated by various authors, aiming to identify characteristics length and time scales and to detect scale dependence of hydrological response processes. Among the various statistical methods that have been proposed to investigate scaling, fractals and multifractals are among the most popular and powerful. Approaches for fractal analysis include: spectral analysis based on 2nd order properties, and trace moments analysis based on a wider range of statistical moments, typically between 0.1 and 4. The universal multifractal framework is based on the identification of scaling
- 10 exponents summarising the changes in flow distributions across a given range of scales^{c3}, (see Schertzer and Lovejoy (1987) and Schertzer and Lovejoy (2011) for a review).

One important drawback of multifractal analyses is that scaling of hydrological flow time series only holds in approximation

^{c1} Several authors have investigated scaling behaviour of river flows. Tessier1996 analysed time series of daily river flows for 30 natural river basins in France, with areas of 40 to 200 km² and time series length ranging from 11 to 30 years. They found a scale break at approximately 16 days and based on this distinguished two regimes, one for 1-16 days and another for 30-4096 days (~ 11 yrs). They suggested a possible explanation of this 16-day period being associated to the atmospheric synoptic maximum, the typical lifetime of atmospheric structures at planetary scale. Koutsoyiannis2003 discussed hydrological statistics in view of climate change and the Hurst phenomenon, i.e. dependence of observed variability on scale. He argues that signals such as trends and jumps detected in long-term time series of atmospheric and hydrological variables using classical statistics are not correct, as they fail to account for this scale dependence. This leads to an underestimation of variance and autocorrelation, especially for large time scales (>10 years), resulting in incorrect detection of trends. Cheng2001 used statistical and multifractal approaches and GIS to characterize stream networks (up to stream order four) and drainage basin systems of 322 drainage basins in southern Ontario, Canada. They detected a scale break for total stream length at a spatial resolution of around 700 m (different scale dependence for higher resolution, mostly covered by lower order streams, than for lower resolution). Based on this analysis, they identified main geological and geomorphological factors including bedrock topography, lithology and slope of drainage basins influencing the evolution of stream networks in the area. Sauquet2008 analysed scaling properties of hourly river flow time series ranging from 16 to 37 years, for 34 rivers in France draining areas between 12.7 and 703 km². They analysed hydrograph shape and flood duration curves across a range of aggregation windows and applied two types of fractal analysis: spectral analysis and moments analysis (for moments q {0.1-4}). In spectral analysis they identified two scaling ranges, with a scale break that ranged from 8.7 hours to 7 days, with a median of 27 hours across stations. They performed the same analysis using daily instead of hourly data and found a scale break at approximately 12 days. Pandey1998 performed spectral analysis for daily river flow data over 9 to 73 years, for 19 river basins in the USA, with basin areas of 5 to 106 km². For most of the rivers, a scale break was observed at approximately 8 days, possibly associated with half of the atmospheric synoptic maximum. Scaling results were independent of basin size and geology. Labat2002 analysed flow from three karstic watersheds located in the Pyrenees in France and found a scale break at 16 days. In Labat2013 they revisited this analysis, using time series of 3 minutes (1.5 year period) and 30 minutes (2-10 years periods), with areas of approximately 13 km². Based on a spectral analysis of 30 minutes time series, they identified a scale break at approximately 1 day; using Trace Moment analysis they found a scale break at 16 hours. Using 3 minute times series, a scale break was identified at approximately 1 hour. Zhou2006 analysed daily flow time series of 4 agricultural watersheds in the USA, incl 31 subwatersheds, ranging in size from 0.01 to 334 km², with approximately 30 years of records. They found a scale break at 365 days, which could be attributed to seasonality, yet the break persisted even if the seasonality effect was removed. These analyses show that scaling behaviour is found in many flow time series, over a restricted range of scales. For daily flow time series, scale breaks are found in most cases at approximately 16 days. For higher resolution time series, different scale breaks are found, indicating that scaling and scale breaks are dependent on the original resolution of the data and method of analysis.

^{c2} Text added.

^{c3} derived from time series analysis over a range of scales and a range of statistical moments

and only over a limited range of scales. Many studies report the existence of “scale breaks” at which scaling parameters change and significant departures from (multi)fractality can be observed. ^{c4} Table 1 ^{c5} summarises findings from selected scaling analyses of flow time series in the literature. It shows that the number and location of the scale breaks as well as the values of the multifractal parameters are sensitive to the method applied to estimate them and the resolution of the data used to conduct the analysis. ^{c6} For example, Labat et al. (2013) ^{c7} performed spectral analysis and trace moments analysis for 30-minute flow times series and identified different flow regimes with scale breaks at 1 day for spectral and 16 hours for trace moments analysis. But when they performed the same analysis at daily and at 3 minute resolution, they identified different scaling regimes, with scale breaks at 16 days and 1 hour for daily and 3 minute resolution. Similarly, Sauquet et al. (2008) ^{c8} found different scaling regimes in their scaling analysis of flows for 34 basins, with scale breaks at 12 days for daily resolution and scale breaks varying between 8.7 hours and 7 days across basins when using hourly data resolution, based on spectral analysis. When they applied trace moments analysis for the same time series at hourly resolution, they found no scale breaks for the lower order moments and scale breaks between 10 and 150 hours for higher order moments. This shows that while most flows exhibit some sort of scaling behaviour, the identified scaling laws are not very robust nor consistent, as they are dependent on analysis methods and data resolution.

15

1.4 Statistical analysis of hydrological response based on adaptive sampling using inter-amount times

^{c1} In this paper, the IAT formalism is applied to flow time series and statistical distributions and scaling properties are compared to the ones obtained using the classical fixed-time framework. To do this, we use flow observations collected in 17 hydrological basins in Charlotte, North Carolina. We aim to investigate what effects an adaptive sampling strategy such as IAT sampling has on statistical properties of the time series, in particular on the tails of the statistical distributions associated with peak flow and low flow extremes. ^{c2} The main problem with a fixed sampling rate, as in traditional flow time series analysis, is that it can only accurately represent frequencies of variations at time scales larger than a certain threshold. When frequencies higher than that exist, errors are introduced as information about the higher frequency variability is lost (Dippe and Wold, 1985). ^{c3} Increasing the sampling resolutions solves this problem, but results in oversampling of base flow values with respect to peak flows. An alternative consists in adopting an adaptive sampling strategy, i.e., one that adapts the sampling rate to the variability of the signal itself (e.g., Feizi et al., 2011). ^{c4} This makes sense for processes that are very unevenly distributed in time (such as rain-fall and hydrological flows), and means taking more samples during periods of high activity (e.g., peak flows following storm

^{c4} ~~Scale breaks were identified in most analyses, indicating that in traditional flow time series, power-law scaling extends only across a limited range of scales and that different power-law scaling was found for small scales compared to large scales.~~

^{c5} Text added.

^{c6} Text added.

^{c7} Text added.

^{c8} Text added.

^{c1} Text added.

^{c2} Text added.

^{c3} Text added.

^{c4} Text added.

events) and fewer during lower activity (e.g., periods of base flow). A well designed adaptive sampling technique lowers the probability of missing an interesting feature like peak flow and avoids oversampling during periods of small flow variations.

^{c5}We examine to what extent IATs influence the variance, skewness and shape of the sample distributions and how they can be used to better characterise basin flashiness and derive more robust scaling laws. Our results show that because IATs give

5 more weight to rare peak flows compared to common base flows, they can provide different insights into flow properties and complement traditional flow time series analyses and metrics. Advantages of IATs sampling compared to conventional time series analysis are that IAT time series contain more information about peak flows and evolve in a more predictable way across ranges of smaller to larger scales. This makes them a more robust and reliable source of information to make predictions about flow characteristics at small, unobserved scales, including crucial information about rapidly evolving peak flows. ^{c6}

10 This paper is organised as follows, in section 2 we present the flow datasets and methods used for analysis. We explain the methodology for deriving normalised IATs and introduce metrics we used to compare properties of flows and IATs time series, to characterise hydrological response and compare response across basins. In section 3, results of the analyses are presented and discussed, first based on results obtained using a daily sampling scale, followed by results obtained a range of sampling scales, from hourly up to seasonal sampling scale. Conclusions and suggestions for future work are summarised in section 4.

15 2 Data and Methods

2.1 Flow datasets

The data used in the study were collected at 17 USGS stream gauging stations in Charlotte-Mecklenburg county, North Carolina. Gauging stations are located at the outlet of hydrological basins that range from 13 km² to 238 km² in size. The area is largely covered by low to high intensity urban development, covering 60% to 100% of basin areas. Percentage impervious
20 cover varies from 8% in the least developed to 48% in the most urbanised basin covering the city centre of Charlotte. Figure 1 shows a map with the location of the area, boundaries of hydrological basins and location of stream gauges used in the analysis, table 2 summarises the main characteristics of the 17 basins.

Stream gage data were collected at 5 to 15 minute intervals over the period 1986-2011. Table 2 summarises the characteristics of the basins associated with each basin as well as the time period covered by the data. The temporal scale of observations
25 changed from 15 to 5 minutes between 2010 and 2014, at different times for each gauge; overall 20-30% of the total observation record was covered by 5 minute intervals. Gauges measure water depth using pressure transducers and flow is derived using

^{c5} Text added.

^{c6} ~~We find that one of the advantages of the inter-amounts approach is that it automatically deals with varying observation time intervals, as the time resolution of flow measurements is often increased during periods of high flow compared to periods of low flow. Also, it allows comparison between basins without the need for normalising by basin area. A problem that arises in analysing catchment response across different events and basins is that hydrological response variables need to be normalised for comparison. This is usually done by dividing flows by basin area. The downside of this approach is that definition of basin boundaries is prone to errors and that, especially for basins with inhomogeneous urbanisation coverage, basin area may imperfectly represent flow generation. A conceptually elegant aspect of the inter-amount-times approach is that it allows comparison of statistical properties of flow times series, independent of basin size.~~

stage-discharge curves. These curves were established based on manual flow measurements during site visits and curves were checked and recalibrated during site visits several times per year. The percentage of missing flow data was smaller than 5% for all gauges included in the analysis; missing data were treated like zeros. ^{c1}The effect of missing data on IATs is difficult to predict as this depends on the pattern of missing values and whether or not they occur during a period of low or peak flow.

- 5 Sensitivity studies by Schleiss and Smith (2016) ^{c2}have shown that the general effect of replacing missing values by zeros is that a few sample IATs will be overestimated. This mostly affects the right tail of the distribution and tends to have limited impact on peak flow characteristics. Another strategy would be to replace missing values by mean or median flow value, which may slightly reduce the overestimation of IATs in case several missing values occur in row. However, in this paper only the worst case scenario will be considered, i.e. missing values were replaced by zeros.

10 2.2 Definition of inter-amount times

^{c1}In this paper we analyse hydrological flow variability, based on the distribution of inter-amount times. We use the following definition of inter-amount time (IATs), based on Schleiss and Smith (2016): Let $\Delta q > 0$ denote a fixed flow amount. We define the series of IATs $\tau_n(\Delta q)$ with respect to Δq as follows:

$$\tau_n(\Delta q) = t_n(\Delta q) - t_{n-1}(\Delta q) \quad (1)$$

- 15 where $t_n(\Delta q)$ denotes the time at which the cumulative flow amount first exceeded n times (Δq) :

$$t_n(\Delta q) = \inf\{u : Q(u) \geq n \cdot \Delta q\} \quad (2)$$

where $Q(u)$ denotes the cumulated flow at time u and $Q(0) = 0$.

A steady flow pattern with constant flow has equal IATs for all values of Δq . A variable flow pattern, on the other hand, is characterized by a more variable IAT distribution.

20 2.3 Normalized inter-amounts

Flow magnitudes strongly vary from one gauge to another. To overcome this scale dependence and compare flow IATs across basins with different sizes and flow amounts, one needs to normalize IATs with respect to a common timescale. A possible way to do this is to fix an average IAT $\bar{\tau}$ (e.g., 24 h) and determine the inter-amount $\Delta q_{\bar{\tau}}$ at this timescale:

$$\Delta q_{\bar{\tau}} = \bar{\tau} \frac{Q_N}{T} \quad (3)$$

- 25 where Q_N denotes the total cumulative flow amount at the considered location and T is the length of the studied time period. In other words, instead of comparing IATs for a fixed accumulation, we choose the mean IAT $\bar{\tau}$ and compute $(\Delta q)_{\bar{\tau}}$ such that

^{c1} Text added.

^{c2} Text added.

^{c1} Text added.

the series of IATs $\{\tau_n(\Delta q_{\bar{\tau}}) : n = 1, \dots, N\}$ has mean $\bar{\tau}$. Two locations with different cumulative flow amounts over a given period of time, e.g. over a year, therefore have different normalized inter-amounts.

2.4 Sample estimates and minimum inter-amount scale

Inter-amount times can be estimated from a sample flow time series q_1, \dots, q_N with temporal observation scale Δt that may vary in time. But for simplicity, only the case with fixed temporal resolution Δt will be considered below. A key step in this procedure is the determination of the first passage times t_1, \dots, t_n in equation (2). This is done by considering the sample accumulated flow amounts $Q_1 < \dots < Q_N$ at times $t_n = t_0 + n\Delta t$:

$$Q_n = \sum_{i=1}^n q_i \quad n = 1, \dots, N \quad (4)$$

The exact first passage times t_1, \dots, t_n for a fixed flow amount $\Delta q > 0$ are likely to be unknown due to the limited temporal resolution of the data. But we can approximate them based on linear interpolation:

$$\hat{t}_n(n\Delta q) = \Delta t \left(i_{n\Delta q} - \frac{Q_{i_{n\Delta q}} - n\Delta q}{q_{i_{n\Delta q}}} \right) \quad n = 1, \dots, N \quad (5)$$

where \hat{t}_n are the estimated passage times and $i_{n\Delta q}$ denotes the index (in the sample) at which the total cumulated flow first exceeded n times (Δq):

$$i_{n\Delta q} = \min\{i \in \mathbb{N} | Q_i \geq n\Delta q\} \quad n = 1, \dots, N \quad (6)$$

The sample IAT estimates are then given by:

$$\hat{\tau}_n(\Delta q) = \hat{t}(n\Delta q) - \hat{t}(n\Delta q - \Delta q) \quad (7)$$

^{c1} Because of the linear interpolation in (5), ^{c2}each sample IAT estimate, regardless of its length and the scale of analysis, will be affected by a small interpolation error $\varepsilon_n(\Delta q) < \Delta t$. ^{c3}This error is random and has little influence on key statistics as long as IATs remain much larger than Δt , as is usually the case for large enough values of Δq and during periods of low to moderate flow. Most of the interpolation errors happen during peak flows, when large flow amounts are accumulated over small periods of time. It is therefore important, for any given gauge, to identify the values of Δq above which reliable IAT estimates can be derived. To identify the range of scales over which IATs can be reliably estimated, we consider the worst case

^{c1} A key point here is to know how the linear interpolation affects the accuracy of sample IAT estimates.

^{c2} Text added.

^{c3} Text added.

scenario in which all interpolation errors are equal to $\pm \Delta t$. In this case, the maximum relative error affecting IAT estimates is given by:

$$\varepsilon_n(\Delta q) = \frac{\Delta t}{\hat{\tau}_n(\Delta q)} \quad (8)$$

The minimum value of Δq for which IATs can be reliably estimated depends on how strictly we want to control the estimation errors in (8). In our analysis, ^{c1}we set the mean of absolute relative errors to be smaller than 50%. This is a rather conservative approach as the estimation errors in (8) represent the worst case scenario and actual errors are likely to be much smaller than that. This leads to the following rule for determination of minimum inter-amounts Δq that can be used for analysis:

$$\Delta q_{min} = \min\{\Delta q > 0 : \bar{\varepsilon}_{\Delta q} < 0.5\} \quad (9)$$

where $\bar{\varepsilon}_{\Delta q}$ represents the arithmetic mean of the maximum relative errors in 8.

In addition to the lower bound, we also impose an upper bound on the inter-amounts used in our analysis. This is necessary to ensure IAT time series are long enough to compute relevant statistical moments. Typically, there should be at least 100 consecutive IATs, which yields the following upper bound for inter-amount Δq :

$$\Delta q_{max} = \lfloor \frac{Q_N}{100} \rfloor \quad (10)$$

where: $\lfloor \rfloor$ denotes the lower integer part and Q_N is the total cumulative flow for the considered time series.

^{c2}

^{c3}It is worth pointing out that the lower bound on the inter-amount in (9) ^{c4}also provides a rough indication of the left-tail properties of IATs, thus of the degree of flashiness of the hydrological response (i.e., the smallest scale at which flow variations can be studied given a fixed temporal observational resolution). More generally, the left tail properties of IAT distributions provide a good indication of what observational resolution is necessary to adequately capture the most extreme flow variations.

^{c4}For more details on this important point, the reader is referred to the results section.

Note also that analyses of IATs were conducted for all gauges over the entire period of available data, without distinguishing between year, season or hour of the day. This was necessary as time series would otherwise be too short to study IATs across different scales. This means we mostly focus on average characteristics of IAT and flow distributions with respect to area size and imperviousness degree and potential influence of flow regulation and stormwater detention facilities, as far as this

^{c1} Text added.

^{c2} ~~After all, the flashier a catchment is, the quicker the flow can rise, resulting in lower inter-amount times during times of heavy rain. Conventionally, flashiness is characterised by (normalised) peak flow magnitude and response time citep[e.g.,][Smith20152365. Based on inter-amount times distribution we were able to define a flashiness indicator that incorporates both the rising and falling components of the hydrological response. In this work we defined a flashiness indicator as follows: basin flashiness (unit: hours) is computed as the ratio of the 1% quantile of inter-amount times with respect to the mean inter-amount time.~~

^{c3} Text added.

^{c4} Text added.

information is available for the 17 basins. We refrain from investigating long-term trends, as our time series are restricted to maximum 30 years and because a recent study by Villarini (2016) showed no signs of long-term trends at 7506 gauges in the contiguous US in the last 30 years. Indeed, our own analyses revealed no significant long-term trend in mean IAT or flow variability over the considered time period.

5 2.5 Distribution of inter-amount times versus flows

Sample histograms of IATs and flows were analysed to investigate what different insights they provide into characteristics of the flow regimes. We plotted sample histograms for all gauges; appropriate bin widths were determined based on Scott's rule (Scott, 1979). We computed the coefficient of variation (CV), defined as the standard deviation divided by the mean, as an indicator for relative spread around the mean. Values of skewness and medcouple (Brys et al., 2004), ^{c1} a more robust skewness metric based on ordered statistics instead of statistical moments, were computed to investigate asymmetry of the histograms and influence of outliers. We compared coefficient of variation, skewness and medcouple values for IATs with those for traditional flow time series and investigated relationships of the three statistics with basin area and imperviousness degree.

^{c2}

2.6 Distribution of changes in inter-amount times

- 15 First-order differences of IATs and flows were computed to look into characteristics of the rising and falling limbs of hydrographs. Because IATs are measured on an inverted scale, positive differences are associated with the falling limb of the hydrograph and negative differences with the rising limb of the hydrograph. Narrow ranges of histogram values for IAT differences indicate slowly varying flow; wide range histograms indicate more flashy behaviour. Positively skewed histograms for IAT differences indicate that the distribution is dominated by values on the rising limb and short recession limbs, while
- 20 negatively skewed histograms indicate a larger part of the flow is associated with flow recession, i.e. long, slowly receding hydrographs, for instance induced by a strong groundwater flow component. Differences were computed at the 24-hour time scale, imposed by the minimum inter-amount scale rule. Similarly to the other histograms, bin widths was chosen based on Scott's rule. ^{c3}

^{c1} Text added.

^{c2} ~~An advantage of inter-amount times analysis is that periods of low flows, which typically constitute a major part of traditional flow time series, form a much smaller part of the inter-amount times distribution. Also, while traditional flow analysis puts a lot of emphasis on peak flows and upper-tail properties of flow rates, the part of inter-amount times that relates to peak flow is located in the lower tail of the distribution. Upper-tail properties of inter-amount times on the other hand are associated with rare occurrences of prolonged low flow. And, although inter-amount times and flow rates are related, they reflect different aspects of flow variability.~~

^{c3} ~~Studying the rising and falling limbs of hydrographs using IATs is facilitated by the fact that, since IAT time series are based on accumulated flow volumes, both limbs are equally sampled, which is not the case for traditional flow analyses based on fixed-sampling time representations.~~

2.7 Flashiness indicator and minimum observable scale

^{c4}As mentioned earlier, the lower bound on the inter-amount provides a rough indication of left-tail properties of IAT distributions (i.e., short waiting times) and can therefore be used to characterise the degree of flashiness of the hydrological response. In flashier catchments, the flow can rise quicker, resulting in lower IATs during times of heavy rain. The minimum observable inter-amount represents the smallest scale at which flow variations can be studied with acceptable interpolation errors, given a fixed temporal observational resolution. By extension, the lower tail of the IAT distribution provides a good indication of what observational resolution is necessary to adequately capture the most extreme flow variations. The IAT flashiness indicator used in this paper is defined as the mean scale μ (expressed in hours) at which the 1% quantile of the IAT distribution equals the observational scale Δt (15 min in our case). That is, the IAT flashiness indicates the average time needed to accumulate the amount of flow that can be accumulated in 15 minutes or less, 1% of the time. The larger the flashiness, the more flow can be accumulated over short amounts of time.

^{c5}To better interpret results, we compared the IAT flashiness index with the classical R-B flashiness index defined in Baker et al. (2004):

$$R - B \text{ index} = \frac{\sum_{i=1}^N |q_i - q_{i-1}|}{\sum_{i=1}^N q_i} \quad (11)$$

where: q_i denotes the flow at time step i . The R-B flashiness index is dimensionless and can vary between 0 and 2. It is 0 for constant flow and 2 for highly variable and continuously changing flow. Its value is independent of the units chosen to represent flow (Baker et al., 2004). However, index values do depend on the time-scale at which they are computed, as will be discussed later in the results section. In our analysis, we computed R-B flashiness indices on daily aggregated flow values.

2.8 Scaling of inter-amount times

Multi-fractal analysis techniques were applied to investigate the scaling behaviour of IAT time series across different inter-amount scales. Multi-fractal analyses are based on the assumption of generalised scale invariance, in which the statistical moments or order $q > 0$ of a stochastic process X_λ at scale ratio λ are related by a power law:

$$\langle X_\lambda^q \rangle = C(q) \lambda^{K(q)} \quad (12)$$

where $\langle X_\lambda^q \rangle$ denote the moments of order q of X measured at a scale ratio λ , $C(q)$ is a constant (for each q) and $K(q)$ is called the moment scaling function. Within the universal multi-fractal framework, $K(q)$ is characterised with the help of only three parameters, α , C_1 and H (Schertzer and Lovejoy, 1987, 2011):

$$K(q) = \begin{cases} \frac{C_1}{\alpha-1} (q^\alpha - q) - qH & \text{if } \alpha \neq 1 \\ C_1 q \ln(q) - qH & \text{if } \alpha = 1 \end{cases} \quad (13)$$

^{c4} Text added.

^{c5} Text added.

^{c1}]The parameter C_1 is referred to as the intermittency and characterises the clustering of the time series at smaller and smaller scales. $C_1 = 0$ for a homogeneous field that fills the embedded space and approaches 1 for an extremely concentrated field. The parameter α is called the multi-fractality index ($0 < \alpha < 2$) and it controls how the moments change when going from one scale to another. Finally, $H = -K(1)$ is called the Hurst exponent. Note that in the case of IATs, the mean inter-amount time $\bar{\tau}$ and scaling ratio λ are inversely proportional to each other (i.e., $\Delta q_{\bar{\tau}} \sim \lambda^{-1}$). So either of them can be used here as a measure of scale. The only difference will be the value of the constant $C(q)$ and the sign of the exponent in (12).

The scaling quality is assessed by noting that if (12) is true, the log-moments for fixed values of q should be a linear function of the log-scale:

$$\ln(\langle X_{\lambda}^q \rangle) = K(q) \ln(\lambda) + \ln(C(q)) \quad (14)$$

^{c1}The extent to which this equality holds can be assessed by fitting a linear regression model and computing the R^2 values, i.e., the coefficient of determination of the log-moments versus the log-scale for each value of q . A R^2 of 1 indicates perfect scaling. The lower the coefficient of determination, the larger the deviations from scale-invariance. The approach was repeated for different values of q and the mean or minimum value of R^2 were chosen as a way to assess the overall quality of the scaling. Based on recommendations by Lombardo et al. (2014), ^{c2}we refrained from using too low or high order moments and only considered values of q between 0.4 and 2.5, with an equal number of moments above and below 1 to avoid favoring one tail of the distribution over the other. The range of IAT scales that was used for the analysis was constrained by the length of the time series and the minimum and maximum inter-amounts defined in (9) and (10). The corresponding scales varied from 0.1 to 0.6 days up to 28 to 100 days for the longest time series.

3 Results

^{c3}In the following sections we compare statistical properties of flow and IAT time series and highlight differences that result from the different sampling strategies. Analyses are first conducted at the 24 hour time-scale and associated mean inter-amount sampling scale. In the second part of this section, we analyse how statistical properties of flow and IAT time series vary across scales and quantify flashiness and scaling behaviour of both time series.

3.1 Time series and variability analysis of inter-amount times and flow values

Figure 2 shows an example of times series for flows and for IATs for the gauge at Taggart Creek, a 13.6 km² basin in the Charlotte catchment, at 24 hour sampling scale. ^{c4} The two graphs bring out different aspects of flow variability: flow time series have most of their data points concentrated in the low flow region, with intermittent peak flows characterising rain

^{c1} Text added.

^{c1} Text added.

^{c2} Text added.

^{c3} Text added.

^{c4} The associated inter-amount Δq is 13,559 m³.

events. For IATs, peak flows appear as minima, while periods of low flow show up as maxima in the time series. ^{c5}The graph illustrates how IAT samples are more evenly distributed across high and low values in the time series compared to flows. The mean inter-amount for Taggart Creek at 24-hour sampling scale is 13,559 m³, equivalent to 0.998 mm when normalised by basin area. Hence, in IAT analysis, the time series is sampled each time 0.998 mm of normalised flow has been accumulated, which amounts to frequent samples during high flows and fewer samples during low flow periods. For instance, a high concentration of IAT samples is clearly visible for the wet year 2003: this year is represented by 802 IAT samples compared to the 365 samples per year we have on average.

c1

Figure 3 illustrates the adaptive sampling strategy based on flow amounts as the sampling unit, instead of fixed time steps. Figure 3b shows cumulative flow over a week, where a storm event occurred on 7 August. In conventional flow time series analysis, flow is sampled daily (in this example), resulting in one sample representing the peak period of the event (i.e., on 7 August). In IAT analysis, flow accumulation determines the sampling frequency, so periods of low flow are sparsely sampled, while the storm event is represented by eight samples. ^{c2}This illustrates how, even for 24-hour mean inter-amounts, sampling frequency can be much higher during periods of peak flow.

Histograms of flow time series and IATs at daily time scale are plotted in figure 4, for two basins, Taggart Creek (13.6 km²) and LSugarA (111 km²). The corresponding inter-amounts are 1 mm and 1.8 mm of normalised flow (for Taggart and LSugarA, respectively). ^{c3}Histograms for the other 15 basins are available in the supplement to this paper. Figure 4 shows that both histograms of flows and IATs are positively skewed. ^{c4}In both cases however, left and right tails represent very different flow characteristics. The left tail of the flow's histogram essentially features common base flow values while the right tail captures rare peak flow events. By contrast, the left tail of IAT distributions, which makes up most of the values, predominantly features short IAT values associated with periods of high flow. The rare samples that make up for the right tail represent long waiting times associated with extended periods of low flow. ^{c5} The low density of the first bin in the flow histogram for LSugarA reflects the effect of low flow regulation for this basin. The same effect is reflected in the bi-modal shape of the IATs histogram. Note that the low density 0-0.5 bin in the flow histogram for LSugarA corresponds to the >3.5 day bins in the IAT histogram.

Tables 2 (6th and 7th columns) and 3 summarise statistics of flow and IAT time series, at 24 hour sampling scale. The results show that mean inter-amounts vary from 12,275 m³ for the smallest to 269,534 m³ for the largest basin in size. Mean normalised inter-amounts vary from 0.68 mm for Irvins Creek, the least urbanised basin (8.2% imperviousness) to 1.79 mm for

^{c5} Text added.

^{c1} ~~Hence small flow amounts correspond to low flow and large amounts correspond to peak flow. Both time series show seasonal variability associated with the summer and winter seasons, as well as inter-annual variability associated with wetter and dryer years. The main difference with the classical sampling is that in the inter-amount times formalism, the sampling rate is adapted to local variations in flow volumes rather than on an arbitrary fixed time interval.~~

^{c2} Text added.

^{c3} Text added.

^{c4} Text added.

^{c5} ~~Rare periods of low flow are represented in the right tail of the inter-amount times distribution.~~

Little Sugar Creek at Archdale, one of the largest basins with a high degree of imperviousness (32%). Coefficients of variation at the daily scale are consistently higher for flows than for IATs (e.g., 1.7 times higher on average), which highlights the more balanced nature of IAT distributions. ^{c6} Skewness values at the daily time scale are 3.6 times higher for flows than for IATs, on average, and even up to a factor of 15 higher for Stewart Creek. ^{c7}By contrast, medcouple values for flows are lower than for IATs by a factor 2.1 on average. This shows that statistical distributions of flows are strongly influenced by the presence of a few very large outliers. Most of the weight, however, lies close to the median (low medcouple). The IAT sampling gives more weight to rare peak flow values and less to common base flow, therefore producing distributions with lower skewness and more information about peak flow values. The larger medcouple values mean that IATs above the median value tend to be much further away from the median than values below the median. In other words, the right part of the distribution, which features long waiting times during low flow conditions can be very stretched.

^{c1}These results show that adaptive sampling based on inter-amounts leads to more balanced representation of high and low flows, resulting in lower coefficients of variation reflecting stabler statistical variance compared to traditional flow time series sampling. We like to point out that these results were obtained at the 24 h sampling scales. In section 3.4, ^{c2}behaviour of the statistical distributions of flows and IATs, as well as associated CV, skewness and medcouple values will be analysed across a range of subdaily to seasonal scales.

3.2 Statistical distribution properties comparison across different hydrological basins

Subsequently, we compared properties of IAT and flow distributions across the 17 basins in relation to basin characteristics. Figure 5 shows scatter plots of mean normalised inter-amounts, CV, skewness and medcouple values for flows and IATs as a function of basin area and imperviousness degree. ^{c3}The results show a positive correlation of 24-hour mean normalised flows or inter-amounts with basin size (Spearman correlation 0.55). This is mainly explained by a lower likelihood of low flows that have a large influence at this scale (24 hours). Mean normalised flows correlate positively with imperviousness degree (Spearman correlation 0.58), which is likely to be explained by a generally growing importance of flow regulation, resulting in maintenance of higher mean base flows in urbanised basins.

Looking at CV-values across all basins (figure 5. c, d), we found that CV-values for both flows and IATs generally decrease with basin size and with imperviousness degree. ^{c4}CV-values are significantly negatively correlated with basin size for flows (Spearman rank correlation -0.75). This can be explained by an increased smoothing effect on flow variation, in particular a lower likelihood of low flow extremes during dry periods for larger basins. CV-values for IAT distributions are significantly negatively correlated with imperviousness (Spearman rank correlation -0.57). Since IAT distributions put more weight on high

^{c6} ~~This is further confirmed by values for skewness and medcouple values of the flow and IAT distributions~~

^{c7} Text added.

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^{c4} Text added.

flows compared to low flows as a result of their adaptive sampling strategy, this probably indicates stronger influence of flow regulation in urbanised basins resulting in more uniform runoff during rainy periods. IATs during these periods concentrate relatively more closely to the mean and show fewer extremes (this is clearly visible for the most urbanised basin, LSugarM, gauge 409). The effect of urbanisation as reflected by imperviousness degree on IAT statistics appears to be more important

5 than basin size.

Scatter plots for skewness and medcouple values (figure 5. e, f, g, h) show generally weak correlation with basin area (Spearman correlations not significant at the 5% level). Skewness of IAT distributions is significantly negatively correlated with imperviousness (Spearman rank correlation -0.63). Similar to CV-values, this probably indicates stronger influence of flow regulation on flows in urbanised basins. Medcouple values for IATs clearly show three low value outliers: for Stewart Creek

10 (970), LSugarP (530) and LSugarA (507). In these basins, active low flow control is applied^{c4} preventing occurrence of low flow extremes and high IAT extremes. The effect shows up more clearly for IAT medcouple values, as a result of the adaptive sampling strategy that gives more weight to peak flows, leading to generally higher medcouple values, but also reflecting more clearly the absence of low flow extremes. ^{c5} ^{c6}Some of the basins in this study are subbasins of each other, which implies that

flows can be correlated. Table 4 ^{c7}summarises CV, skewness and medcouple values for three sets of subbasins in the Char-

15 lotte catchment. The results show that variability in skewness and medcouple values is unrelated to inter-basin connections. The same applies for flow CV-values, while CV-values for IATs seem to be clustered by group of subbasins, indicating that inter-basin correlation plays a role in explaining IAT 2nd order variability. The fact that the effect is only visible for IAT, not for flows, indicates that correlation is mainly associated with occurrence of peak flows, that receive more weight in IAT than in flow statistics

20

In this section we discussed distributions of IATs and flows at the 24 hour scale. Results showed that larger basins are generally characterised by stronger smoothing of flows, resulting in higher mean flow, lower CV and lower skewness of the flow histograms. Flow variability is clearly correlated with basin size, which is mainly a result of smoothing of low flows, in the left tail of the flow histogram. Results showed that larger imperviousness is associated with higher mean flows and

25 significantly lower CV-values for IATs, which is mainly associated with stronger flow regulation by dams and detention ponds in urbanised basins. CV and skewness values are much higher for flows than for IATs, while medcouple values are lower for

^{c4}USGS, water year reports

^{c5} ~~The effect of flow regulation is clearly reflected in the skewness (medcouple) values for IATs times and shows up in associated histogram that have a bi-modal shape for these basins. Interestingly, the effect is not visible in the histograms of flow values.~~

^{c6} Text added.

^{c7} Text added.

flows, indicating strong asymmetry of the flow distributions and low representation of high flow extremes in the statistical distribution. ^{c1} ^{c2}

3.3 Distribution of changes in inter-amount times

Figure 6 shows histograms of first-order differences in IATs and flows at the 24 hour analysis scale, for Irwins Creek, the least urbanised basin, LSugarM the most impervious basin, Stewart Creek, a basin with low flow regulation and McAlpine, the largest of all studied basins. In the flow histograms, negative differences are associated with recession, positive differences with flow rise. Conversely, negative differences in IATs occur during flow rise, positive differences during flow recession. Most flow differences are concentrated in the 0 to -0.5 mm bin, associated with slow flow recession of 0.5 mm/day. Most IAT differences are concentrated in the 0 to 0.1 or 0.2 day bin, associated with steeper flow recession of approximately 5 to 10 mm per day. This reflects the relatively higher sampling of rapid flow response for IATs compared to conventional flow sampling. ^{c1} Skewness and medcouple values of the histograms provide indications of hydrograph shape, in particular of the steepness of the hydrograph recession limb: higher skewness, thus more weight of the distribution concentrated in one of the tails, indicates slow flow recession compared to relatively rapid flow rise. Figure 7 shows scatter plots for skewness and medcouple values versus basin size and imperviousness, for all basins. The three basins with low flow regulation (970, 530, 507) can be recognised by their low medcouple values for IAT difference indicating near symmetrical histograms, i.e. flow rise and recession occur at similar rates. ^{c2} Most IAT differences histograms are negatively skewed, with a longer left tail than right tail, i.e. IATs generally decrease quicker (flow rise) than they increase (flow recession). Strongest negative skewness for IAT differences was found for the least urbanised basin (Irwins Creek, gauge 975), indicative of steep flow rise occurring in this basin. ^{c3} ^{c4} Significant positive correlation was found between skewness of IAT difference histograms and imperviousness (Spearman correlations 0.75), indicating lower probability of steep flow rise in higher urbanised basin. Negative correlation was found between medcouple and imperviousness (Spearman correlation -0.55), thus relatively more symmetrical hydrographs with flow rise and recession at similar rates occur for urbanised basins. Here, subbasin correlation appears to play role: medcouple values are higher overall in the McAlpine subbasins than in Little Sugar Creek and Irwin subbasins (see table 4). Significant correlations of IAT differences skewness and medcouple with imperviousness show that urbanisation is associated with more regulated flows, confirming findings in section 3.1.

^{c1} ~~These could be interpreted as a result of anomalous flow behaviour, occurring under extreme conditions (river bank overtopping or other conditions really different from normal flow), but given that IAT distributions are much more stable/less characterised by such outliers, one could conclude it is (at least partly) a result of the sampling strategy.~~

^{c2} ~~Medcouple values for IATs identified three basins where active regulation of low flows is applied, resulting in bi-modal shape of the IAT histograms. No significant correlations were found for medcouple values of flow distributions versus area or imperviousness pointing toward possible flow control. The IAT analyses on the other hand appear to be much more sensitive to these types of flow control measures.~~

^{c1} Text added.

^{c2} Text added.

^{c3} Medcouple values for IAT differences tend to decrease with imperviousness, indicating lower occurrence of steep flow recession.

^{c4} Text added.

3.4 Inter-amount times variability across scales, from sub-daily to seasonal sampling scale

In this section we analyse the variability of IATs and flows across a wide range of sampling scales. ^{c5}We investigate how the statistical distributions and hydrological response characteristics change when moving from inter-event (multiple days) to intra-event (sub-daily) scales. Figure 8 shows ^{c6}quantile plots for normalised flows and IATs at scales between 12 hours and 64 days, for Taggart Creek. ^{c7} On the horizontal axis is the sampling scale, i.e. fixed sampling time for conventional flow statistics or, equivalently, mean inter-amounts for IAT statistics. Note that for the IAT analysis, mean inter-amounts are normalised by basin area size and reported in mm to allow easier interpretation of flow magnitudes and to allow easier comparison between basins. For instance, the normalised inter-amount Δq for Taggart Creek at the daily scale is 0.998 mm. The vertical axis shows quantiles of normalised flows respectively IATs corresponding to the sampling scale in time or Δq . ^{c8}Values on both x- and y-axes are plotted on log scales to allow easier visualisation of quantile values that vary by 2 to 4 orders of magnitude. The bold black line denotes the mean, dotted black line shows median values. The central part of the quantile plots represents the 25-75 percentile range, upper and lower whiskers 10-90 percentiles and crosses the 1 and 99 percentiles.

^{c1}We can see that mean values of normalised flows and IATs decrease log-linearly with sampling scale, as indicated by a straight line in the log-log plot, i.e. the sampling mean follows power-law scaling. As histogram analysis at the 24 hour scale already showed, statistical distributions of both flows and IATs are highly skewed. Moreover, skewness increases at smaller scales as indicated by an increasing distance between mean and median values. Median values for flows follow close to log-linear scaling (albeit steeper compared to the mean) but exhibit stronger departures from log-linear scaling for IATs. In particular, the median of IATs shifts from close to log-linear scaling between 16-64 mm (associated with about 16 to 64 days) to non-log-linear scaling between 1-14 mm scales (1-14 days) and again to near-log-linear scaling below 1 mm. Coincidentally, these transitions correspond to the range of scales over which IATs generally transition from being inter-event to intra-event dominated. Indeed, IATs at coarser scales mostly combine the properties of multiple storms, resulting in a more symmetric distribution. This effect is much stronger in IAT than in flow distributions, because it is mainly associated with changes in sampling of peak flows which are more frequently sampled in the IAT framework than in the conventional fixed time approach.

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^{c5} Text added.

^{c6} Text added.

^{c7} ~~boxplots of flows for scales between 12h and 60 days.~~

^{c8} Text added.

^{c1} Text added.

^{c2} ~~At smaller scales, individual storm properties and intra-event variability play an increasingly dominant role. For example, 10-percentiles of inter-amount times in figure8b change more or less log-linearly with scale below a sampling scale of 7 days (7.1 mm of normalized flow). The frequency of occurrence of this amount of flow at the event scale, i.e. at sub-daily scale, is 2.9%, as shown in the plot of flow quantiles. The 10-90% range of inter-amount times associated with this scale is approximately 1.8 to 513 hours. This indicates that inter-amounts of 7.1 mm can occur over a range of 1.8 hours during peak flow to 513 hours during low flow, equivalent to 3.9 mm/h resp. 0.014 mm/h. At the daily scale, the 10-90% range of inter-amount times is approximately 0.18 to 74 hours, for 1 mm inter-amounts, or 5.6 to 0.014 mm/h. At coarser scales however, the 10-90% range decreases, as variations at sub-daily scale can rarely be captured. This is evidence of different scaling at large and small scales, depending on whether inter-amount times are mostly inter- or intra-event dominated. In between lies a transition range where aggregated flow values increasingly relate to a smaller number of events, down to single events. Flows sampled over fixed time intervals as shown in figure 8a do not exhibit such a strong transition. The fixed temporal sampling strategy means only phenomena occurring above~~

Comparing the 10-90 and 1-99 percentile ranges in figures 8a and 8b we see that ^{c3}the 10-90 percentile range of IATs gradually increases towards smaller scales. For flows, the 10-90 percentile range remains approximately constant, however, distance between 90 and 99 percentile values rapidly increases towards smaller scales. This reflects the highly skewed nature of flow distributions caused by oversampling of low flows compared to high flows; an effect that increases progressively towards smaller scales. By contrast, 10-90 and 1-99 percentile ranges for IATs increase more or less similarly with scale, for sampling scales ranging from 0.51 mm to approximately 10-16 mm. This indicates that the tails of IAT distributions are more or less equally sampled, at least up to the 1 and 99 percentiles.

The upper 75, 90 and 99 IAT percentiles of IATs, associated with low flow periods, change approximately log-linearly with scale, showing that upper tail percentiles of IAT values refer to the same low flow periods across all scales, up to 8-16 mm scale. Associated low flows are approximately 0.1 mm/day. The 1-percentiles for flows are associated with approximately 0.02 mm/day, for the 12 hour to 4 day scale, showing that the distribution tail associated with low flows captures lower flow extremes in conventional sampling than in IAT sampling. This is a result of the relatively high frequency at which low flows are sampled. Conversely, peak flows, associated with the right tail of the flow distribution are sampled less frequently in conventional flow sampling: the 99 percentiles are associated with peak flows of 0.78 to 0.38 mm/h for 12 hour to 4 days scale. The 1-percentiles of IATs are associated with peak flows of about 20 mm/h, at the 0.5 to 4 mm inter-amount scale, associated with mean IATs of 12 hours to 4 days. This shows that the IAT distribution captures more extreme peak flow values than conventional flow sampling, at the same sampling scale. ^{c4}

Quantile plots of inter-amounts over range of scales were created for all 17 gauges included in our analysis ^{c1}(results are added as a supplement to this paper). This allowed us to compare transition ranges between inter-event dominated and intra-event dominated IAT distributions for all basins. Results show that for 10% IAT quantiles, the lower end of the transition range, where intra-event characteristics start to be mixed with inter-event phenomena, lies roughly between 10 mm and 25 mm mean inter-amounts, being accumulated in about 1 hour in most of the basins. Lower values are found for basins with higher urbanisation degree and for basins where low flow control is applied, reflecting the smoothing influence of flow control measures on peak flows. ^{c2} Similarly, one can compare the amount of flow that is being generated in an hour, compared to the mean flow. This can be derived from the IAT quantile plots by looking at the scale at which a given IAT quantile, for instance 10% or 1%, equals 1 hour. For Taggart Creek, the IAT 1-percentile equals 1 hour at sampling scale of 18 mm of mean normalised flow or the sampling scale can be resolved. Peak flow variability remains poorly sampled, even at small scales where the vast majority of the samples come from base flow. Inter-amount times adapt the sampling rate depending on the level of activity and therefore still capture a fair amount of peak flow statistics and intra-event properties at coarser scales.

^{c3} Text added.

^{c4} Stronger departures from log-linear scaling can be seen in the left tail of the interamount time distribution (1%, 10%, 25% and median). These indicate a rapid shift of the inter-amount time distribution towards lower values. The range over which this happens depends on the percentile and considered gauge.

^{c1} Text added.

^{c2} High values generally indicate slow response and are associated with low imperviousness. They imply response to single rainfall events is stretched over multiple days. Results for basins with low flow control show deviating results, due to the multi-modal nature of inter-amount times histograms for these basins, as was discussed in section 3.1.

equivalently, 18 days of mean IAT. This means there is a 1% probability of exceeding 18mm of flow accumulation in 1 hour or less. Or, in terms of time it implies that there is a 1% chance to accumulate the amount of flow measured on average over a period of 18 days in 1 hour or less. Thus, higher values of 1 hour, 1-percentiles indicate stronger flashiness of basin response. Comparing values across basins, we found that higher values of 1%, 1 hour accumulations were strongly correlated with basin area, while no significant correlation with imperviousness was observed.

^{c1}Subsequently, we investigated scaling behaviour from the perspective of statistical moments, by looking at coefficients of variation for flows and IATs across scales. For the purpose of statistical analysis and downscaling applications, it is important to have a robust scaling model, that predicts how distributions change when going from one scale to another. Scale invariance means that a distribution can be derived at any scale, especially small scales, by shifting and scaling the distribution at larger scales. One way to assess the property of scale invariance is to check if the statistical moments of distributions follow a power-law of scale. Figure 9 shows coefficients of variation, computed as the ratio of the 2nd over the 1st order moment, for four gauges, across a range of sub-daily (3 to 12 hours) up to bi-monthly (60-68 days) scales. Results show that coefficients of variation for flows vary non-linearly with scale, while they approximately follow a power law with scale for IATs. For Irwins Creek, the most natural basin in this study (8.2% imperviousness, figure 9a), CV-values of IATs and flows are similar over a range of 10 to 50 days. ^{c2}At smaller scales, CV-values for flows increase more rapidly than for IATs, indicating that IAT variance remains more stable at smaller scales, while variance rapidly increases at small scales for flows, as a result of growing skewness of the statistical distribution, caused by relative oversampling of low flows, or conversely, undersampling of high flows. CV-values for Upper LSugar Creek, the most urbanised basin are lower than for Irwins Creek, especially at smaller scales (figure 9b). ^{c3}This is explained by the influence of flow control measures in this basin, as flows are constrained by the stormwater drainage system. The difference is more pronounced for IATs, because IAT variance is more sensitive to peak flows as a result of the adaptive sampling strategy. Figure 9c shows that for LMcAlpine, the largest basin (238.4 km²), CV-values for flow are more or less stable between 3 and 24 hour scale, due to strong smoothing of peak flows at this intra-event scale. In contrast, CV-values for IATs increase over this range, due to scale sensitivity of the upper tail of the IAT distribution, where long IATs at this small scale (0.1 to 1.1 mm for 3 to 24 hours) are broken up more unevenly, creating increased CV and skewness. This shows that for analysis of low flows, especially in basins characterised by strongly smoothed flow variability, IAT analysis offers little advantage and conventional flow statistics are more suitable. CV-values for Stewart Creek in figure 9d show very low CV-values for IATs that vary little with scale, while CV-values for flows are much higher and strongly sensitive to scale. Stewart Creek is a small, semi-urbanised basin (33% imperviousness) where active low flow control is applied. This results in very low variability in IATs across the entire range of scales, while CV-values for flows are lower than those for similar basins, but highly sensitive to scale, probably due to unbalanced sampling of peak flows compared to very stable low flows.

^{c1} Text added.

^{c2} Text added.

^{c3} Text added.

In section 3.1 ^{c4}we analysed skewness and medcouple values of flow and IAT distributions at the 24-hour scale and found that skewness values were lower and medcouple higher for IATs than for flows. This was explained by the sensitivity of flow distributions to rare peak flows compared to frequently sampled low flows. Initial analyses of skewness and medcouple values across scales showed that results are highly sensitive to the sampling scale. While CV-values show a stable pattern across scales, results for skewness and medcouple are much more variable, across scales and across basins. Explanation of this scale sensitivity of skewness metrics and what information can be derived from this about the tails of the distributions requires deeper analysis that will be part of future work.

3.5 Flashiness indicators and minimum observable scale

^{c1}Two flashiness indicators were computed, as explained in section 2, ^{c2}the classical R-B flashiness index and an IAT flashiness indicator based on characteristics of the IAT distribution. Table 3 summarises flashiness values for all gauges, as well as minimum and maximum observable inter-amounts, as defined in equations (9) and (10). IAT flashiness indicators vary between 12.5 and 165 hours; higher values are generally associated with smaller basins. ^{c3}R-B flashiness values vary between 0.8 and 1.3, indicative of moderately variable flows (R-B flashiness can vary between 0 and 2). Values are in the same range as those reported by Baker et al. (2004) ^{c4}for smaller basins: they found R-B flashiness values larger than 1 for basins smaller than 50 km²). R-B flashiness is strongly correlated with CV-values (figure 10c, ^{c5}Spearman correlation 0.77); this confirms that R-B flashiness is essentially a metric of flow variability. Figure 10a shows that ^{c6}IAT-based flashiness and R-B flashiness are moderately correlated (Spearman rank correlation 0.55), yet there are some striking differences. The three low-flow-regulated basins have very low R-B flashiness values, while IAT flashiness values are in line with values for other basins. This is explained by R-B flashiness being strongly sensitive to low flow variability, while IAT flashiness is more sensitive to occurrence of peak flow values. For instance, McAlpine basin (gauge 255) has a very high IAT flashiness as a result of high occurrence of peak flows. On the other hand LSugarM (gauge 409), the most urbanised basin, has low IAT flashiness as a result of peak values being capped by maximum capacity of pipes in the drainage network.

Figure 10b and 10d ^{c7}shows scatter plots of IAT flashiness (left y-axis) and R-B flashiness (right y-axis) versus basin area and imperviousness, for all gauges. They show a clear relationship between flashiness and basin area (Spearman correlation -0.83 for IAT, -0.71 for R-B flashiness), with a large range of flashiness values for the smallest basins (< approx. 30 km²). Here, clearly other processes than basin size play a role in explaining flashiness. Correlations between R-B and IAT flashiness versus impervious degree are not significant at the 5% level. For R-B flashiness, the most pervious and the most impervious basins (gauges 975 and 409 respectively) are both in the high range of flashiness values, showing that other influences, such as basin

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size and presence or absence of low flow regulation play a more important role than imperviousness degree. IAT flashiness tends to decrease for a combination of higher imperviousness and large basins, basin size playing a stronger role than urbanisation. The most urbanised basin, LSugarM (gauge 409, 31.7 km², 48% imperviousness) has a relatively low flashiness value of 48.8 hours, while the least impervious basin, Irvins Creek (gauge 975, 21.8 km², 8% imperviousness) has a high flashiness value of 102.8 hours. As discussed in section 3.1,^{c8} the effect of urbanisation on flow patterns for the basins in the study area seems to be mainly determined by increased flow regulation associated with introduction of dams, stormwater detention basins and stormwater drains with capacity limitations. While higher imperviousness leads to higher mean runoff flows (for instance, 1.5 mm for LSugarM versus 0.68 mm for Irvins Creek, at 24 hour scale), rainfall in impervious basins tends to run off relatively more quickly and uniformly, depending on the degree of flow regulation. The leads to a mixed effect of basin size, imperviousness and flow regulation on IAT flashiness and peak flows.

^{c1}In this study, IAT flashiness values were defined as the time that is needed on average to accumulate the amount of flow that is accumulated in 15 minutes or less, 1% of the time. R-B flashiness indices were computed at the daily scale, to allow comparison with results obtained by Baker et al. (2004).^{c2}For a fair comparison, both flashiness indices should be computed at similar scales, as far as possible, given that definitions used in the two approaches are different. We aimed to compute both indices at hourly scale, as this is an appropriate scale in relation to the size of most of the basins in our analysis and a reasonable compromise between the 15 minute and 24 hour time scales used for IAT flashiness and R-B flashiness index respectively. Note that Baker et al. (2004)^{c3}stated that the hourly scale would be more suitable for smaller basins (<30 km²), but never computed R-B flashiness values at this scale, only Richard's pathlengths. When we computed R-B flashiness indices at the hourly scale, using the same definition, we found lower flashiness than at the daily scale, which is rather counterintuitive, as one would expect higher flashiness at smaller scales due to the fact that Richard's pathlengths increase from daily to hourly scales. However, R-B flashiness is based on absolute differences of flow values, not gradients (i.e., differences per unit of time). And since flow differences decrease when moving toward smaller scales, R-B index also decreases. Alternatively, one could use discharges instead of flow amounts, but then values could grow much larger than 2. Regardless of the used approach, R-B flashiness index appears to be rather sensitive to the scale of analysis. By contrast, the IAT flashiness index proposed in this paper tends to be much more robust. Additional sensitivity analyses (not shown) revealed almost no changes in IAT flashiness estimates for 15 minutes to 3-6 hours aggregation scales. Beyond that, significant underestimation started to occur as the resolution is not sufficient anymore to correctly capture peak flow variability. For data aggregated at 24 h resolution (instead of the original 15 min), IAT flashiness values were underestimated by 20-80%, depending on the considered gauge.

^{c4}Quantile plots of IAT distributions furthermore provide information about the minimum observable scale at a given observational resolution (15 minutes, in the data series used in our analysis), i.e. the degree of flow variability that occurs at scales

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smaller than the observation scale. When moving towards smaller sampling scales, a growing percentage of flow accumulations occurs in less than 15 minutes, hence cannot be analysed at the given observational resolution. This typically coincides with peak flows and implies that during peak events, the observational resolution is too low to measure flow variability. IAT analysis can thus be used to identify a critical resolution for flow observations, if a given peak flow accumulation is of interest. This

5 could be associated with for instance the capacity of detention ponds or flooding caused by exceedance of stormwater drainage capacity. For the example of Taggart Creek (figure 8b), the scale at which 1% of flow accumulations occurs in less than 15 minutes is associated with inter-amount sampling scale of 4.76 mm. ^{c5}This implies that flows that exceed 4.76 mm in 15 minutes, i.e. peak flows above 19.0 mm/h, cannot be observed 1% of the time. If correct observation of peak flows of this magnitude or larger is important, flow data need to be collected at a higher than 15 minute resolution during times of peak flows. This is

10 typically the case of urban basins, where stormwater drainage systems are often designed for peak flows associated with 10 to 50 year return periods. ^{c6}

3.6 Scaling of inter-amount times across scales: multifractal analysis

As explained in section 2, ^{c1}log-log plots of statistical moments versus sampling scale can be used to study scaling behaviour of time series. In the following, we plotted the moments $\langle X_\lambda^q \rangle$ of order q of IATs as a function of mean inter-amount scale

15 Δq (proportional to the inverse of the scaling ratio λ), on a log-log scale, for moments of order 0.6 to 2.4. We applied the same procedure for flow time series over the same range of equivalent scales. Figure 11 shows examples of log-log plots for flow volumes and IATs for McAlpine Creek (gauge 750). ^{c2}They show that log-linear fits are better for IATs than for flows, especially for higher order moments; minimum R^2 values, that are associated with fits for higher order moments, are 0.9972 and 0.9993 for flows and IAT respectively.

20 Plots in figure 11 ^{c3}show stronger departures from linearity in the log-log plots for flows than for IATs, especially for higher order moments. Figures 11c and 11d illustrate this for log-log curves of moment $q = 2.4$, where a scale break was detected at 22 hours for flows and subtle departures from linearity were found at 20.4 days for flows as well as 11.2 hours and 17.3 days for IATs. Similar analyses were conducted for all gauges, table 5 ^{c4}summarises minimum R^2 values for log-moments fits

25 for flows and IATs. Log-moments for IATs show near perfect fits for all gauges, with minimum R^2 values between 0.995 and

^{c5} Text added.

^{c6} Flashiness values of the order of 100 hours imply that, as for Taggart Creek, flow peaks upward of 15-20 mm/h cannot be measured 1% of the time, at 15 minute observational resolution. This value can be interpreted as an indicator for flashiness of basin response: higher flashiness values indicate larger differences between the rising and recession components of the hydrological response. As explained in the Methods section, a threshold for minimum observable inter-amounts was determined for our analysis, based on a maximum acceptable error for estimated IATs. Results in table 3 show that the minimum observable scales vary between 2.75 and 13.75 hours and generally decrease for larger basins. Correlation between minimum observable scale and flashiness is high (Spearman correlation 0.9), as for most gauges the 90-percentiles (for minimum observable scale) and 99-percentiles (for flashiness) decrease log-linearly with scale along a similar gradient.

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1.000. Quality of log-moments is consistently lower for all basins; minimum R^2 values are between 0.990 and 0.997, lower quality fits generally occurring for smaller basins.

Investigation of departures from linearity showed that for flows, most gauges exhibited a scale break between 8 and 20 days. Similar scale breaks, between time scales of 8 to 16 days, were found in scaling analyses of flow data by other authors based on flow data at daily resolution (Tessier et al., 1996; Labat et al., 2002; Sauquet et al., 2008). Labat et al. (2013) and Sauquet et al. (2008) found scale breaks in the range of 16 to 27 hours, for 30 minutes respectively hourly resolution. We did not detect any strong departures from linearity in the IAT framework except for the 3 gauges where low flow regulation is applied (LSugarA, 507, LSugarP, 530, Stewart Creek, 750).

Using the empirical log-moments, we fitted the multifractal parameters C_1 and α for IATs and flow amounts. Table 5 summarises C_1 and α values for all basins, for flows and for IATs. Results show that C_1 - values, characterising intermittency of the time series, are lower for IATs than for flows. This makes sense and can be explained by the adaptive sampling strategy of IATs, especially the fact that low flows are sampled less often than in the classical fixed-time framework. Values of the multi-fractality index α are generally lower for IATs, with the exception of four basins. Two of these basins are characterised by low flow regulation, one basin has anomalous land-use distribution with a high concentration of imperviousness in the upper part of the basin. Time series of the 4th basin is short (8 years), which might influence outcomes of the scaling analysis. C_1 and α values for flows are in the range of values found by other authors. Figure 12 shows scatter plots of values for C_1 and α for flow and for IATs versus basin size and imperviousness. C_1 - values are clearly negatively correlated with basin area. Rank correlations for IATs are -0.67 and -0.85 for flows. No significant correlation of C_1 with imperviousness was found, but the three basins with low flow control stand out with lower than average C_1 values. This shows up both in the IAT analyses and in the classical approach based on flows. The α values for IATs are positively correlated with area (0.6) and negatively with imperviousness (-0.56). No significant correlation with area nor imperviousness was detected. For IATs, negative correlation of α with imperviousness comes from the fact that IATs in highly impervious basins are redistributed more evenly when moving from large to small scales (due to high imperviousness).

4 Summary and conclusions

^{c1}In this study, we introduced an alternative approach for analysis of hydrological flow time series, using an adaptive sampling framework based on inter-amount times (IATs). The main difference between flow time series and time series for IATs is the rate at which low and high flows are sampled; the unit of analysis for inter-amount times is a fixed flow amount, instead of a fixed time window. Thus, in IAT analysis, sampling rate is adapted according to the local variability in flow time series, as opposed to time series sampling using fixed time steps. We aimed to investigate the effect of adaptive IAT sampling on flow statistics, especially on the tails of the statistical distributions associated with peak flow and low flow extremes. We analysed and compared statistical distributions of flows and IATs across a wide range of sampling scales to investigate sensitivity of statistical properties such as distribution quantiles, variance, scaling parameters and flashiness indicators to the sampling scale.

^{c1} Text added.

We did this based on streamflow time series for 17 (semi)urbanised basins in North Carolina, US. The following conclusions were drawn from the analyses:

1. ^{c1}Adaptive sampling of flow time series based on inter-amounts leads to higher sampling frequency during high flow periods compared to conventional sampling based on fixed time windows. This results in a more balanced representation of low flow and peak flow values in the statistical distribution. While conventional sampling gives a lot of weight to low flows, as these are most ubiquitous in flow time series, IAT sampling gives relatively more weight to high flow periods, when given flow amounts are accumulated in shorter time. As a consequence, IAT sampling gives more information about the tail of the distribution associated with high flows, while conventional sampling gives relatively more information about low flow values.
2. ^{c2}Statistical analysis of IATs and flows at the 24 hour scale showed that coefficient of variation (CV) and skewness values were much higher for flows than for IATs, while medcouple values were lower for flows, indicating strong asymmetry of the flow distributions and low representation of high flow extremes in the statistical distribution. Larger basins were generally characterised by stronger smoothing of flows, resulting in higher mean flow, lower CVs and lower skewness of the histograms. Flow variability was clearly correlated with basin size, which is mainly a result of smoothing of low flows, in the left tail of the flow histogram. Larger imperviousness was associated with higher mean flows and significantly lower CV-values for IATs, which was mainly associated with stronger flow regulation by dams and detention ponds in urbanised basins
3. ^{c3}Comparison of coefficients of variation (CV) across the 17 basins showed that CV values of flows were negatively correlated with basin size, due to an increased smoothing effect for larger basin areas on flow variation. CV values of IAT distributions were not significantly correlated with basin size, which indicates that basin size has a stronger smoothing effect on low flow variability, strongly represented in conventional flow time series, than on peak flows which are more frequently represented in IAT time series. By contrast, CV values of IAT distributions were negatively correlated with imperviousness, while correlation between CV values for flows and imperviousness was not significant. Negative correlation between IAT CV values and imperviousness probably indicates stronger influence of flow regulation in more urbanised basins resulting in more uniform runoff during rainy periods. IATs during these periods concentrate relatively more closely to the mean and show fewer extremes. This result is contrary to findings in many other studies, where urbanisation tends to be associated with higher peak flows. In the basins analysed in this study, flow regulating measures as indicated by a higher number of dams for detention basins and high drainage connectivity associated flow capacity limitations explains the strong influence of flow regulation associated with urbanisation. ^{c4}

^{c1} Text added.

^{c2} Text added.

^{c3} Text added.

^{c4} ~~We found a bi-modal histogram shape for three basins, which could be attributed to low flow regulation applied for these basins. This was also clearly reflected in the skewness (medcouple) values for inter-amount times. The effect was not visible in histograms or skewness values for flows.~~

4. ^{c5}Histograms of first-order differences showed negative skewness for IATs and positive skewness for flows, for most of the basins, indicating prevalence of slow flow recession compared to flow rise. The three basins with low flow regulation could be recognised by their relatively low medcouple values (<0.4) for IAT differences, showing that hydrographs tend towards being symmetrical in these basins. Significant correlations were found between skewness and medcouple of IAT differences and imperviousness (Spearman correlations 0.75 and -0.55), showing that urbanisation is associated with more regulated flows, thus relatively more symmetrical hydrographs with flow rise and recession at similar rates and lower frequencies of steep flow rise. Here, subbasin correlation appears to play a role: medcouple values were higher overall in the McAlpine subbasins than in Little Sugar Creek and Irwin subbasins. No significant correlations were found for differences in flows.
5. ^{c1}Quantile plots of flows and IATs plotted over a range of subdaily to seasonal scales showed the influence of the different sampling strategy for IATs compared to conventional flow sampling on median, 25-75, 10-90 and 1-99 percentile ranges of the distributions. The 25-75 and 10-90 percentile ranges for flows remained approximately constant, but the distance between 90 and 99 percentile values rapidly increased towards smaller scales. This reflects the highly skewed nature of flow distributions caused by oversampling of low flows compared to high flows; an effect that increased progressively towards smaller scales. By contrast, 10-90 and 1-99 percentile ranges for IATs increased more or less similarly with scale, for sampling scales ranging from 0.51 mm to approximately 10-16 mm, largely associated with intra-event flow variability. This indicates that the tails of IAT distributions are more or less equally sampled, at least up to the 1 and 99 percentiles.
6. ^{c2}Quantile plots for IATs showed different scaling at small scales (up to inter-amount scale 8-10mm) and large scales (roughly exceeding 20 mm inter-amounts), with a transition range in between. At smaller scales, IATs are mostly dominated by intra-event variability, while at large-scales IATs span multiple events. Flows sampled over fixed time intervals did not clearly exhibit this transition, probably because peak flow variability is being poorly sampled by fixed time window sampling. Because IATs adapt the sampling rate depending on the level of activity, they still capture a fair amount of peak flow statistics and intra-event properties, even at coarser scales.
7. ^{c3}Comparison of the tails of flows and IAT distributions showed that the distribution tail associated with low flows captures lower flow extremes in conventional sampling than in IAT sampling (0.02 mm/day compared to 0.1 mm/day). Conversely, IAT distributions capture more extreme peak flow values than conventional flow sampling, at the same sampling scale: the 99 percentiles for flows are associated with peak flows of 0.38 to 0.78 mm/h (sampling scales 12 hours to 4 days), while 1 percentiles of IATs are associated with peak flows of about 20 mm/h (sampling scales 0.5 to 4 mm inter-amounts, associated with IATs of 12 hours to 4 days).

^{c5} Text added.

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^{c2} Text added.

^{c3} Text added.

8. ^{c4}Analysis of CV-values of flow and IAT distribution across scale showed that at smaller scales, CV-values for flows increase more rapidly than for IATs, indicating that IAT variance remains more stable at smaller scales, while variance rapidly increases at small scales for flows. This is as a result of growing skewness of the statistical distribution of flows, caused by relative oversampling of low flows, or conversely, undersampling of high flows. This shows that for analysis of peak flows, IAT analysis offers advantages of the fixed-time sampling framework, as it samples peak flows more frequently and results in stabler variance across scales. For analysis of low flows, especially in basins characterised by strongly smoothed flow variability, IAT analysis offers little advantage and conventional flow statistics are more suitable.
9. ^{c1}An IAT flashiness indicator was defined as the inter-amount scale at which 1% of flow accumulations occur in less than 15 minutes. Comparison between IAT-based flashiness and the commonly applied R-B flashiness index showed that indices were moderately correlated (Spearman rank correlation 0.55), yet there were some striking differences. R-B flashiness was shown to be strongly sensitive to low flow variability, while IAT flashiness was more sensitive to occurrence of peak values. Both flashiness indices showed strong correlation with basin area. R-B flashiness showed no clear relationship with imperviousness. IAT flashiness tends to decrease for a combination of higher imperviousness and larger basin size, basin size playing a stronger role than urbanisation. The effect of urbanisation on flow patterns for the basins in the study area is a mixture of faster runoff flows due to imperviousness and stronger flow regulation by dams and detention basins. This leads to a mixed effect of basin size, imperviousness and flow regulation on IAT flashiness and peak flows.
10. ^{c2}A minimum observable inter-amount scale was defined as the smallest scale at which flow variations can be studied given a fixed temporal observational resolution. At higher sampling scales, a growing percentage of flow accumulations occurs in less than the given observational resolution, 15 minutes in this study. This typically coincides with peak flows and implies that during peak events, the observational resolution is too low to measure flow variability. IAT analysis can thus be used to identify a critical resolution for flow observations, if a given peak flow accumulation is of interest. If correct observation of peak flows of a given magnitude is important, flow data need to be collected at a higher than 15 minute resolution during times of peak flows. This is typically the case of urban basins, where stormwater drainage systems are often designed for peak flows associated with 10 to 50 year return periods.
11. ^{c3}Multifractal analysis of IATs and flows was applied over a range of sub-daily to seasonal scales. Flows exhibited departures from multifractality for most basins, while IATs systematically scaled better than flows and showed departures from multifractality only for three basins subject to low flow regulation. This showed that IATs can help better predict peak flow characteristics at small unobservable scales based on coarse resolution data. Additionally, they provide new interesting alternatives for the stochastic modelling and downscaling of flow data.

^{c4} Text added.

^{c1} Text added.

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^{c3} Text added.

^{c4}This study showed that properties of statistical distributions of flow time series are very sensitive to the scale at which the statistics have been derived. This influences values of summary statistics that are used to characterise flow patterns of hydrological basins, like peak flows at given recurrence intervals and flashiness indices. Adaptive sampling based on inter-amount times helped to achieve stabler variance across scales, yet the behaviour of other statistical properties such as skewness, median couple is less clear. Further investigations are needed to interpret changes of statistics across scales. Future work will focus on multi-scale analysis, on how to compare results at different scales and what can be learnt from behaviour at different scales about flow variability in hydrological basins in relation to basin characteristics.

-Analyses in this study identified minimum observable scales below which flow variability cannot be captured at a given measurement resolution. The combination of being able to identify these minimum observable scales and to downscale flow data based on IATs is an interesting area for future investigation. Results showed that scaling parameters for IAT time series were more reliable than those based on fixed-time sampling because of smaller departures from linearity in log-log plots. Future work will focus on possible ways to use IATs to downscale coarse resolution flow data with the help of multifractals and multiplicative random cascades, to see if this leads to more robust and reliable results than downscaling based on conventional flow time series.

Another aspect that remains to be investigated is how IATs computed on flow data compare to IATs computed on associated rainfall time series. Because flow is linked to rainfall, the comparison of the two could help better distinguish which aspects of flow variability are due to rainfall and which relate to basin characteristics and stormwater management.

c1

^{c4} Text added.

^{c1} Results of this study showed that inter-amount times distributions can be used to identify characteristics of hydrologic response that conventional flow time series sampling could not reveal. In particular, effects of low flow regulation on hydrological response could clearly be identified. The lower tail of inter-amount time distribution was shown to provide a good indication of what resolution is necessary to adequately capture the most extreme flow variations for given basin; conversely to estimate the error made in flow measurement given the observational resolution. Sampling frequency in inter-amount times analysis being tuned to variability in the flow pattern results in more balanced statistical distributions. More balanced sampling of flow rise and recession of flow peaks allowed to shed new light on characterising the flashiness of watersheds. Conventionally, flashiness is characterised by (normalised) peak flow magnitude and response time. Based on inter-amount times distribution we were able to define a flashiness indicator that incorporates both the rising and falling components of the hydrological response. More balanced distributions were also reflected in better scaling behaviour of inter-amount times compared to conventional flow sampling. Scaling analysis showed that inter-amount times provide a promising way to better predict peak flow characteristics at small unobservable scales from coarse resolution data. Analyses in this study identified minimum observable scales below which flow variability cannot be captured at the given measurement resolution, leading to errors in especially peak flow estimation. The combination of being able to identify these minimum observable scales and downscale flow data based on inter-amount times is an interesting area for future investigation. Another aspect that remains to be investigated is how inter-amounts times computed on flow data compare to inter-amount times of rainfall. Because flow is linked to rainfall, the comparison of the two could help better distinguish which aspects of flow variability are due to rainfall and which relate to storm water management. Future work will mostly focus on these issues and on possible ways to use inter-amount times to downscale coarse resolution flow data with the help of multifractals and multiplicative random cascades.

Acknowledgements. The authors would like to acknowledge USGS for making available the datasets of flow gauges in Charlotte. The first author would like to thank NWO Aspasia and Delft University of Technology for the grant that supported this research collaboration. The second author acknowledges the funding provided by the Swiss National Science Foundation, grant P300P2_158499 (project STORMS).

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- Table 1: Summary of results reported on the literature for (multi)fractal analysis of hydrological flows. MA: moments analysis, MFA: multifractal analysis, SA: Spectral analysis, TMA: Trace Moment analysis
- Table 2: Summary of hydrological basins in the Charlotte area: basin area [km^2], imperviousness [%], average 24 h flow [m^3], average 24 h flow normalized by basin area [mm] and length of observation in years.
- 10 Table 3: Summary statistics of time series for flows and inter-amount times, at 24 hour sampling scale: coefficient of variation (CV), skewness (skew) and medcouple (mc).
- Table 4: Summary statistics of time series for flows and inter-amount times, at 24 hour sampling scale: coefficient of variation (CV), skewness (skew) and medcouple (mc), for three sets of connected subbasins in the Charlotte catchments: Irwin, Little Sugar and McAlpine
- 15 Table 5: Minimum and maximum observable scales (in hours), flashiness index for 15 min observation time (in hours) and fitted multifractal parameters α and C_1 for inter-amount times respectively flows
- 20 Figure 1: Map with the location of the area, boundaries of hydrological basins and location of stream gauges used in the analysis (NB use figure1 from Joyce's paper: 1a + 1g)
- Figure 2a, 2b: Example of times series for flow (2a) and for associated inter-amount times (2b) for the flow gauge at Taggart Creek, a 13.6 km^2 basin in the Charlotte catchment.
- 25 Figure 3: Illustration of inter-amount data sampling for cumulative flow over a period of 7 days, for Taggart Creek. 3a Flow data series at original 15 min observational resolution; 3b Cumulative graph for flows and IATs at the same mean sampling resolution, illustrating how adaptive sampling based on IATs differs from classical fixed-time sampling)
- 30 Figure 4a, 4b: Histograms of flow time series (4a) and time series of inter-amount times (4b) for Taggart Creek and Little Sugar Creek at Archdale (LSugarA), for 24 hour scale.
- Figure 5a-f: Scatter plots for mean normalised flows inter-amounts (a, b), coefficient of variation (c, d) and medcouple values (e, f) for flows and inter-amount times versus basin area and imperviousness degree. Grey triangle symbols represent inter-amount times, black circles represent flows.

Figure 6: histograms of first-order differences in inter-amount times and flows, at 24 hour sampling scale, for Irvins Creek and LSugarM Creek

Figure 7a-d: Scatter plots of skewness (a, b) and medcouple values (c, d) of histograms for differences in flows and inter-amount times, plotted versus basin size and imperviousness degree. Grey triangle symbols represent inter-amount times, black circles represent flows.

Figure 8a, 8b: Quantile plots of flows (a) and inter-amount times (b) for Taggart Creek for a range of scales, from 12 hours to 60 days. The bold black line denotes the mean values. The dotted black line shows median values. The central part of boxplots represents the 25-75 percentile range, upper and lower whiskers 10-90 percentile range, crosses the 1-99 percentile range.

Figure 9: Coefficients of variation for flows and inter-amount times scales across a range of sub-daily (3 to 12 hours) up to bi-monthly (60-68 days) scale, for Irvins Creek, LSugarM, Stewart Creek and McAlpine

Figure 10: Scatter plots of flashiness versus basin area and imperviousness, for all gauges

Figure 11: Example of log-log plots for flows and inter-amount times (11a, 11b), for Mc Alpine Creek, illustrating departures from linearity at high order moments. Curve for moment $q=2.4$ illustrating scale breaks for flows and inter-amount times (11c, 11d)

Figure 12: Multifractal parameters $C1$ and α for scaling analysis of flows and inter-amount times, as a function of drainage area and imperviousness degree

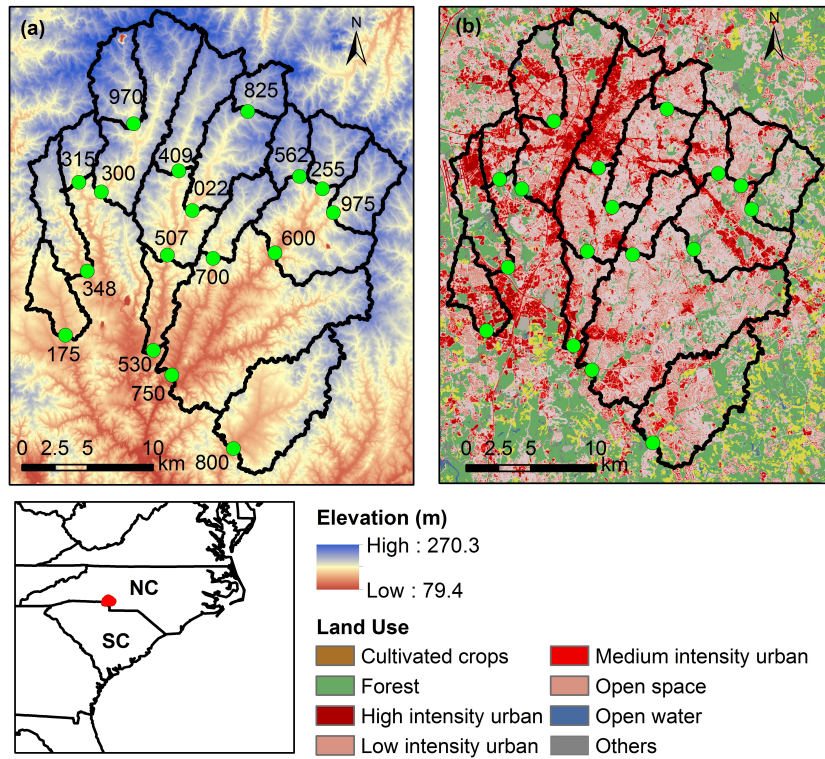


Figure 1. Map with the location of the area, boundaries of hydrological basins and location of stream gauges used in the analysis

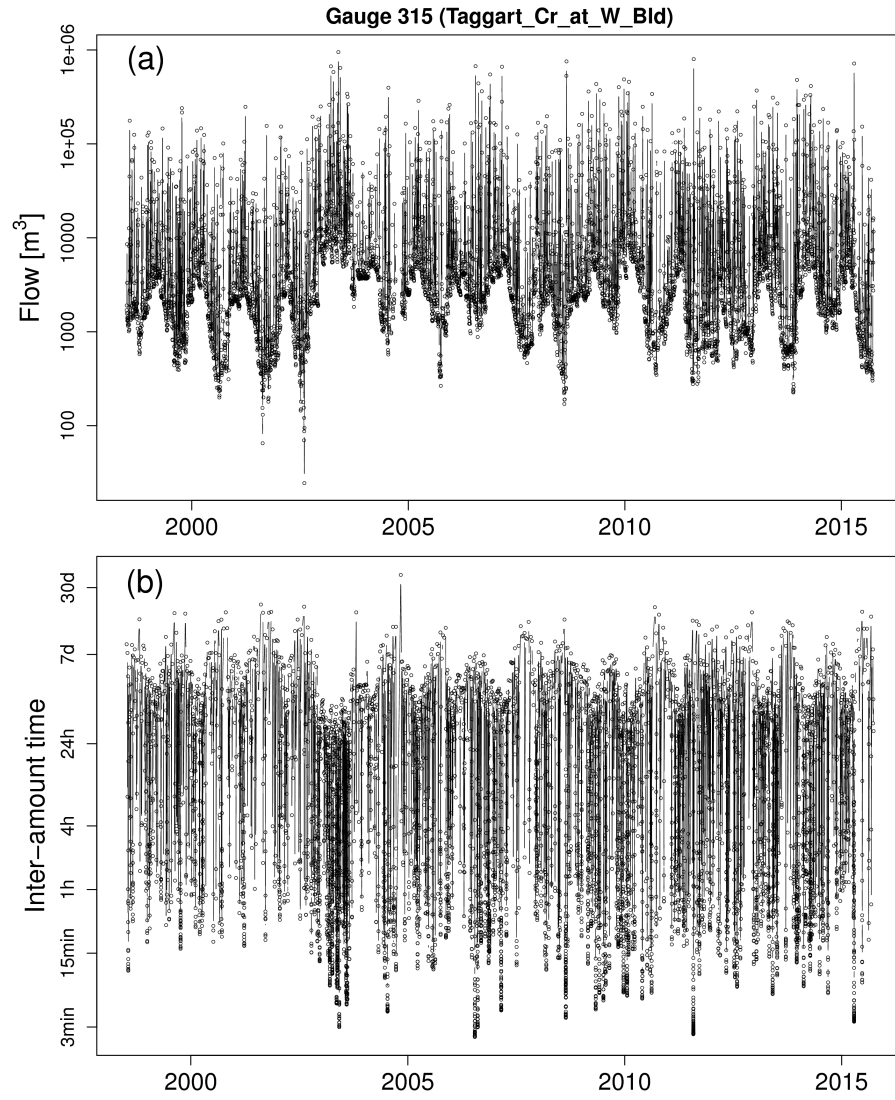


Figure 2. Example of times series for flow (a) and for associated inter-amount times (b) for the flow gauge at Taggart Creek, a 13.6 km^2 basin in the Charlotte catchment.

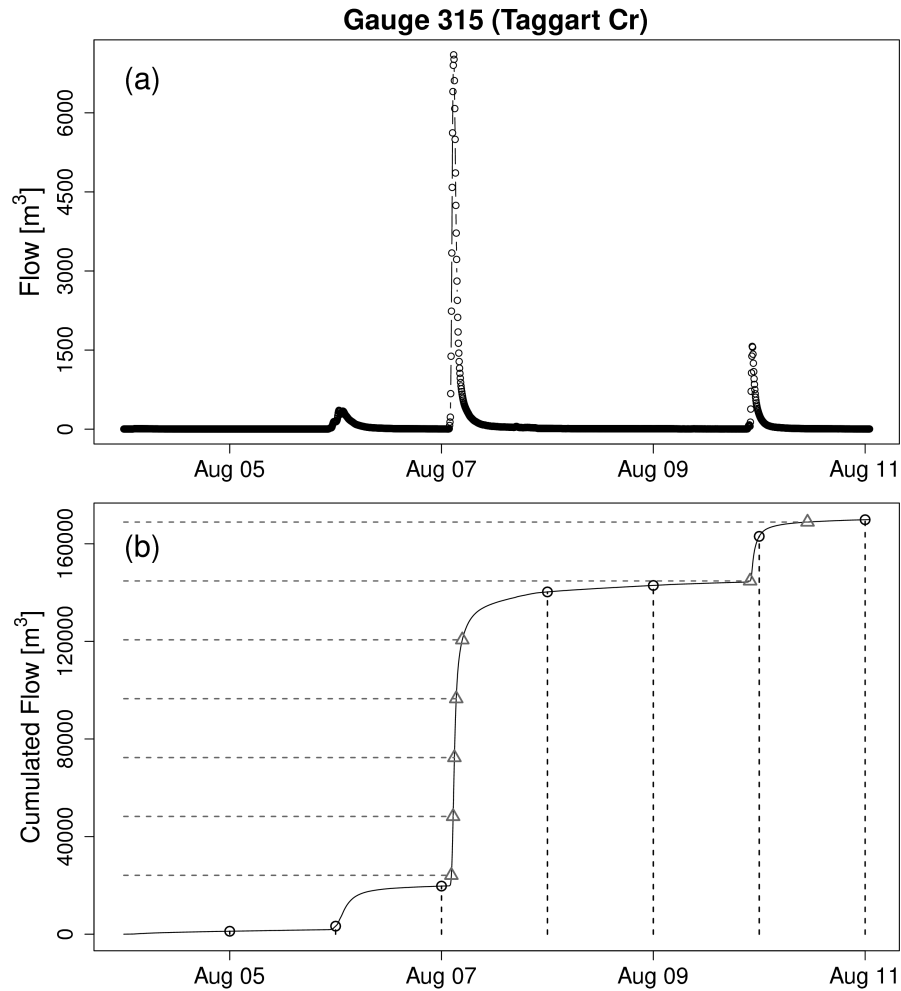


Figure 3. Illustration of inter-amount data sampling for cumulative flow over a period of 7 days, for Taggart Creek. 3a Flow data series at original 15 min observational resolution; 3b Cumulative graph for flows and IATs at the same mean sampling resolution, illustrating how adaptive sampling based on IATs differs from classical fixed-time sampling.

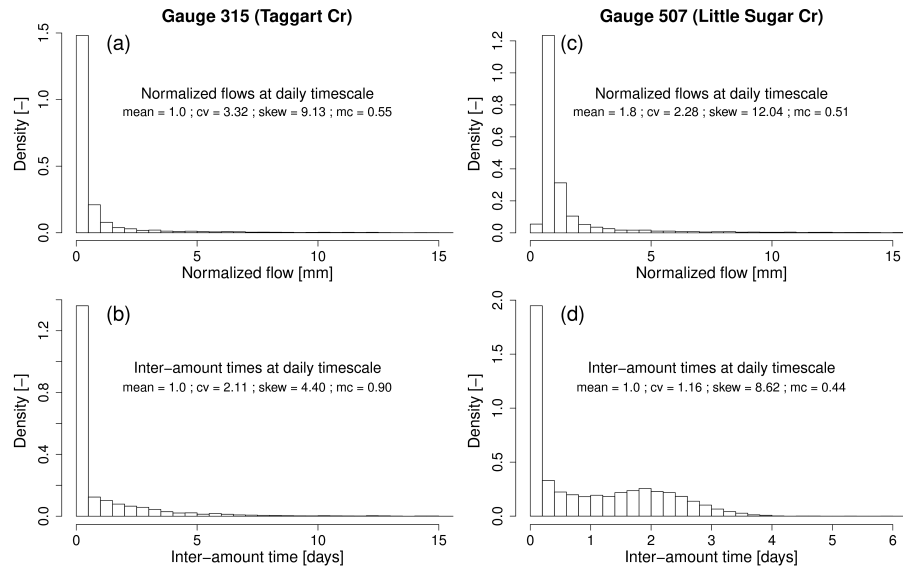


Figure 4. Histograms of flow time series (a) and time series of inter-amount times (b) for Taggart Creek and Little Sugar Creek at Archdale (LSugarA), for 24 hour scale.

Table 1. Summary of results reported on the literature for (multi)fractal analysis of hydrological flows. MA: moments analysis, MFA: multifractal analysis, SA: Spectral analysis, TMA: Trace Moment analysis

Reference	Method	Data sampling scale	Basins	Time series length	Scale break break	Value C1	Value alpha
Tessier et al. (1996)	MFA	day	30 basins in FR 40-200 km ²	11-30 yrs	16 days 16 days	1-16d: 0.2 +/- 0.1 30-4096d: 0.2 +/- 0.1	1-16d: 1.45 +/- 0.25 30-4096d: 1.45 +/- 0.2
Sauquet et al. (2008)	SA	Hour	34 basins in FR	16-37 yrs	8.7h-7 d	-	-
Sauquet et al. (2008)	MA	Hour	12.7-703 km ²	16-37 yrs	10h-6.25d*	-	-
Sauquet et al. (2008)	SA	Day	Idem	Idem	12 days	-	-
Pandey et al. (1998)	SA	Day	19 basins USA 5 -1.8 10 ⁶ km ²	9-73 yrs 9-73 yrs	8 days 8 days	1-8d: 0.2 +/-0.1 1-8d: 0.2 +/-0.1	1-8d: 1.65 +/-0.12, 1-8d: 1.65 +/-0.12
Labat et al. (2013)	SA	30 min	3 basins in FR	-	1 day	-	-
Labat et al. (2013)	TMA	30 min	ca. 13 km ²	-	16 hours	30min-16h: 0.22 >16h: 0.35	30min-16h: 1.18 >16h: 0.79

* only for higher order moments

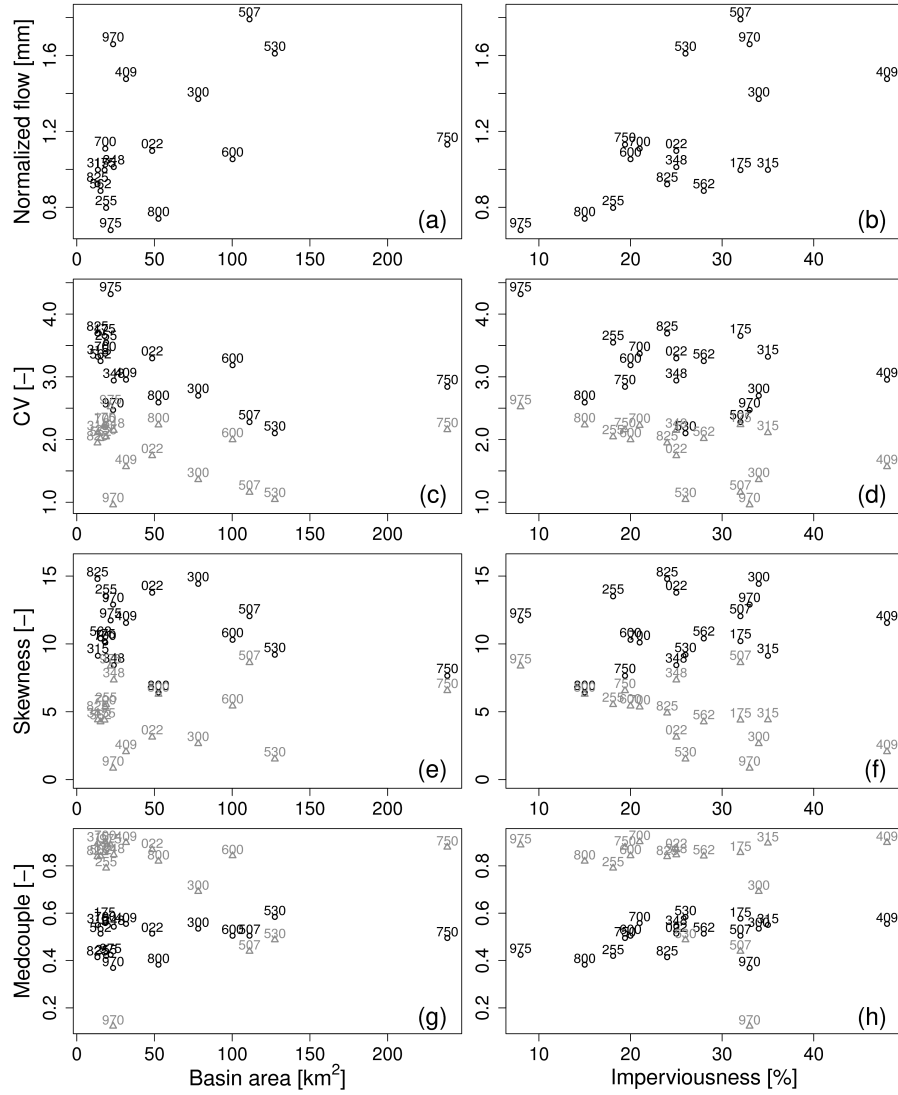


Figure 5. Scatter plots for mean normalised flows inter-amounts (a, b), coefficient of variation (c, d) and medcouple values (e, f) for flows and inter-amount times versus basin area and imperviousness degree. Grey triangle symbols represent inter-amount times, black circles represent flows.

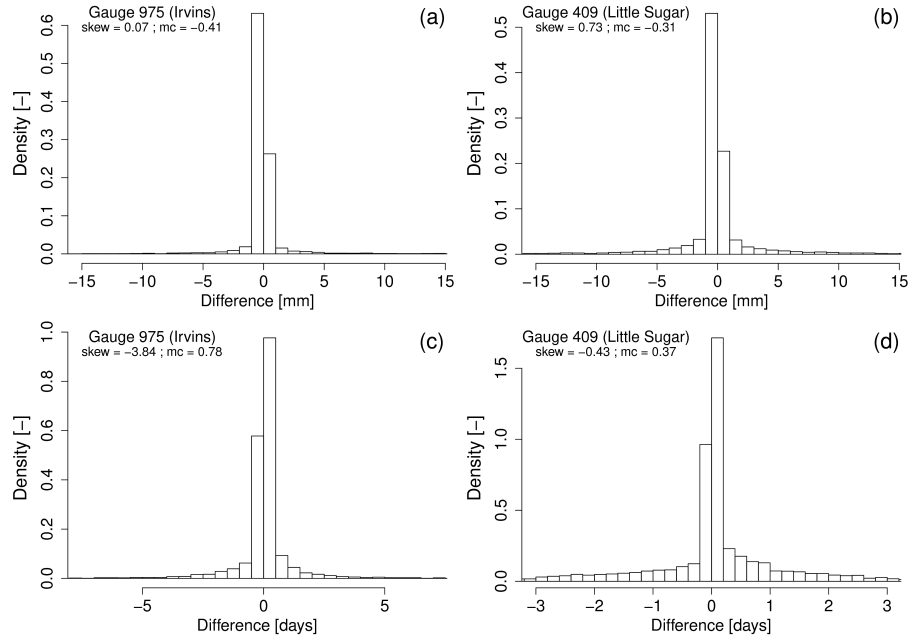


Figure 6. histograms of first-order differences in inter-amount times and flows, at 24 hour analysis sampling scale, for Irwins Creek and LSugarM Creek.

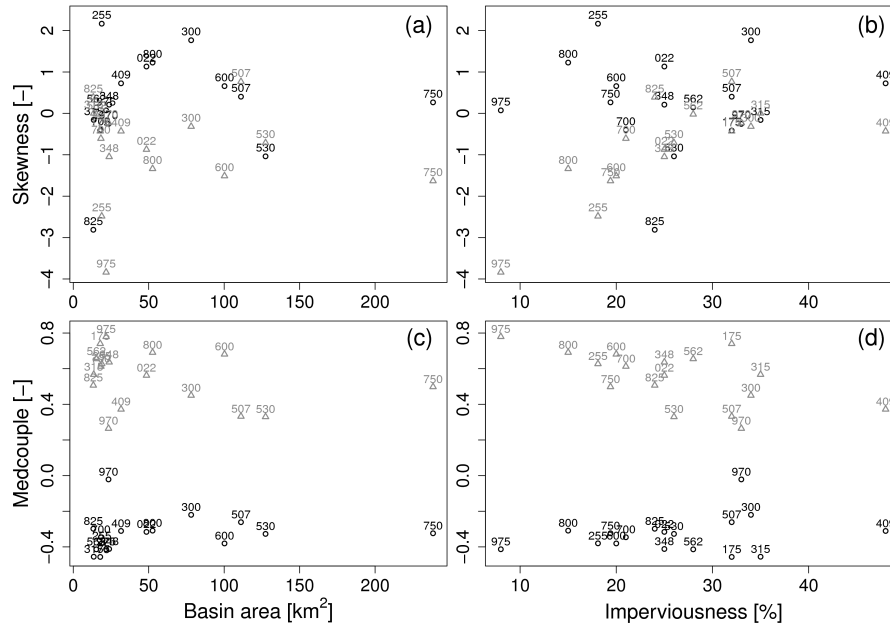


Figure 7. Scatter plots of skewness (a, b) and medcouple values (c, d) of histograms for differences in flows and inter-amount times, plotted versus basin size and imperviousness degree. Grey triangle symbols represent inter-amount times, black circles represent flows.

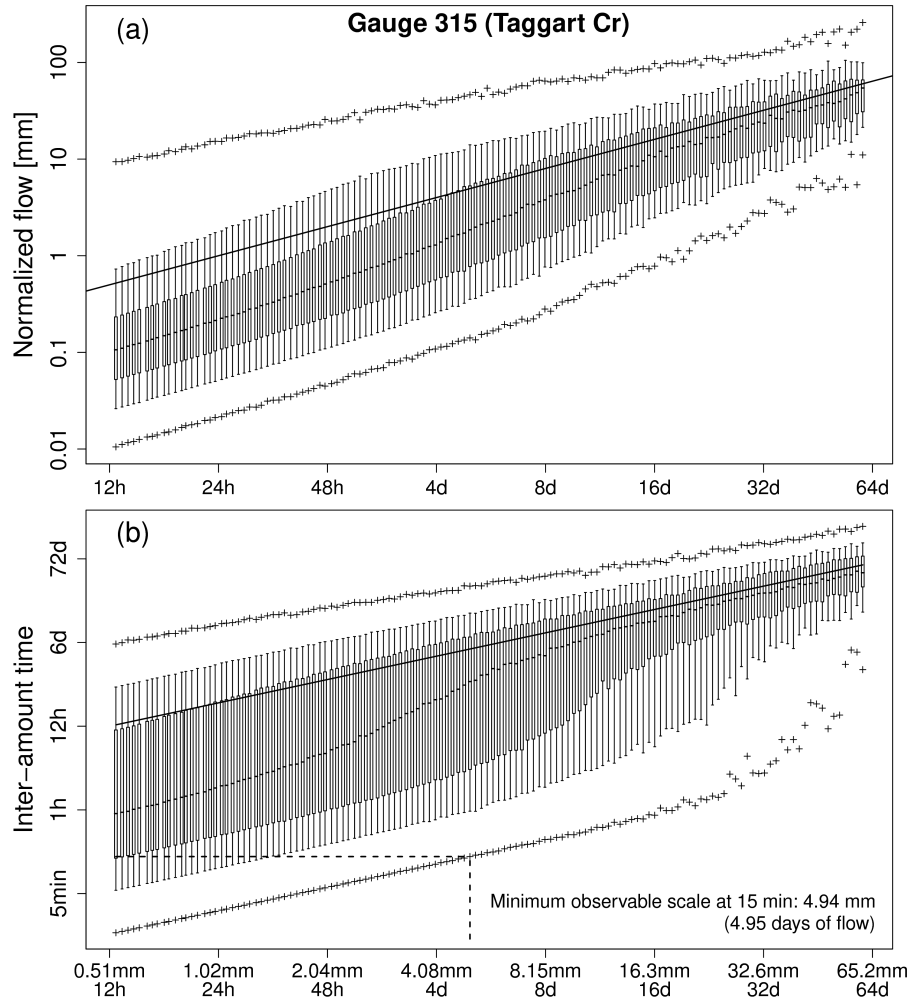


Figure 8. Quantile plots of flows (a) and inter-amount times (b) for Taggart Creek for a range of scales, from 12 hours to 60 days. The bold black line denotes the mean values. The dotted black line shows median values. The central part of boxplots represents the 25-75 percentile range, upper and lower whiskers 10-90 percentile range, crosses the 1-99 percentile range.

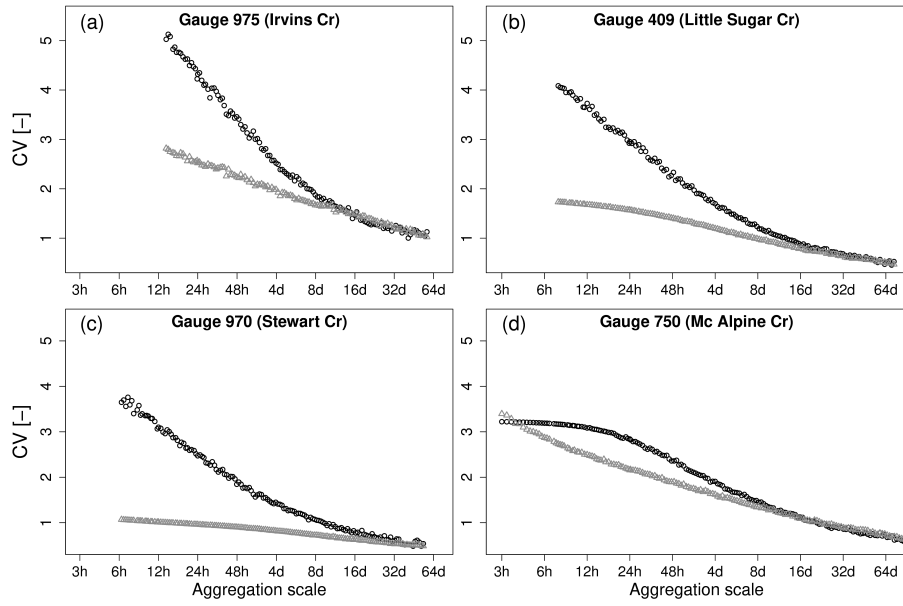


Figure 9. Coefficients of variation for flows and inter-amount times scales across a range of sub-daily (3 to 12 hours) up to bi-monthly (60-68 days) scale, for Irvins Creek, LSugarM, Stewart Creek and McAlpine. Grey triangle symbols represent inter-amount times, black circles represent flows.

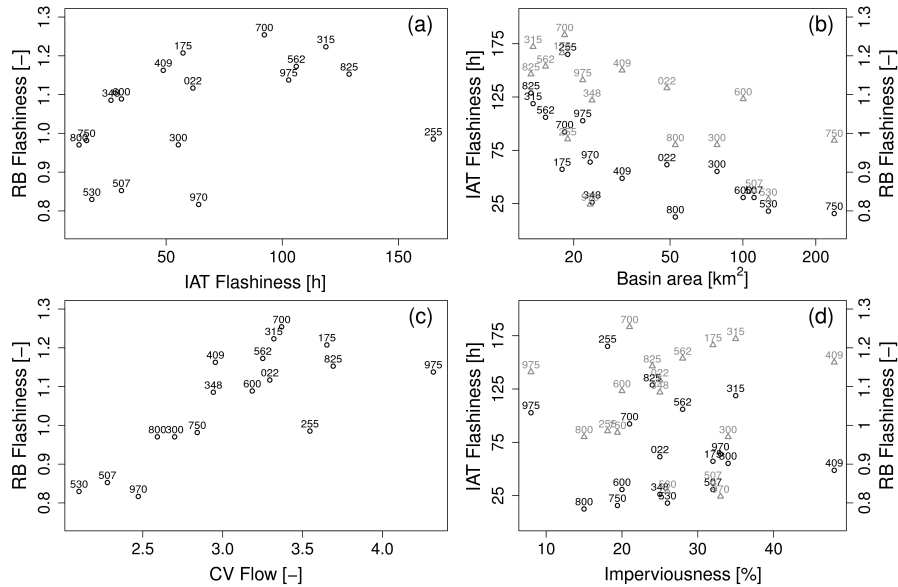


Figure 10. Scatter plots of flashiness versus basin area and imperviousness, for all gauges. Grey triangle symbols represent inter-amount times, black circles represent flows.

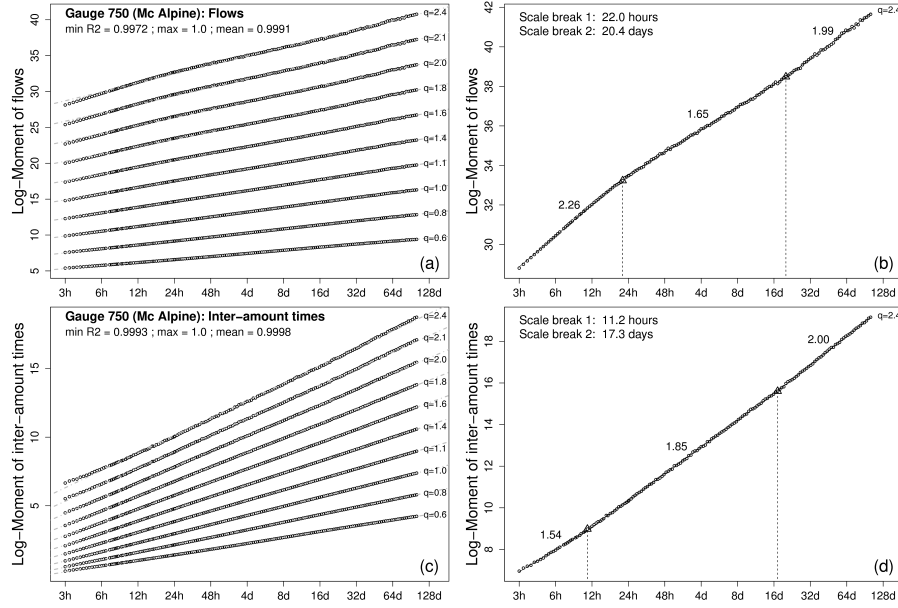


Figure 11. Example of log-log plots for flows and inter-amount times (a, b), for Mc Alpine Creek, illustrating departures from linearity at high order moments. Curve for moment $q=2.4$ illustrating scale breaks for flows and inter-amount times (c, d).

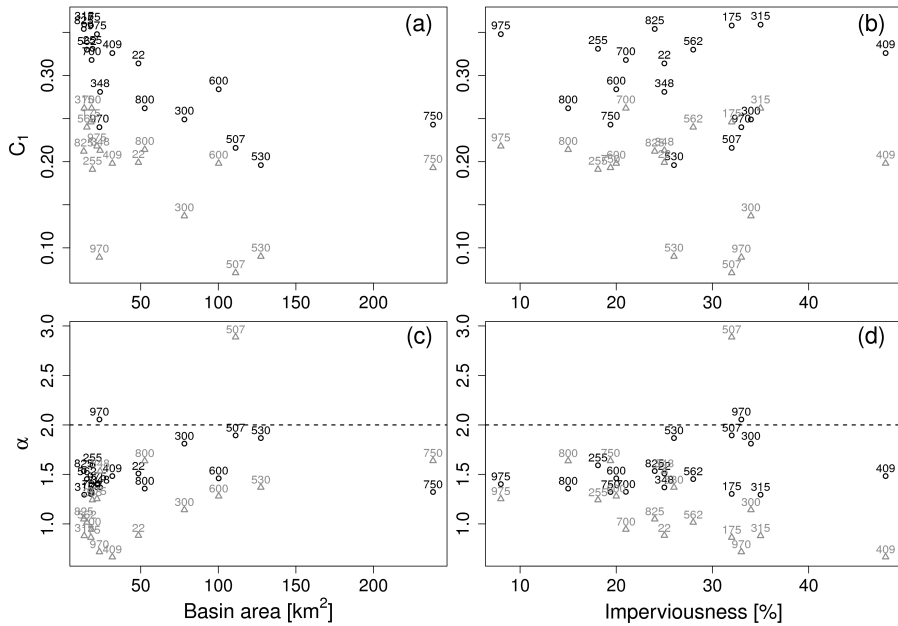


Figure 12. Multifractal parameters C_1 and α for scaling analysis of flows and inter-amount times, as a function of drainage area and imperviousness degree. Grey triangle symbols represent inter-amount times, black circles represent flows.

Table 2. Summary of hydrological basins in the Charlotte area: basin area [km²], imperviousness [%], average 24 h flow [m³], average 24 h flow normalized by basin area [mm] and length of observation in years.

ID	name	area	imperv	dams	mean flow	mean norm. flow	Nyears
825	UBriar	13.3	24.0	22	12275	0.92	17.4
315	Taggart	13.6	35.0	3	13559	1.00	17.2
562	Campbell	15.3	28.0	48	13567	0.89	16.2
175	Steele	17.9	32.0	21	17838	1.00	17.4
700	McMullen	18.3	21.0	15	20348	1.11	29.0
255	UMcAlpine	18.9	18.1	100	15061	0.80	16.3
975	Irvins	21.8	8.0	62	14821	0.68	16.3
970	Stewart	23.4	33.0	55	38800	1.66	15.3
348	Coffey	23.8	25.0	72	24104	1.01	17.0
409	LSugarM	31.7	48.0	2	46775	1.48	21.0
022	LBriar	48.5	25.0	17	53246	1.10	19.8
800	SixMile	52.6	15.0	-99	38914	0.74	8.0
300	UIrwin	78.1	34.0	39	107119	1.37	29.0
600	MMcAlpine	100.2	20.0	51	105640	1.05	29.0
507	LSugarA	111.1	32.0	24	199002	1.79	29.0
530	LSugarP	127.4	26.0	-99	205202	1.61	18.3
750	LMcAlpine	238.4	19.4	-99	269534	1.13	29.0

Table 3. Summary statistics of time series for flows and inter-amount times, at 24 hour sampling scale: coefficient of variation (CV), skewness (skew) and medcouple (mc).

name	CV IAT	CV flow	skew IAT	skew flow	mc IAT	mc flow	skew dIAT	skew dflow	mc dIAT	mc dflow
UBriar	1.95	3.69	4.91	14.79	0.84	0.41	0.39	-2.81	0.51	-0.30
Taggart	2.11	3.32	4.40	9.13	0.90	0.55	0.00	-0.16	0.57	-0.46
Campbell	2.02	3.25	4.26	10.40	0.84	0.51	-0.02	0.15	0.66	-0.41
Steele	2.24	3.65	4.39	10.21	0.86	0.58	-0.43	-0.41	0.74	-0.46
McMullen	2.22	3.37	5.35	10.10	0.90	0.56	-0.61	-0.40	0.61	-0.35
UMcAlpine	2.04	3.55	5.53	13.51	0.79	0.42	-2.48	2.17	0.63	-0.38
Irvins	2.52	4.32	8.37	11.74	0.89	0.42	-3.84	0.07	0.78	-0.41
Stewart	0.96	2.47	0.84	12.90	0.12	0.37	-0.23	-0.25	0.26	-0.02
Coffey	2.15	2.94	7.34	8.44	0.85	0.54	-1.05	0.21	0.64	-0.41
LSugarM	1.57	2.95	2.06	11.55	0.90	0.55	-0.43	0.73	0.37	-0.31
LBriar	1.74	3.30	3.13	13.77	0.87	0.51	-0.87	1.13	0.56	-0.32
SixMile	2.23	2.59	6.29	6.42	0.82	0.38	-1.34	1.23	0.69	-0.31
UIrwin	1.36	2.70	2.65	14.43	0.69	0.53	-0.32	1.77	0.45	-0.22
MMcAlpine	2.00	3.19	5.42	10.30	0.84	0.50	-1.51	0.66	0.68	-0.38
LSugarA	1.16	2.28	8.62	12.04	0.44	0.51	0.77	0.40	0.33	-0.26
LSugarP	1.04	2.10	1.52	9.20	0.49	0.58	-0.71	-1.04	0.33	-0.33
LMcAlpine	2.16	2.84	6.56	7.65	0.88	0.50	-1.63	0.27	0.50	-0.32

Table 4. Summary statistics of time series for flows and inter-amount times, at 24 hour sampling scale: coefficient of variation (CV), skewness (skew) and medcouple (mc), for three sets of connected subbasins in the Charlotte catchments: Irwin, Little Sugar and McAlpine

ID	name	CV IAT	CV flow	skew IAT	skew flow	mc IAT	mc flow	skew dIAT	skew dflow	mc dIAT	mc dflow
970	Stewart	0.96	2.47	0.84	12.90	0.12	0.37	-0.23	-0.25	0.26	-0.02
300	UIrwin	1.36	2.70	2.65	14.43	0.69	0.53	-0.32	1.77	0.45	-0.22
825	UBriar	1.95	3.69	4.91	14.79	0.84	0.41	0.39	-2.81	0.51	-0.30
022	LBriar	1.74	3.30	3.13	13.77	0.87	0.51	-0.87	1.13	0.56	-0.32
409	LSugarM	1.57	2.95	2.06	11.55	0.90	0.55	-0.43	0.73	0.37	-0.31
507	LSugarA	1.16	2.28	8.62	12.04	0.44	0.51	0.77	0.40	0.33	-0.26
530	LSugarP	1.04	2.10	1.52	9.20	0.49	0.58	-0.71	-1.04	0.33	-0.33
562	Campbell	2.02	3.25	4.26	10.40	0.84	0.51	-0.02	0.15	0.66	-0.41
255	UMcAlpine	2.04	3.55	5.53	13.51	0.79	0.42	-2.48	2.17	0.63	-0.38
975	Irvins	2.52	4.32	8.37	11.74	0.89	0.42	-3.84	0.07	0.78	-0.41
600	MMcAlpine	2.00	3.19	5.42	10.30	0.84	0.50	-1.51	0.66	0.68	-0.38
750	LMcAlpine	2.16	2.84	6.56	7.65	0.88	0.50	-1.63	0.27	0.50	-0.32

Table 5. Minimum and maximum observable scales (in hours), flashiness index for 15 min observation time (in hours) and fitted multifractal parameters α and C_1 for inter-amount times respectively flows.

ID	min scale	max scale	flash	RB	min R2 IAT	min R2 flow	alpha IAT	alpha flow	C1 IAT	C1 flow
UBriar	13.75	1462	128.75	1.15	0.999	0.994	1.05	1.53	0.21	0.35
Taggart	12.50	1443	118.75	1.22	0.999	0.993	0.88	1.30	0.26	0.36
Campbell	9.25	1360	106.00	1.17	1.000	0.993	1.01	1.45	0.24	0.33
Steele	9.50	1457	57.25	1.21	1.000	0.991	0.86	1.30	0.25	0.36
McMullen	11.00	2420	92.25	1.25	0.999	0.992	0.94	1.32	0.26	0.32
UMcAlpine	10.00	1367	165.00	0.99	1.000	0.990	1.24	1.59	0.19	0.33
Irvins	13.75	1367	102.75	1.14	0.999	0.991	1.25	1.40	0.22	0.35
Stewart	6.25	1284	64.00	0.82	1.000	0.994	0.72	2.06	0.09	0.24
Coffey	4.75	1422	26.25	1.09	0.999	0.997	1.53	1.37	0.21	0.28
LSugarM	7.50	1752	48.75	1.16	1.000	0.996	0.66	1.48	0.20	0.33
LBriar	6.75	1658	61.50	1.12	1.000	0.996	0.88	1.51	0.20	0.31
SixMile	3.00	672	12.50	0.97	0.999	0.995	1.64	1.36	0.21	0.26
UIrwin	5.00	2420	55.25	0.97	1.000	0.995	1.14	1.81	0.14	0.25
MMcAlpine	5.50	2420	30.75	1.09	1.000	0.996	1.28	1.46	0.20	0.28
LSugarA	3.50	2420	30.75	0.85	0.995	0.996	2.89	1.89	0.07	0.22
LSugarP	2.75	1532	18.00	0.83	1.000	0.996	1.37	1.87	0.09	0.20
LMcAlpine	3.00	2420	15.75	0.98	0.999	0.997	1.64	1.32	0.19	0.24