RESPONSE TO COMMENTS OF REVIEWER #2 ON HESS-2016-456 "GOVERNING EQUATIONS OF TRANSIENT SOIL WATER FLOW AND SOIL WATER FLUX IN MULTI-DIMENSIONAL FRACTIONAL ANISOTROPIC MEDIA AND FRACTIONAL TIME" by M. L. Kavvas et al.

The authors thank Reviewer #2, Prof. T. Yamada, for his insightful and constructive comments. Responses to the two particular issues that are raised by him are given below.

1. " As the paper mentioned, fractional differential equation is an important approach to explain the non-Fickian dispersion in transport phenomena. It would be helpful if the authors give some details about whether the new govern equations can simulate the dispersion well or not."

Authors' Response:

In the context of solute transport in heterogeneous porous media Meerschaert, Benson, Baumer, Schumer and their co-workers (Meerschaert et al. 1999, 2002; Benson et al. 2000a,b; Baumer et al. 2005; Schumer et al. 2001), have shown by theoretical and numerical studies that the fractional advection-dispersion equation (fADE) which has fractional derivative powers both for the spatial derivatives and time derivatives in the governing equation of transport, has a nonlocal structure that can model well the heavy tailed non-Fickian dispersion. However, very few studies (mentioned in the paper) have addressed the underlying porous media flow in fractional time-space. The previous studies, cited in the paper, on the governing equations of soil water flow only treat time with fractional dimension, while keeping space with integer dimension. Furthermore, these previous studies address only one spatial dimension. Accordingly, our study attempts to develop a fractional continuity equation and a fractional water flux (motion) equation for unsaturated soil water flow in both fractional time and in multi-dimensional fractional space as the model for time-space fractional soil water flow that will provide the necessary flow information to the above-mentioned fADE non-Fickian model of transport in the case of soil media. At the moment we are working on the numerical application of the developed fractional soil water flow equations, and hope to complete this numerical application by the end of 2017. We hope that this numerical application will shed light into the question whether the dispersion can be simulated better by means of the new fractional governing equations, developed in our study.

2. "The authors suggested that some former work had been done in the same topic like He (1998), but He’s govern equation is not dimensionally-consistent. It would be interesting if the authors explain the difference between their work and He’s, and how they solve the dimensionally-consistent problem."

Authors' Response:

He (1998) was the first scholar who proposed a fractional form of Darcy's equation for water flux in porous media. Based on this fractional water flux equation, in his pioneering work He (1998) then proposed a fractional governing equation of flow through saturated porous media. However, the main objective of He's study was to develop a variational iteration method for the solution of fractional differential equations, which he successfully accomplished in his paper. Hence, he proposed a fractional form of the water flux in porous media rather than deriving it. He (1998) expressed the fractional Darcy flux as:

$$q_i(\overline{x},t) = -K_i(\overline{x},\theta)\frac{\partial^{\beta_i} h(\overline{x},t)}{(\partial x_i)^{\beta_i}} \qquad , 0 < \beta_i < 1 , \text{ i } =1,2,3 \tag{1}$$

where q_i is the water flux within the porous medium in the i-th direction (i=1,2,3), K_i is the hydraulic conductivity of the porous medium in the i-th direction, h is the hydraulic head in the i-th direction, x_i is the displacement in the i-th direction, and β_i is the fractional power of the hydraulic gradient in the i-th direction. Denoting the length dimension by L, and the time dimension by T, for i = 1,2,3, the dimension of q_i is L/T, the dimension of K_i is L/T, and the dimension of h is L. Hence, the fractional derivative $\frac{\partial^{\beta_i} h(\overline{x},t)}{(\partial x_i)^{\beta_i}}$ of the hydraulic head h has the dimension of $L^{1-\beta_i}$, that is, length to the power (1- β_i), i=1,2,3. As such, the dimension of the left-hand-side (LHS) of Eqn. (1) is L/T while the dimension of the right-hand-side (RHS) of Eqn. (1) is $L^{2-\beta_i}/T$, for i = 1,2,3. The fractional soil water flux equation that was derived in our study is in the form:

$$q_i(\bar{x},t) = -K_i(\overline{x},\theta)\frac{\Gamma(2-\beta_i)}{{x_i}^{1-\beta_i}}\frac{\partial^{\beta_i} h(\overline{x},t)}{(\partial x_i)^{\beta_i}}, \qquad 0 < \beta_i < 1 , \text{ i } = 1,2,3 \tag{2}$$

Performing a dimensional analysis on our soil water flux equation (2), one obtains for the left and the right hand sides of the equation respectively,

$$[q_i(\bar{x},t)] = {}^{L}/_{T} \qquad \text{and} \qquad \left[K_i(\overline{x},\theta)\frac{\Gamma(2-\beta_i)}{{x_i}^{1-\beta_i}}\,\frac{\partial^{\beta_i}h}{(\partial x_i)^{\beta_i}}\right] = \frac{L}{T}\frac{L}{L^{1-\beta_i}L^{\beta_i}} = \frac{L}{T} \quad .$$

At the moment we are working on the numerical application of the developed fractional soil water flow equations, and hope to complete this numerical application by the end of 2017. We hope that this numerical application will shed light into the applicability of the developed fractional equations to the solution of soil water flow problems in fractional time and multi-fractional soil space.

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