RESPONSE TO COMMENTS OF REVIEWER #1 ON HESS-2016-456 "GOVERNING EQUATIONS OF TRANSIENT SOIL WATER FLOW AND SOIL WATER FLUX IN MULTI-DIMENSIONAL FRACTIONAL ANISOTROPIC MEDIA AND FRACTIONAL TIME" by M. L. Kavvas et al.

The authors thank Reviewer #1 for the insightful and constructive comments. Responses to the particular issues that are raised by the reviewer are given below.

REVIEWER #1:

The Reviewer #1 comments "I am also curious that the physical meanings of the fractional governing equation. If the authors have any idea or even speculation about them, it would be very interesting to hear."

Authors' Response:

The physical meaning of the fractional governing equation may be explained most easily in the case of vertical soil water flow. In the context of upstream-to-downstream vertical soil water flow from the soil surface downward, in the integer form of the soil water flow mass conservation equation (the traditional equation) the time rate of change of mass within the control volume grid (i) is determined by the mass flux coming from the upstream neighbour grid (i-1) into (i), and the mass flux that is moving from the control volume grid (i) to the downstream neighbour grid (i+1). This framework holds also for the soil water flow in the two horizontal directions. As such, the mass evolution in the case of the integer governing equation of soil water flow is local (at the scale of the specific computational grid), only due to interaction among neighbouring computational grids. On the other hand, in the case of the fractional governing equation of mass of vertical upstream-to-downstream soil water flow from the soil surface downward, the time rate of change of mass within the control volume grid (i) is due to fluxes of mass coming not only from the immediate nearest upstream neighbour grid (i-1) but from all possible upstream grids (i-1, i-2, i-3,…) (representing the intervals (x-Δx, x), (x-2Δx, x-Δx), (x-3Δx, x-2Δx)…) into control volume grid (i). As such, the fractional governing equations of soil water flow are nonlocal. In fact, in our paper the Caputo fractional derivative

$$\frac{\partial^{\beta} f(x)}{(\partial x)^{\beta}} = D_0^{\beta} f(x)$$

that is used for the fractional derivatives in the fractional governing equations of soil water flow in fractional time-space, is defined by (Odibat and Shawagfeh, 2007; Momani and Odibat, 2008; Podlubny, 1999),

$$D_0^{\beta} f(x) = \frac{1}{\Gamma(1-\beta)} \int_0^x \frac{f^{`}(\xi)}{(x-\xi)^{\beta}} d\xi \qquad 0 < \beta < 1, \quad x \geq 0.$$

As such, the Caputo fractional derivative superimposes nonlinearly (with weights $(x-\xi)^{-\beta}$) the effects of the local derivatives $f^{`}(\xi)$ at each location ξ in the interval (0,x) to the location x. Within this framework, for example, in the case of one-dimensional downward vertical soil water flow in fractional time-space, the effect of the upstream boundary condition is still accounted for at any depth x below the soil surface by means of the fractional spatial derivatives that appear in the corresponding governing equation (Equation (39) or Equation (46) in the paper).

REFERENCES:

Momani, S. and Odibat, Z.: A novel method for nonlinear fractional partial differential equations: Combination of DTM and generalized Taylor's formula, J. of Computational and Applied Mathematics, 220, pp.85-95, 2008.

Odibat, Z.M., and Shawagfeh, N.T.: Generalized Taylor formula, Appl. Math. Comput., 186(1), 286–293, 2007.

Podlubny, I.: *Fractional Differential Equations*, Academic Press, San Diego, 340pp, 1999.