~~Effect of restoration vegetation on the stochasticity of soil erosion in a semi-arid environment~~

An integrated probabilistic assessment to analyze stochasticity of soil erosion in different restoration vegetation

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Abstract:

The interaction between vegetation and soil erosion is a core problem in ecohydrological research. Although the effects of vegetation on soil erosion have been widely studied, the stochasticity of soil erosion in restoration vegetation types in water-limited environment is less investigated. Based on monitoring soil erosion over five rainy seasons, we employed probabilistic trait analysis framework (OCIRS-Bayes) to assess the stochasticity of runoff and sediment generation in three typical restoration vegetation types (*Armeniaca sibirica* (T1), *Spiraea pubescens* (T2) and *Artemisia copria* (T3)) in the Loess Plateau of China, and applied binomial and Poisson distribution functions to predict the probability distribution of erosion random events. The results indicated that, in OCIRS-Bayes system, 130 rainfall events were subdivided into four types. Two types with relative high average precipitation (27.6 and 69.0 mm respectively) could cause larger probability of soil erosion in all vegetation types than other type with average precipitation being 5.0 mm. Under the same rainfall condition, T1 with largest crown structure have lowest average probability of runoff (23.1 %) and sediment (10 %) generation; T2 with thicker litter layer and denser root system have moderate runoff (34.6 %) and sediment (14.6 %) occurrence probability; the probability of runoff (34.6 %) and sediment (25.4 %) generating in T3 were relative higher. The probability distribution of numbers of times soil erosion events in all restoration vegetation could be well predicted by binominal and Poisson probabilistic models, however, parameter analysis implied that Poisson model is more suitable for predicting stochasticity of soil erosion over larger temporal scale. This study could be meaningful to apply more effectively restoration on protecting the soil and water resources in the water-limited environment.

Stochasticity of soil erosion reflects the variability of soil hydrological response to precipitation under complex environment. Assessing this stochasticity is important to conserve soil and water resources, however stochasticity of erosion event in restoration vegetation types in water-limited environment is less investigated. In this study, we constructed an event-driven framework to quantify the stochasticity of runoff and sediment generation in three typical restoration vegetation types (*Armeniaca sibirica* (T1), *Spiraea pubescens* (T2), and *Artemisia copria* (T3)) at closed runoff plot over five rainy seasons in the Loess Plateau of China. The results indicated that, under the same rainfall condition, the average probabilities of runoff and sediment in T1 (3.8% and 1.6%) and T3 (5.6% and 4.4%) were lowest and highest, respectively. The Binomial and Poisson probabilistic model were two effective ways to simulate the frequencies distribution of times of erosion events occurring in all restoration vegetation. The Bayes model indicated that relative longer duration and stronger intensity rainfall events respectively become the main probabilistic contributors of one stochastic erosion event occurring in T1 and T3. Logistic regression modeling highlighted that the higher-grade rainfall intensity and canopy structure were as two most important factors to respectively improve and restrain the probability of stochastic erosion generation in all restoration vegetation types. Bayes, Binomial, Poisson and logistic regression models

constituted an integrated probabilistic assessment to systematically simulate and evaluate soil erosion stochasticity. It may be an innovative and important complement in understanding of soil erosion from stochasticity view, and also provide an alternative to assess the efficacy of ecological restoration on conserving soil and water resource in a semi-arid environment.

Key words:~~ stochasticity~~ stochasticity of soil erosion~~,~~ restoration vegetation, soil erosion, ~~Poisson distribution, ~~Binomial and Poisson, logistic regression model,

1. Introduction

~~The climate change and anthropogenic activities accelerate soil erosion triggering soil deterioration, and degrading terrestrial ecosystem over worldwide (Marques et al., 2008;Portenga and Bierman, 2011). The stochasticity of soil erosion reflects the effect of environmental elements such as stochastic rainfall on the erosive variability (Kim. J et al., 2016). As one of important environment factors, vegetation plays an important role on disturbing the impact of rainfall on soil erosion. The interaction between plant and erosion processes is still a research frontier in ecohydrology (Ludwig et al., 2005;Rodríguez-Iturbe et al., 2001). Actually, how plant affect the stochasticity of soil erosion implies the risk of erosion generation in complex natural conditions. Exploring the effect is meaningful to assessing the efficacy of soil control practices as well as~~

corresponding ecosystem service in semi-arid regions (Fu et al., 2011).

Soil erosion is one of globe environmental problems. In the recent centuries, the erosion rate over worldwide has been accelerating by the climate change and anthropogenic activities, causing soil deterioration and terrestrial ecosystem degradation (Jiao et al., 1999; Marques et al., 2008; Fu et al., 2011; Portenga and Bierman, 2011). The uncertainty and intensity of soil erosion constitute the main feature of erosive phenomenon. Although many studies have been concentrating on the intensity of erosion under different spatiotemporal scales (Cantón et al., 2011; Puigdefábregas et al., 1999), the uncertainty of soil erosion generation is another challenge of researchers expecting to improve the accuracy of erosion prediction. To some extent, the stochasticity of environment and spatiotemporal heterogeneity of soil loss mainly affected the randomness of runoff production and sediment transportation in natural conditions (Kim. J et al., 2016). Meanwhile, the complex mechanism of erosion generation also increased the uncertainty and variation of erosion processes (Sidorchuk, 2005, 2009). Therefore, how to effectively describe the erosive stochasticity and to reasonably assess its impacting factors is necessary and important for understating soil erosion science from the perspective of randomness.

The stochasticity approach based on probability theory is a crucial tool to describe the random phenomenon and their ecohydrologic effects in natural condition. Precipitation is one of most important source of environmental stochasticity to directly affect the uncertainty of soil erosion. As early as 1978, Eagleson, (1978) applied probabilistic trait methods to simplify the randomness of rainfall event. He predicted the distribution of annual precipitation from observed storm sequences by Poisson and Gamma probability distribution functions. Due to the obvious disturbance of rainfall events on environment, especially on the water-limited condition, many hydrological responses which are closely related to rainfall has also expressed different randomness, and indicated by various probabilistic models. For instance, Verma et al, (2011) applied probabilistic methods to assess the influence of daily precipitation distribution on dynamic of soil moisture. Rodriguez-Iturbe et al, (1999) described the dynamics of soil moisture by probability distribution functions depending on water balance at point scale. Wang and Tartakovsky, (2011) employed probability density function to reduce the complexity of infiltration rate in heterogeneous soils. Additionally, the susceptibility of some disasters trigged by some extreme rainfall events – such as flood (Mouri et al., 2013), slope instability (Li et al., 2014), and landslide (Ya and Chi, 2011) – have also assessed by probabilistic models.

First, the combination of various probabilistic, conceptual and physical models have been reported as different integrated approaches to describe the stochasticity of soil erosion intensity (Table 1). As one form of erosion intensity, the runoff processes was proved as a stochastic process by different mathematic simulation models. Some studies (Moore, 2007; Janzen and McDonnell, 2015) have also simulated the stochastic processes, and further quantified the randomness of runoff production and its connectivity dynamics in hillslope and catchment scales by using different probabilistic distribution functions and conceptual models. In these studies, the theory-driven conceptual models simplified main hydrological behaviors related to runoff production, highlighting the stochastic effects of infiltration and precipitation on runoff processes. Based on above precondition, the data-driven probabilistic models further simulated the stochastic runoff production by mapping or calibrating the difference between observed and predicted probabilistic values. As a results, the stochastic-conceptual approaches have formed an effective framework to model the rainfall-runoff processes (Freeze, 1980), as well as to assess flood forecasting (Yazdi et al., 2013)

~~As to the soil erosion which is typical hydrological response of soil to rainfall, Moore, (2007) predicted runoff production through probability models of soil storage capacity, and Sidorchuk, (2005, 2009) combined the probabilistic and deterministic soil erosion components to analyze the stochasticity of interaction between soil structure and overflow during erosion process. These probabilistic-trait approaches closely related to the theory of water balance and some typical hydrological assumptions. This optimized the hydrological models to more precisely represent the randomness of hydrological responses, which could more effectively describe complex hydrological processes (Bhunya et al., 2007). However, under the framework of probability theory, there are still few studies to explore the probabilistic method to analyze the stochasticity of soil erosion. Especially, little effort has been made to systematically investigate how the signal of stochastic rainfall is transmitted to soil erosion in different restoration vegetation types based on observational data rather than on other model assumptions. In fact, this investigation deriving from specific experiment results probably have more practical meaning for understanding the stochastic interaction between rainfall and erosion.~~

The stochasticity of soil erosion rate which is another pattern of erosion intensity was generally investigated by probabilistic and physical models in some studies. The theory-driven physical models in these studies (Sidorchuk, 2005) integrated hydrological and mechanical mechanism of overflow and soil structure with sediment transpiration processes, stressing the stochastic effect of physical principles on erosion rate in different spatial scales (Table 1). Especially Sidorchuk in 2005 further introduced stochastic variables and parameters into probabilistic models by randomizing the physical properties of overflow and soil structure. This approach developed the understanding of uncertainty of sediment transpiration processes, leading the randomness simulation to be better fit the reality of stochastic erosion rate (Sidorchuk, 2009). Additionally, the stochasticity of soil erosion rate also reflected the erosion risk which was assessed by the integration of theory-driven empirical model with geostatistics (Jiang et al., 2012; Wang et al., 2002; Kim. J et al., 2016). Erosion risk analysis generally concentrated on the uncertainty or variability of soil erosion rate in catchment and regional scales. It highlighted the impact of the spatiotemporal heterogeneous rainfall and other environment conditions on the stochastic erosion rate. In a word, these probabilistic and physical models constituted a systematical analysis framework which closely related to the principle of water balance and basic hydrological assumptions. It effectively described the randomness of soil erosion rate affected by complex hydrological processes (Bhunya et al., 2007). However, few studies has been made to analyze the stochasticity of soil erosion events. Especially, there are little effort to systematically investigate how the signal of stochastic rainfall is transmitted to erosion event occurring in different restoration vegetation types based on observational data rather than on other model assumptions. In fact, this event-based investigation deriving from specific experiment results probably have more practical meaning for understanding the stochastic interaction between rainfall and erosion events.

Morphological structures of plant including canopy structure, root system, and litter layer formation were endowed with controlling-erosion functions (Gartner, 2007;Jost et al., 2012;Wang et al., 2012;Woods and Balfour, 2010). Due to these function, vegetation acts as an important role on reinfiltrating overland flow, storing runon and restructuring sediment fluxes (Ludwig et al., 2005;Moreno-de las Heras et al., 2010). This significantly restricts the capacity of surface flow for delivering erosive particle out of a soil-plant system during rainfall processes (Bautista et al., 2007;Puigdefábregas, 2005). How vegetation affects soil erosion was also further interpreted and predicted by some conceptual and empirical models (Kumar and Kushwaha, 2013;Mallick et al., 2014;Prasannakumar et al., 2011). Both of vegetation-driven spatial heterogeneity (VDSH) (Bautista et al., 2007) and trigger-transfer-reserve-pulse (TTRP) (Ludwig et al., 2005) conceptual frameworks have stressed the driving role of vegetation on controlling erosion. Wischmeier and Smith, (1978) defined the land use conditions as a factor in universal soil loss equation (USLE) to imply the importance of vegetation on predicting erosion module. However, the effect of vegetation on stochasticity of soil erosion was less studied. Theoretically, soil erosion generation triggered by the stochastic precipitation, indispensably expressed the randomness. This ubiquitous property in hydrological processes could also be affected by the hydrological function of plant. Therefore, the application of stochasticity method on analyzing the interaction between plant and soil erosion, could be meaningful to understand the mechanism of erosion generation as well as to improve the accuracy of prediction.

Secondly, the probabilistic approaches have also been reported as a crucial tool to describe the stochasticity of factors affecting soil erosion rate (Table 1). The randomness of soil water content (Ridolfi et al., 2003), antecedent soil moisture (Castillo et al., 2003), infiltration rate (Wang, P and Tartakovsky 2011) and soil erodibility (Wang et al., 2001) in heterogeneous soil types were all modelled by different probability distribution functions. These stochasticity of soil hydrological characteristics related to erosion rate mainly acted as various roles on impacting the spatiotemporal distribution of erosion rate especially generating in regional or even larger spatial scales. Meanwhile, as the main driving force of soil erosion generation, the uncertainty of rainfall event, to some extent, represents the environment stochasticity (Andres-Domenech et al., 2010). Eagleson in 1978 applied probabilistic-trait models to characterize the stochasticity of rainfall event by using Poisson and Gamma probability distribution functions. The stochastic rainfall distribution in different spatiotemporal scales has also been applied to examine the effect of runoff and sediment yield (Lopes, 1996), to calibrate the runoff-flood hydrological model (Haberlandt and Radtke, 2014), as well as to evaluate the sewer overflow in urban catchment (Andres-Domenech et al., 2010).

It has been well recognized the role of spatial distribution of vegetation in controlling the soil erosion rate under different spatiotemporal scales (Wischmeier and Smith, 1978; Puigdefabregas, 2005). How the plants reduce soil erosion rate was also illuminated and interpreted by various physical and empirical models (Liu, 2001; Mallick et al., 2014; Prasannakumar et al., 2011). In theory, Puigdefábregas in 2005 proposed Vegetation-Driven-Spatial-Heterogeneity (VDSH) to explain the relationship between vegetation patterns and erosion fluxes, which improves the understanding of hydrological function of plant on erosion processes. Moreover, Trigger-Transfer-Reserve-Pulse (TTRP) framework proposed by Ludwig in 2005, systematically explored the responses and feedback between vegetation patches and runoff-erosion during whole ecohydrological processes. Theoretically, the stochastic signals of different rainfall events could also be disturbed by the hydrological function of plant, which finally affects the randomness of runoff and sediment events occurring in various vegetation types. However, little effort has been made to explore the effect of different vegetation types on the stochasticity of corresponding soil erosion events. In particular, less approaches have been used to analyze how the properties of rainfall, soil and vegetation impact on the stochastic erosion events through event-based investigation deriving from observational data rather than on theory-based models. Actually, logistic regression modeling (LRM) probably deal with above problems. LRM evaluates the causal effects of categorical variables on dependent variables, and quantifies the probabilistic contribution of influencing factors on the randomness of responsive random events in terms of odds ratio (Hosmer et al., 2013). It could be regarded as another probabilistic model to explore the probability-attribution of influencing factors. However, little literature is available on making LRM to explore the probabilistic attributing of stochastic erosion events under complex environmental conditions.

In this study, we monitored soil erosion in three typical restoration vegetation types over five years' rainy seasons in the Loess Plateau of China, and aim to (1) construct assessment frameworks to characterize the random events in stochastic environment, (2) investigate how the stochastic signal of rainfall transmit into soil erosion in different restoration vegetation types; and (3) assess the effect of probability modellings on predicting the stochasticity of soil erosion in vegetation types. By exploring the stochastic property of soil erosion from more comprehensive and objective aspects, this study could enrich the methodology of sensitivity analysis of soil erosion, and probably be meaningful for the selection of reasonable restoration vegetation for conserving the soil and water resources in the Loess Plateau, China.

Table 1

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2. Materials and methods

2. Method

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2.1 Study region description

2.1 Definition and classification of random events

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Each observed stochastic weather condition with different durations in field monitoring period was defined as a random experiment. All possible outcomes of a random experiment constituted a sample space (Ω) defined as a random observational event (short for O event). Two mutually exclusive random event types—random rainfall event (short for I event) and random non-rainfall event (short for C event)—constituted the O event. Precipitation is a necessary condition of runoff generation, and the random runoff production event (short for R event) is a subset of I event. Similarly, R event is also a necessary condition of random sediment migration event (short for S event), which lead to S event be a subset of R event. As a result, O, C, I, R, and S events constituted a random events framework (OCIRS) to reflect the stochasticity of environment in which soil erosion events generation.

The random event duration in OCIRS is an important property of stochastic weather conditions. In particular, the duration property of I event was closely related to the transmission of stochastic signals of rainfall into the R and S events. According to the rainfall duration patterns in China (Wei et al., 2007), the time interval between two adjacent individual I events is set to be more than 6 hours, forming the criteria for individual rainfall classification. Meanwhile, based on the observation of random events over five consecutive rainy seasons, we summarized duration property of all I events and further classified them into four mutually exclusive I event types. They were a random extreme long rainfall event type (short for Ie event), a random general long duration rainfall event type (short for Il event), a random spanning rainfall event type (short for Is event) whose duration spans two consecutive days, and a random within rainfall event type (short for Iw event) generated in a day. Additionally, the C event can also be divided into two types at daily scale. They are random non-rainfall event type lasting a whole day (short for Cd event), and random non-rainfall event type whose duration is less than 24 hours (short for Ch event) which is interrupted by I event.

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Table 2 indicated the physical, probabilistic properties and implications of all random event types in OCIRS. The classification process of all random event types was sketched by figure 1a, the Venn diagram of all random event types in OCIRS was showed in figure 1c. Considering the observed longest duration of Ie event approximating 72 hours, in figure 1b, we summarized a series of random event sequences in terms of different combing patterns of I and C events in every three consecutive days during the whole monitoring period.

Figure 1

Table 2

2.2 Probabilistic description of erosion event

2.2.1 Conditional probability of erosion event

In the sample space Ω, for any random event type *E* in OCIRS, we defined *P(E)* as the proportion of time that *E* occurs in terms of relative frequency:

$$P(E) = \lim_{n\to\infty} \frac{n(E)}{n} = p_E, \quad p_E \in [0,1] \qquad (1)$$

Theoretically, $n(E)$ is the number of times in n outcomes of observed random experiment that the event *E* occurs. According to the law of total probability (Robert et al., 2013), the probability of R event is defined as:

$$P(R) = P(RI) = P(R|\cup_{m=1}^{4} I_m)P(\cup_{m=1}^{4} I_m) = \sum_{m=1}^{4} P(R|I_m)P(I_m) = p_R \qquad (2)$$

I_m, m=1, 2, 3 and 4 represent the Ie, Il, Is, and Iw respectively, and $P(R|I_m)$ represents conditional probability that R event occur given that m^{th} I event type has occurred. Similarly, the probability of S event is defined as:

$$P(S) = P(SI) = P(S|\cup_{m=1}^{4} I_m)P(\cup_{m=1}^{4} I_m) = \sum_{m=1}^{4} P(S|I_m)P(I_m) = p_S \qquad (3)$$

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Equation (2) and (3) quantify the stochastic soil erosion events by using conditional probability.

2.2.2 Probability distribution functions of erosion event

We defined X, Y as two discrete random variables, representing two real-valued functions defined on the sample space (Ω). Let X, Y denote the numbers of times of R and S events occurrence respectively, and assign the sample space Ω to another random variable Z. $X(R) = x, Y(S) = y, Z(\Omega) = z, y \leq x \leq z$. *x, y, z* are integers. The ranges of X and Y are $R_X = \{all\ x: x = X(R), all\ R \in \Omega\}$ and $R_Y = \{all\ y: y = Y(S), all\ S \in \Omega\}$. The probability of x_i or y_j numbers of times of R or S events can be quantified by probability mass function (PMF) as follow:

$$pmf_X(x_i) = P[\{R_i: X(R_i) = x_i,\ \ x_i \in R_X\}] \tag{4}$$

$$pmf_Y(y_j) = P[\{S_j: Y(S_j) = y_j,\ \ y_j \in R_Y\}] \text{ for } i \geq j \tag{5}$$

PMF in Equation (4), (5) describe the general expression of probability distribution of all possible numbers of times of R or S events.

The random variables X, Y obey the Binominal distribution with *n* independent Bernoulli experiments (Robert et al., 2013). Therefore, the PMF of X, and Y can be defined as follow:

$$pmf_{Xbin}(x) = P_{Xbin}(X = x) = \begin{cases} \binom{n}{x} p_R^x (1 - p_R)^{n-x} & x = 0,1,2,\dots,n \\ 0 & elsewhere \end{cases} \tag{6}$$

$$pmf_{Ybin}(y) = P_{ybin}(Y = y) = \begin{cases} \binom{n}{y} p_S^y (1 - p_S)^{n-y} & y = 0,1,2,\dots,n \\ 0 & elsewhere \end{cases} \tag{7}$$

where *x* and *y* indicate all possible numbers of times of R and S occurring over *n* I events. However, when the Bernoulli experiment is performed infinite independent times ($n \to \infty$), the Binomial PMF can be transformed into Poisson PMF (proved by appendix A), and finally expressed as follow:

$$pmf_{Xpoi}(x) = P_{Xpoi}(X = x) = \begin{cases} \dfrac{\lambda_R^x e^{-\lambda_R}}{x!} & x = 0,1,2,\dots \\ 0 & elsewhere \end{cases} \quad (8)$$

$$pmf_{Ypoi}(y) = P_{Ypoi}(Y = y) = \begin{cases} \dfrac{\lambda_S^y e^{-\lambda_S}}{y!} & y = 0,1,2,\dots \\ 0 & elsewhere \end{cases} \quad (9)$$

where the parameter $\lambda_R \approx np_R, \lambda_S \approx np_S$. Equation (6) ~ (9) reflect two PMF models to simulate the probability distribution of R or S events.

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2.3 Probabilistic attribution of erosion events

2.3.1 Bayes model

Based on the Bayes forumula theroy (Sheldon, 2014), if we want to evaluate how much the probabilistic contributions of k^{th} type of random rainfall event on one stochastic runoff or sediment event which has been generated and observed, the Bayes model can quantify the results as follow:

$$P(I_k|R) = \frac{P(I_k R)}{P(R)} = \frac{P(R|I_k)P(I_k)}{\sum_{m=1}^{4} P(R|I_m)P(I_m)} \quad (10)$$

$$P(I_k|S) = \frac{P(I_k S)}{P(S)} = \frac{P(S|I_k)P(I_k)}{\sum_{m=1}^{4} P(S|I_m)P(I_m)} \quad (11)$$

In fact, the Bayes model provides an important explanation that how the priori stochastic information ($P(I_k)$) was modified by the posterior stochastic information ($P(R)$or$P(S)$). The application of Bayes model in equation (10) ~ (11) reflects the feedback of random erosion events on the stochastic rainfall events. It could also be regarded as one pattern of probabilistic attribution to assess the effect of different random rainfall events on the uncertainty of soil erosion events without considering the diversity of restoration vegetation.

2.3.2 Logistic regression model

Firstly, we constructed event-driven logistic function, and defined Y_R and Y_S as two dichotomous dependent variables. When we denoted 1 or 0 to Y_R and Y_S respectively, it means that a R and S event has occurred or not occurred. Given Y_R is a dichotomous dependent variable of R event in linear probability model to be expressed as follow:

$$Y_{R_i} = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_n x_{ni} + \xi_i = \alpha + \sum_{n=1}^{n} \beta_n x_{ni} + \xi_i \qquad (12)$$

Then further transforming equation (12) into conditional probability of R event which has generated in i^{th} observation time as follow:

$$\begin{aligned} P\left(Y_{R_i} = 1 \middle| \cap_{n=1}^{n} x_{ni}\right) &= P\left[\left(\alpha + \sum_{n=1}^{n} \beta_n x_{ni} + \xi_i\right) \geq 0\right] \\ &= P\left[\xi_i \leq \left(\alpha + \sum_{n=1}^{n} \beta_n x_{ni}\right)\right] \\ &= F\left(\alpha + \sum_{n=1}^{n} \beta_n x_{ni}\right) \qquad (13) \end{aligned}$$

α, β are constants, $F(\alpha + \sum_{n=1}^{n} \beta_n x_{ni})$ is the cumulative distribution function of ξ_i when $\xi_i = \alpha + \sum_{n=1}^{n} \beta_n x_{ni}$. Equation (12) and (13) quantified the stochasticity of Y_{R_i} depending on the linear combination of n influencing factors x_n and measurement error ξ under i^{th} observation times of stochastic runoff generation.

Secondly, assuming the probabilistic distribution of ξ_i satisfies logistic distribution and $P\left(Y_{R_i} = 1 \middle| \cap_{n=1}^{n} x_{ni}\right) = p_i$, then the logistic regression modeling (LRM) expression of $Y_{R_i} = 1$ is deduced as follow:

$$p_i = F\left(\alpha + \sum_{n=1}^{n} \beta_n x_{ni}\right) = \frac{1}{1 + e^{-(\alpha + \sum_{n=1}^{n} \beta_n x_{ni})}} = \frac{e^{\alpha + \sum_{n=1}^{n} \beta_n x_{ni}}}{1 + e^{\alpha + \sum_{n=1}^{n} \beta_n x_{ni}}} \qquad (14)$$

Correspondingly, the LRM of $Y_{R_i} = 0$ can be express as:

$$P\left(Y_{R_i} = 0 \middle| \cap_{n=1}^{n} x_{ni}\right) = 1 - p_i = \frac{1}{1 + e^{\alpha + \sum_{n=1}^{n} \beta_n x_{ni}}} \qquad (15)$$

The ratios of equation (14) to (15) is defined as odds of R event:

$$\mathrm{Odds} = \frac{p_i}{1-p_i} = \frac{\frac{e^{\alpha+\sum_{n=1}^{n}\beta_n x_{ni}}}{1+e^{\alpha+\sum_{n=1}^{n}\beta_n x_{ni}}}}{\frac{1}{1+e^{\alpha+\sum_{n=1}^{n}\beta_n x_{ni}}}} = e^{\alpha+\sum_{n=1}^{n}\beta_n x_{ni}},\quad \mathrm{odds}\ \in\ [0,1] \tag{16}$$

In this study, the odds in equation (16) is a probabilistic attribution index to quantify how much the *n* influencing factors to affect the generation of i^{th} stochastic runoff event. Specifically, when the odds of an influencing factor is greater (less) than 1, it means that the corresponding influencing factor exerts positively (negatively) effects on the probability of R generation.

Finally, taking the natural logarithms of the both sides of equation (16), we transform the odds of stochastic runoff event into linear equation (17) reflecting the standard expression of LRM:

$$ln\left[\frac{P\left(Y_{R_i}=1\middle|\bigcap_{n=1}^{n}x_{ni}\right)}{P\left(Y_{R_i}=0\middle|\bigcap_{n=1}^{n}x_{ni}\right)}\right] = ln\left(\frac{p_i}{1-p_i}\right) = \alpha + \sum_{n=1}^{n}\beta_n x_{ni} \tag{17}$$

LRM could be regarded as another probabilistic attribution pattern to evaluate the effect of mutiple impacting factors—such as soil, vegetation, and rainfall—on the randomness of soil erosion events occuring in different restoration vegetation types.

3. Experimental design and data analysis

3.1 Study area

The study was implemented in the Yangjuangou Catchment (36°42'N, 109°31'E, 2.02 km^2) which is located in the typical hilly-gully region of the Loess Plateau in China (Figure ~~1a~~2a). A semi-arid climate in this area is mainly affected by the North China monsoon. Annual average precipitation reaches approximately 533 mm, and the rainy season here spans from June to September (Liu et al., 2012). ~~When the rainy season comes, some high-intensity precipitation more easily cause soil erosion as the Calcaric~~

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~~Cambisol (FAO-UNESCO, 1974) soil type has relative higher potential erodibility. Soil erosion was one of most environmental hazard and cause the ecosystem degradation in the Loess Plateau before 1980s (Wang et al., 2015). And after 1998, as a crucial soil and water resource protection project, the Grain-for-Green Project was widely implemented in the Loess Plateau. A large number of steeply sloped croplands were abandoned, restored or natural recovered by shrub and herbaceous plants(Cao et al., 2009;Jiao et al., 1999). And in the Yangjuangou Catchment, the main restoration vegetation distributed on hillslopes includes *Robinia. pseudoacacia Linn*, *Lespedeza davurica*, *Aspicilia fruticosa*, *Armeniaca sibirica*, *Spiraea pubescens*, and *Artemisia copria*, etc. All the restoration vegetation was planted over 20 years ago.~~ The Calcaric Cambisol soil type (FAO-UNESCO, 1974) with weak structure and higher erodibility in the Loess Plateau is vulnerable to water erosion. For these reasons, soil and water loss was one of most environmental problems to seriously degrade the ecosystem in the Loess Plateau before 1980s (Miao et al., 2010; Wang et al., 2015). After that, as a crucial soil and water resource protection project, the Grain-for-Green Project was widely implemented in the Loess Plateau. A large number of steeply sloped croplands were abandoned, restored or natural recovered by local shrub and herbaceous plants (Cao et al., 2009; Jiao et al., 1999). In the Yangjuangou Catchment, the main restoration vegetation distributed on hillslopes includes *Robinia. pseudoacacia Linn*, *Lespedeza davurica*, *Aspicilia fruticosa*, *Armeniaca sibirica*, *Spiraea pubescens*, and *Artemisia copria*, etc. All the restoration vegetation was planted over 20 years ago.

3.2 Design and monitoring

~~2.2 Experimental design and measurement~~

In the Yangjuangou Catchment, we have had conducted a systematic long-term field experiments, including the monitoring of soil erosion (Liu et al., 2012; Zhou et al., 2016), observation of soil moisture dynamic (Wang et al., 2013; Zhou et al., 2015) and assessment of soil controlling service in this typical water-restricted environment (Fu et al., 2011).

In this study, we first monitored the soil erosion events occurring in three typical restoration vegetation (*Armeniaca sibirica* (T1), *Spiraea pubescens* (T2) and *Artemisia copria* (T3)) from rainy season of 2008 to 2012 (figure 2b). Each restoration vegetation type was designed in three 3 m by 10 m closed runoff-plot distributing on southwest facing hillslopes with a 26.8% aspect. The boundaries of each runoff-plot were perpendicularly fenced by impervious polyvinylchloride (PVC) sheet with 50 cm depth. Collection troughs and storage buckets were installed at the bottom boundary to collect the runoff and sediment (Zhou et al., 2016). Under natural precipitation condition, we recorded the number of times of stochastic runoff and sediment events generating in each runoff-plot over five rainy seasons. Meanwhile, we collected runoff and sediment, and separated them after settling the collecting bottles for 24 hours, dried at 105℃over 8 hours and weighted.

Secondly, we systematically monitored the hydrological properties of soil in different restoration vegetation types. In the rainy season of 2010, the dynamic of soil moisture

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was started to be measured in the study region (Wang et al., 2013). The real-time dynamic data of soil water content with interval of 10 minutes were recorded by the S-SMC-M005 soil moisture probes (Decagon Devices Inc., Pullman, WA), and were collected by HOBO weather station logger (figure 2c). These data provided the information about average antecedent soil moisture (short for ASM) before every rainfall events generating in the two rainy seasons from 2010 to 2012. We further measured the field saturated hydraulic conductivity (short for SHC) in all vegetation types by Model 2800 K1 Guelph Permeameter (Soilmoisture Equipment Corp,. Santa Barbara, CA, USA) to determine the average infiltration capability of soil matrix (figure 2d).

Thirdly, we also investigated the morphological properties of different vegetation types in each runoff-plot for 2-3 times over different periods of rainy season. We measured the average crown width, height and the thickness of litter layer in three restoration vegetation by setting 60×60 cm quadrats in each runoff plot (Bonham, 1989) (figure 2e).

Finally, two tipping bucket rain gauges were installed outside of runoff-plot to automatically record the rainfall processes over the five rainy seasons with an accuracy of 0.2 mm precipitation. Table 3 summarized the properties of four types of random rainfall event, and all the basic characteristic of soil and vegetation was showed in Table 4.

Figure 2

Table 3

Table 4

In the Yangjuangou Catchment, systematic long-term field monitoring experiments were conducted. We have mainly concentrated on the runoff production and sediment yield in designed runoff plots (Liu et al., 2012;Zhou et al., 2016), dynamic of soil moisture in different restoration vegetation (Wang et al., 2013;Zhou et al., 2015), and the ecosystem service assessment in the typical water-restricted environment (Fu et al., 2011). In this study, we monitored the soil erosion in three typical restoration vegetation (*Armeniaca sibirica* (T1), *Spiraea pubescens* (T2) and *Artemisia copria* (T3)) over five years' rainy seasons from 2008 to 2012 (figure 1b). Each restoration vegetation type was designed in three 3 m by 10 m closed runoff plot all of which were distributed on southwest facing hillslopes with a 26.8% aspect. The boundaries of each runoff plot were perpendicularly fenced by impervious polyvinylchloride (PVC) sheet with 50 cm depth. And a collection trough and storage bucket was installed at the bottom boundary to compose the collection transmission system of runoff and sediment (Zhou et al., 2016). Two tipping bucket rain gauges were installed outside of runoff plot to automatically record the precipitation with accuracy of 0.2mm. We counted the number of times of runoff and sediment generation in each runoff plot based on natural precipitation stochastically generating in the experiment area over five rainy seasons. Meanwhile, we stored runoff and sediment in collection-transmission system, separated them after settling the collecting bottles for 24 hours, dried at 105℃over 8 hours and weighted. We further measured the field saturated hydraulic conductivity in three restoration vegetation types by Model 2800 K1 Guelph Permeameter (figure 1c) (Soilmoisture Equipment Corp,. Santa Barbara, CA, USA) to determine the infiltration capability of soil matrix. And visually estimated the restoration vegetation cover by thirty 1 m^2 quadrats distributed over each runoff plot for 2-3 times over different periods of rainy season (figure 1d). At last, we measured the average height, crown width, leaf area index, and the thickness of litter layer in T1 to T3 (Bonham, 1989). More information was showed in table 1.

Figure 1

Table 1

2.3 Analysis framework for erosion stochasticity

2.3.1 Construction of random events system

Each observed stochastic weather condition is defined as a random experiment. All the possible outcomes of a random experiment constitute a sample space (Ω) defined as observation random event (short for O event, the same as follow). O event is subdivided into two mutually exclusive random event types, one is rainfall random event (I event) and the other is non-rainfall random event (C event). Precipitation is a necessary condition of runoff production, therefore, the runoff production random event (R event) is a subset of I event. Similarly, R event is also a necessary condition of sediment migration random event (S event). As a result, S event is contained in R event. Above defined O, C, I, R, and S events could be regarded as five different elements constituting the OCIRS random events system which is a basic framework for quantifying environment stochasticity.

Precipitation is a crucial disturbance environmental factor to transmit their stochastic signals into the R and S events. Therefore, it is necessary to investigate and classify the characteristics of all I events. Firstly, the time interval between two adjacent individual I events is set to be more than 6 hours, which is a criteria for the classification of individual I event according to its duration. And secondly, considering the typical rainfall eigenvalues including precipitation, intensity and duration as well as the main rainfall patterns in the Loess Plateau (Wei et al., 2007), we used Ward's method of hierarchical cluster analysis to classify 130 individual I events into four types (figure 2c). They are I_A events with lowest average precipitation and intensity; I_B events with second largest average precipitation and intensity; I_C events whose average precipitation and duration are largest; and I_D event which was an individual extreme rainfall event. Table 2 summarizes the physical and probabilistic properties all the elements in OCIRS system. Finally, the whole confirming process of all elements in OCIRS system is sketched by figure 2a, and Venn diagrams in figure 2b explored the relationships of all elements in OCIRS. In fact, various combinations of I and C events formed different random event sequences which finally constituted the whole field monitoring period.

Figure 2

Table 2

2.3.2 Quantification of erosion stochasticity

In the sample space Ω, for each random event E which could be regarded as any elements of OCIRS system, we define *P(E)* as the proportion of time that E occurs in terms of relative frequency:

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n} = p_E \qquad (1)$$

Theoretically, $n(E)$ is the number of times in n outcomes of observed random experiment that the event E occurs, and $p_E \in [0,1]$. Let I_m, m=1, 2, 3 and 4 be the I_A, I_B, I_C, and I_D which are mutually exclusive random event types composing I event. According to the law of total probability, the probability of R event *P(R)* is defined as follow:

$$P(R) = P(RI) = P(R|\cup_{m=1}^{4} I_m)P(\cup_{m=1}^{4} I_m) = \sum_{m=1}^{4} P(R|I_m)P(I_m) = p_R \qquad (2)$$

And $P(R|I_m)$ is conditional probability that R event occur given that m^{th} I event type has occurred. Similarly, the probability of S event *P(S)* are showed as follow:

$$P(S) = P(SI) = P(S|\cup_{m=1}^{4} I_m)P(\cup_{m=1}^{4} I_m) = \sum_{m=1}^{4} P(S|I_m)P(I_m) = p_S \qquad (3)$$

Equation (2) and (3) quantify the effect of stochastic signal of rainfall on soil erosion. On the other hand, supposing an R or S event has occurred stochastically, based on Bayes formula, we furtherly deduces two equations as follow:

$$P(I_k|R) = \frac{P(I_kR)}{P(R)} = \frac{P(R|I_k)P(I_k)}{\sum_{m=1}^{4} P(R|I_m)P(I_m)} \qquad (4)$$

and

$$P(I_k|R) = \frac{P(I_kR)}{P(R)} = \frac{P(R|I_k)P(I_k)}{\sum_{m=1}^{4} P(R|I_m)P(I_m)} \qquad (5)$$

Equation (4) and (5) quantify how much the contributions of k^{th} type of I event on a R or S event stochastically generating at month or seasonal scale, which reflect the feedback of soil erosion to rainfall stochasticity. Equation (2)~(5) characterize the interaction of rainfall and erosion by means of probability theory and expression. Consequently, we designs the OCIRS-Bayes framework combining OCIRS system with Bayes method. It systematically describe the stochasticity of soil erosion in different restoration vegetation types through the monitoring experiment, which indicates the interaction of rainfall and soil erosion.

We defined X, Y as two discrete random variables which are real-valued functions defined on the sample space Ω. Let X, Y denote the numbers of times of R and S events occurrence respectively. And let another random variable Z assign the sample space Ω to z. $X(R) = x, Y(S) = y, Z(\Omega) = k, y \leq x \leq z$. x, y, k are integers. The ranges of X and Y are $R_X = \{all\ x: x = X(R), all\ R \in \Omega\}$ and $R_Y = \{all\ y: y = Y(S), all\ S \in \Omega\}$. The probability of x_i or y_j times of R or S events could be quantified by the probability mass function (PMF) as follow:

$$pmf_X(x_i) = P[\{R_i: X(R_i) = x_i,\ \ x_i \in R_X\}] \quad (6)$$

$$pmf_Y(y_j) = P[\{S_j: Y(S_j) = y_j,\ \ y_j \in R_Y\}] \text{ for } i \geq j \quad (7)$$

PMF in equation (6), (7) describe the general expression of probability distribution of all possible numbers of times of R or S events.

Actually, according to the property of Bernoulli experiment (Robert et al., 2013), the random variables X, Y obey binominal distribution. The PMF of X, and Y were defined as follow:

$$pmf_{XBin}(x) = P_{XBin}(X = x) = \begin{cases} \binom{n}{x} p_R^x (1 - p_R)^{n-x} & x = 0,1,2,\dots,n \\ 0 & elsewhere \end{cases} \quad (8)$$

and

$$pmf_{Ybin}(y) = P_{ybin}(Y = y) = \begin{cases} \binom{n}{y} p_S^y (1 - p_S)^{n-y} & y = 0,1,2,\dots,n \\ 0 & elsewhere \end{cases} \quad (9)$$

And the expectation and variance of X and Y are equation (10) and (11):

$$E_{Xbin}[X] = np_R,\ V_{Xbin}[X] = np_R(1 - p_R) \quad (10)$$

$$E_{Ybin}[Y] = np_S,\ V_{Ybin}[Y] = np_S(1 - p_S) \quad (11)$$

where x and y indicate all possible numbers of times of R and S occurring over n independent I events which are also characterized as n Bernoulli experiments. However, when the Bernoulli experiment is performed infinite independent times ($n \to \infty$), the binomial PMF can be transformed into Poisson PMF, which is proved by appendix A. Therefore, equation (8) and (9) can be transformed as follow:

$$pmf_{Xpot}(x) = P_{Xpot}(X = x) = \begin{cases} \frac{\lambda_R^x e^{-\lambda_R}}{x!} & x = 0,1,2,\dots \\ 0 & elsewhere \end{cases} \quad (12)$$

and

$$pmf_{Ypot}(y) = P_{Ypot}(Y = y) = \begin{cases} \frac{\lambda_S^y e^{-\lambda_S}}{y!} & y = 0,1,2,\dots \\ 0 & elsewhere \end{cases} \quad (13)$$

And expectation and variance of X and Y are :

$$E_{Xpot}[X] = V_{Xpot}[X] = \lambda_R \quad (14)$$

$$E_{Ypot}[Y] = V_{Ypot}[Y] = \lambda_S \quad (15)$$

where the parameter $\lambda_R \approx np_R, \lambda_S \approx np_S$. As a result, equation (8)~(11) reflect two PMF models to construct the prediction system of stochasticity of soil erosion.

3.3 Statistics

2.4 Statistics

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We employed nonparametric statistical tests—one-way ANOVA and post hoc LSD—

to determine the significant difference of soil, vegetation and erosive properties in the three restoration vegetation types, and took Spearman's rank correlation coefficients to analyze how the vegetation coverage affect the probability of soil erosion generation under three grouped precipitation types. At last, the maximum likelihood estimator (MLE) and uniformly minimum variance unbiased estimator (UMVUE) (Robert et al., 2013) were explored to compare the suitability of the binomial PMF and Poisson PMF for predicting the uncertainty of runoff and sediment generation over long term.

4. Results

4.1 Environmental stochasticity in different rainy seasons

The probabilistic distribution of random rainfall events (I events) and random non-rainfall events (C events) forms the environmental stochasticity which is a background of stochastic soil erosion generation. In the OCIRS, the stochastic environment at monthly and seasonal scales over five rainy seasons was described by figure 3. From the rainy season of 2008 to 2012, the probability of I event generation firstly increased with the increasing of monitoring period and then decreased in the last two rainy seasons. In the rainy season of 2008, the average probability of I event was lower than other four rainy seasons, with being less than 15%. However, the types of I events was most complex in 2008. The random extreme long rainfall event (Ie event) only appeared in this rainy season, with the probability even reaching to 2.5% On the other hand, the average probability of I event was the highest in the rainy season of 2010, with being larger than 18%. But, there only existed two types of I events (Iw and Is events) in this rainy season. Over the five rainy seasons, the average probability of Iw (12.3%) and Ie (0.8%) events generation were the highest and lowest, respectively. The average probability of Is (1.7%) and Il (1.3%) events ranged between Iw and Ie. The probability of Cd event was higher than Ch in each month of rainy season, with average probability being 54.4% and 29.4%, respectively. Moreover, in the table 3, the difference of average precipitation and duration in the four types of I events was significance. But the average rainfall intensity of Iw and Is events were nearly twice that of Il and Ie events.

Figure 3

3.

3.1 Stochasticity of classified rainfall

The stochasticity of I event in OCIRS system is a direct source of randomness of soil erosion. According to cluster analysis, all I events were classified into four categories including I_A, I_B, I_C and I_D (figure 2c). Firstly, I_A type was characterized as lowest average precipitation (5 mm), intensity (0.015 mm/min) and duration (365 minutes) in the four categories types. The proportion of I_A to all I events reaches to 72% with its higher reoccurrence in each rainy seasons (figure 3). Especially, in 2010, nearly 90% of I events was I_A. However, due to its small rainfall erosivity, the times of R and S events occurring in three vegetation restoration types was lowest under I_A condition (table 3). Secondly, characterized as high average rainfall intensity (0.072 mm/min), I_B

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event has the second higher occurrence probability in each rainy season (figure 3). Even in 2008, the proportion of I_B to all I events (50%) was larger than that of I_A (33%). Although the average probability of I_B event occurrence approximated to 5% in all O events of five rainy seasons, I_B can more easily lead to soil erosion in three restoration vegetation types. Especially, when each I_B event occurred stochastically in five rainy seasons, then it would nearly trigger R event in type 2 and 3 restoration vegetation (table 3). Thirdly, the probability of I_C event with highest average precipitation (69 mm) occurring in each rainy season is 1% in all O events of five rainy seasons. In the rainy season of 2010, there was even no I_C occurrence. However, if each I_C event stochastically generated in rainy seasons, the R event would occurred in all restoration vegetation types. On July, 2008, there was a specific I event with extreme high rainfall intensity (0.78 mm/min) which was classified I_D event. I_D event was very rare, because it was observed one times over five rainy seasons. Under this precipitation condition, soil erosion generated in all restoration vegetation types.

Figure 3

Table 3

4.2 Stochasticity of soil erosion events

4.2.1 Probability of erosion events in vegetation types

The stochasticity of erosion events was quantified by the probability of runoff and sediment generation in three restoration vegetation types (T1, T2 and T3) under monthly and rainy season scales (figure 4). Over the five rainy seasons, the probability of soil erosion occurring in all vegetation types generally decreased with the increasing of monitoring period, and then increased in 2012. At early period of erosion monitoring (2008), the randomness of erosion events is similar, and the probability of R and S event ranged from 6% to 13% and from 3% to 13% respectively. After that, from rainy season of 2009 to 2011, the highest probabilities of erosion events in each vegetation type all concentrated in the July and August of each season. As to runoff production, the average probability of R event in T1 (3.78%) was less than that of T2 (5.60%) and T3 (5.58%) under same precipitation condition. With respect to sediment yield, the average probability of S event in T1 (1.65%) was also the lowest in all restoration vegetation types. Especially, in the last two rainy seasons, there was no S event occurring in T1, but, the average probability of S event in T2 and T3 reached to 1.83% and 3.36% respectively in corresponding rainy seasons. Consequently, affected by the same stochastic signal of rainfall events, T1 and T3 have the lowest and highest probability of erosion event generation over the five rainy seasons respectively.

Figure 4

3.2 Stochasticity of soil erosion in vegetation types

Based on OCIRS system, the stochasticity of soil erosion in three restoration vegetation types (T1, T2 and T3) at month and seasonal scales is described by figure 4. At early period of erosion monitoring, the stochasticity of soil erosion in all restoration vegetation types is similar, with probability of R and S event generation ranging from

6% to 13% and from 3% to 13% respectively. From rainy season of 2009 to 2011, the highest probabilities of soil erosion in each vegetation type all appeared in the middle of rainy season (July and August). However, these probabilities were observed to be different extents of decrease with the increasing of experiment period. As to runoff production, the probability of R event generation in T1 was generally less than that of T2 and T3 under same precipitation condition, with it being less than 7% in the last four rainy seasons. The randomness of R events occurring in T2 and T3 have similar distribution in each month of rainy season. With respect to sediment yield, the probability reduction of S event generating in T1 was more obvious than that of other types, with it being only less than 3% in the last four rainy seasons. Especially, in the rainy season of 2011 and 2012, there was no S event occurrence in T1, however, the corresponding average probability of S event in T2 and T3 was near 1.5% and 4% respectively. Generally, influenced by the same stochastic signal of I events, T1 and T3 have the lowest and highest probability of soil erosion respectively.

According to the Bayes formula, figure 5 indicated that given one R or S event has stochastically generated in some restoration vegetation type at specific month or rainy season, how much the probabilistic contribution of different types of I events on the corresponding soil erosion occurrence. In the rainy season of 2008, as to all restoration vegetation types, the contributing types of I events on soil erosion was more complex than other rainy seasons, but also concentrated on relative high precipitation and intensity classified I events such as I_B, I_C events. With the increasing of experiment duration from 2009 to 2011, the complexity seemed to be reduced, and the probabilistic contribution of I_A event on soil erosion have different extent increase in three restoration vegetation types. If one R event has stochastically occurred in T1, the probabilistic contribution on this runoff production were generally I_B and I_C events, which they ranged from about 50% to 100% and near 20% to100% respectively. And I_A and I_B events have even no probabilistic contribution on one S event occurring on T1 stochastically over the last four rainy seasons. However, I_A and I_B events have been the main probabilistic contributors for one statistical soil erosion generation on T2 and T3, which they ranged from about 10% to 100% and 30% to 100% respectively. Consequently, the contribution pattern of I events on soil erosion in T1 was relative simple and mainly focused on I events type with higher rainfall erosivity than that of in T2 and T3.

Figure 4

Figure 5

4.2.2 Probabilistic distribution of erosion events in vegetation types

More detailed stochastic information of erosion events in different vegetation types was simulated by Binominal and Poisson mass functions (PMFs) under the monthly scales. It also compared the frequencies distribution of different numbers of observed erosion events with the corresponding simulated results by the two PMFs in figure 5. Firstly, as to the detailed stochastic information of R events, the two PMFs generally provided a better simulation to the observation in T1 than that of in T2 and T3. When comparing the simulated and observed values, Binomial PMF supplied better simulation to the observed numbers of time of R events with larger frequency (such as 2~4 time) than that of Poisson PMF. However, Poisson PMF simulated the observed numbers of time of R events with the lower frequency (such as 6~8 times) better than that of Binomial PMFs. Secondly, as to the detailed stochastic information of S event, the two PMFs provided better simulation to the observation in T3 than that of in T1 and T2. In particular, when the number of times of S event generation reaches two in T1 and T2, the corresponding simulated probability values were all nearly 2 times larger than the observed frequencies, reflecting the most simulation error of the two PMFs. Moreover, with the restoration vegetation types changing from T1 to T3, both of the simulated and observed numbers of time of R and S events with largest probability or frequency increased in consistence. In a word, comparing with the observed frequency of numbers of erosion events, both PMFs indicated well-simulating effect to detail the stochasticity of runoff and sediment events under monthly scale.

Figure 5

4.3 Stochastic attribution of soil erosion events

4.3.1 Effect evaluation of stochastic erosion events by Bayes model

The Bayes model was applied to analyze the effect of random rainfall events (including Iw, Is, Il and Ie) on stochastic erosion events in different restoration vegetation types. Specifically, Bayes model evaluated the different probabilistic contributions of four types of I events on one observed erosion event which has stochastically generated in specific vegetation type under monthly and rainy seasonal scales (figure 6). In the rainy season of 2008, the types of I events driving one stochastic erosion event was most complex than other rainy seasons. In contrast, one stochastic soil erosion generation in three vegetation types attributed to only Iw and Is events in the rainy season of 2010. In other three rainy seasons, when one R or S event stochastically generated on T1, the main contributing I event types concentrated on Is and Il events both of which have relatively higher precipitation and longer duration, respectively. On the other hand, if one R or S event occurred in T2 or T3 randomly, the main contributing I event types was Iw event which, however, have no contribution to S event occurred on T1.

In general, over five rainy seasons, the composition of I event driving one R event was more complex than that of driving one S event. The relative longer duration rainfall events (Il and Ie) became the main probabilistic contributors of one stochastic erosion event occurring in T1, and the relative stronger intensity rainfall events (Iw and Is) mainly caused one random erosion event generating in T2 and T3.

Figure 6

4.3.2 Effect evaluation of stochastic erosion events by LRM

According to the results of significant difference analysis in table 4, we defined the properties of soil and plant as ordinal variables, and classified them into four grades (Table 5). Meanwhile, based on previous studies (Liu et al., 2012; Wei et al., 2007) and rainfall properties in this study area, we further subdivided all precipitation and rainfall intensity into four grades with different scores.

First, the intensity of positive and negative effects of single influencing factor on the probability of runoff and sediment generation in all restoration vegetation types was quantified in terms of odds ratio of erosion events by LRM (table 6). In the LRM, the highest and lowest odd ration appeared in rainfall intensity ordinal variable (INT) and average crown width ordinal variable (CRO). The increasing INT and CRO (from middle to extreme grade) significantly increased and decreased the odds ratio of erosion events, respectively. This means that INT and CRO acts as two most important roles on improving and restraining the probability of stochastic erosion generation in all restoration vegetation types. Additionally, the increasing of antecedent soil moisture ordinal variable (ASM) and the filed saturated hydraulic conductivity ordinal variable (SHC) (from middle to high grade) in the LRM, also significantly increased and decreased the odds ratio of R and S events, respectively. However, the average thickness of litter layers ordinal variable (TLL) has not exerted significant effect on the odds ratio of erosion events. Table S-1 and S-2 in supplementary information systematically describe the whole processes of LRM to evaluate the effect of single factor on odds ratio of erosion event.

Secondly, we further applied LRM to evaluate the interactive effects of multiple influencing factors on the odds ratio of R and S events in all restoration vegetation types (table 7). As to the interactive effect of two soil hydrological properties, the interaction between low-grade of SHC and increasing-grade of ASM significantly raised the odds ratio of erosion events. Such that the odds ratio of R and S events affected by the interactive effects of low-grade of SHC and extreme-grade of ASM were respectively 7.02 and 1.82 times larger than that interactive effects of low-grade of SHC and low-grade of ASM. Similarly, as to the effect of two vegetation properties, the interactive effect of low-grade of CRO and increasing-grade of TLL would reduce the odds ratio of erosion events. Such that the odds ratio of R and S events influenced by the interaction between low-grade of CRO and high-grade of TLL were respectively only 0.12 and 0.33 times larger than that interactive effects of low-grade of CRO and low-grade of TLL. Additionally, with respect to the interaction between soil and plant properties, the interactive effect of low-grade of CRO and increasing-grade of ASM properties also significantly raised the odds ratio of erosion events. The whole processes of LRM to evaluate the interactive effect of multiple factors on odds ratio of erosion event were indicated by the table S-3,4 and 5 in the supplementary information.

Table 5

Table 6

Table 7

Table S-1,2,3,4,5

3.3 Prediction of soil erosion stochasticity

We defined ten consecutive stochastic I events as an stochastic environment unit of the background of soil erosion, which indicates that *n* =10 in the binomial and Poisson distribution functions (equation (8~9, 12~13)). Under this assumption, figure 6 describes binomial and Poisson PMFs to predict the probability distributions of numbers of times of soil erosion events in three restoration vegetation types. It also compares the predictions with the frequencies of numbers of times of observed R and S event in vegetation types. Firstly, as to the probability distribution of R event, it seems that the binomial and Poisson PMFs provide a better fit to the observation in T1 than that of in T2 and T3. More specifically, in all restoration vegetation types, binomial PMFs supply better fit to the observed numbers of time of R events with larger frequency (such as 2~4 time) than that of Poisson PMFs. However, Poisson PMFs fit the observed numbers of time of R events with the lower frequency (such as 6~8 times) better than that of binomial PMFs. The frequencies of observed numbers of time of R events in T2 and T3 have similar distribution patterns. Secondly, with respect to probability distribution of S event, the predictions about the observed probability distribution of S events in T1 by both PMFs do not fit very well. Especially, when the frequency of number of times of no-sediment in T1 is nearly two times larger than the corresponding predication of binomial and Poisson PMFs. However, the two PMFs are seemed to provide better fit to the observation in T3 and T2 than that of in T1. With the restoration vegetation types changing from T1 to T3 in figure 6, the predicted or observed numbers of time of R events with largest probability or frequency increased in consistence. Generally, Poisson PMF seems to provide better probability distribution prediction about observed numbers of times of R events in all restoration vegetation types than that of Binomial PMF.

Figure 6

~~4.~~5. Discussion

~~4.1 OCIRS-Bayes framework for erosion stochasticity~~

5.1 The integrated probabilistic assessment to erosion stochasticity

The probabilistic attribution and description of stochastic erosion events constituted the framework of integrated probabilistic assessment (IPA).

First, as to one pattern of probabilistic attribution in the IPA, Bayes model supplies a supplementary view and algorithm about how to evaluate the feedback of a result which had stochastically occurred on all possible reasons (Wei and Zhang, 2013). Under the conditions of insufficient information about an occurred result, Bayes model can determine which reasons have the relative greater probability to trigger the occurrence of the result through some prior information. Specific to this study, Bayes model was used to evaluate the probabilistic contribution of four types of I events on one stochastic R ($P(I_k|R)$) and S ($P(I_k|S)$) event generated in each restoration vegetation. Although there were no more specific information about a stochastic soil erosion event, the prior information ($P(R|I_m), P(S|I_m), P(I_m)$) can provide assistance for us to assess the feedback of the stochasticity of soil erosion on different random rainfall events by Bayes model. Meanwhile, ($P(I_k|R)$) and ($P(I_k|S)$) also reflect the different probability threshold values of four rainfall event types triggering soil erosion. Bayes model integrated with total probability theory to systematically quantify the interactive relationship between the stochasticity of precipitation and soil erosion, forming a relative simple and practicable risk assessment of soil erosion event occurring in complex restoration vegetation conditions.

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Secondly, as a pattern of probabilistic description in the IPA, Binomial and Poisson PMFs are two crucial probabilistic functions to characterize many random hydrological phenomena and to model their ecohydrological effects in natural condition (Eagleson, 1978, Rodriguez-Iturbe et al, 1999, 2001). In this study, the two PMFs were found to have good simulations of the frequency of times of soil erosion events in three restoration vegetation types. However, it is necessary and meaningful for the reliability and accuracy of the IPA to assume whether the two PMFs can both stably and reasonably simulate the erosion stochasticity at closed-runoff-plot over longer monitoring period. Therefore, based on above assumption, two important point estimations methods—the maximum likelihood estimator (MLE) and uniformly minimum variance unbiased estimator (UMVUE) (Robert et al., 2013)—were applied to evaluate the stability of erosion stochasticity estimation by means of analyzing the unbiasedness and consistency of p_R, p_S, λ_R and λ_S. Taking parameter analysis of random runoff event for example, we defined X_i as the number of times of R event occurring in some specific restoration vegetation in i^{th} rainy season (i = 1,2,3,4 and 5). The five independent and identical (*iid*) random variables satisfies the same and mutually independent binomial or Poisson PMFs as follows:

$$\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_5 \overset{iid}{\rightarrow} binomial\ (p_R) \text{ or } \mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_5 \overset{iid}{\rightarrow} Poisson\ (\lambda_R) \quad (18)$$

Considering longer monitoring periods, we supposed that the numbers of corresponding I events (n) and rainy seasons (i) would approach infinity (n, $i \rightarrow \infty$), and (18) can be transformed as follow:

$$\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_i \overset{iid}{\rightarrow} binomial\ (p) \text{ or } \mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_i \overset{iid}{\rightarrow} Poisson\ (\lambda) \quad (19)$$

We take MLE and UMVUE methods to search for the best reasonable population estimators $\hat{p}$ and $\hat{\lambda}$ to approximate the unknown p and λ in (19), and finally obtain more comprehensive stochastic information about the randomness of R event over *i* rainy seasons. The Appendix B proved that the best estimator $\hat{p}$ in Binomial PMF is the unbiasedness and consistency of the MLE of p. However, proved by the Appendix C, the best estimator $\hat{\lambda}$ in Poisson PMF have more reliability as it is not only the unbiasedness and consistency of the MLE of λ, but also the UMVUE of MLE. The UMVUE in Poisson PMF implied that lowest variance unbiased estimator can make the Poisson PMF to be more steadily and accurately stimulate the stochasticity of soil erosion events over long-term observation than binomial PMF.

Thirdly, besides having better simulation of the stochastic soil erosion events at larger temporal scale, the Poisson PMF could also be more suitable for simulating the randomness of S event in the closed-design plot system than that of binomial PMF.

As the hypothesis of Boix-Fayos et al in 2006, the closed runoff-plot design forms an obstruction to prevent the transportable material from entering the close monitoring system, which, in particular, lead the transport-limited erosion pattern to gradually transform into detachment-limited pattern in the closed-plot over time (Boix-Fayos et al., 2007; Cammerraat, 2002). Consequently, with the extension of monitoring period, this closed runoff-plot design would cause the sediment more and more difficult to migrate out of plot, which also reduce the probability of observed S events under the same precipitation condition. In fact, the effect of closed runoff-plot on stochastic sediment event could also be successfully implied by the algorithm of Poisson PMF.

Specifically, in order to satisfying the fact that $\lambda = np$ in Poisson PMF is an unknown constant, the extension of monitoring period could lead to the numbers of times of I events (n) approach infinity, then the probability (p) of R or S events generation have to approach to zero. Above inference coincides with the assumption about the decreasing of sediment generation in closed-plot system, and further proves that Poisson PMF could be more reliable to simulate the stochastic erosion events at longer temporal scale.

The OCIRS designing and Bayes method in this paper constitute an innovative analysis framework for soil erosion study. Environmental stochasticity is an inevitable factors to affect the variability of soil erosion, which is also a non-negligible obstacle for the understanding of soil erosion and its modelling prediction (Kim. J et al., 2016). OCIRS-Bayes framework formed a random event system to evaluate the stochasticity of environment, but also analyze the transmission of stochastic signal of rainfall into soil erosion. In this framework, the stochastic weather conditions were defined as a series random events with various physical and probabilistic meanings, which have direct or indirect relevance to stochasticity of soil erosion (table 2). There also exist many modelling systems to evaluate the effect of influencing factors on soil erosion, and universal soil loss equation (USLE) is a typical one which models intensity of influencing factors to be predicted the erosion module by empirical formula (Wischmeier and Smith, 1978). But, there are less analysis frameworks like OCIRS-Bayes to model the stochasticity of soil erosion and its influencing factors totally

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depending on the long-term experimental data and fundamental probability theories. In order to stressed that the stochastic signals of rainfall events are the most important disturbances and sources of uncertainty and variability of soil erosion, OCIRS-Bayes further subdivides all rainfall events into various subsets (from I_A to I_D event) representing different rainfall erosivities which was similar with the typical rainfall patterns in rainy seasons of the Loess Plateau (Wei et al., 2007). Therefore, OCIRS-Bayes become a more practicable and simplification system to supplement to the studies on evaluating effect of rainfall properties on soil erosion in semi-arid environment.

In this study, OCIRS-Bayes framework discovered that the probability of soil erosion is closely related to the complexity of rainfall event types distributing in rainy season, which affected by the transmission of stochastic signals of high-erosivity rainfall events (such as I_C and I_D). This systematically analyzed how the stochastic signals of different rainfall events transmits to the soil erosion in restoration vegetation types in the water-limited natural condition at different temporal scales (showed in figure 4). Meanwhile, this framework also explored that the only relative high-erosivity rainfall events can make a contribution for the stochastically soil erosion generating in T1, which implied the feedback of rainfall properties to stochasticity of soil erosion. Therefore, the interactive relationship between rainfall and soil erosion under restoration vegetation condition was characterized by OCIRS-Bayes framework. This supplies a new and meaningful aspect to understanding the soil erosion properties especially under the background of climate change transmitting more stochastic and extreme environmental

~~signals into soil-plant system.~~

5.2 The effect of influencing factors on erosion stochasticity

The effects of rainfall, soil and vegetation properties on erosion stochasticity in different restoration vegetation types were evaluated by LRM. It integrated stochastic rainfall events with their precipitation and intensity grades, and connected the ecohydrological functions of soil and plant with their classified hydrological and morphological features. Just as serving as previous studies (Verheyen and Hermy, 2001a, 2001b; Verheyen et al., 2003 and Hermy, 2001a; 2001b; Verheyen et al., 2003), LRM in this study explored the relative importance of morphological features disturbing on the transmission of stochastic signal of I events into R and S events in different restoration vegetation types. These disturbances are closed related to the complex hydrological functions owned by different morphological structures, which finally affect the whole processes of runoff production and sediment yield (Bautista et al., 2007; Puigdefábregas, 2005).

First, many previous field experiments and mechanism models have proved that canopy structure has capacity for intercepting intercept precipitation. This specific hydrological function could potentially prevent the rainfall from directly forming overland flow or splashing soil surface particles (Liu, 2001; Mohammad and Adam, 2010; Morgan, 2001; Wang et al., 2012). The precipitation retention owned by canopy structure was regarded as an indispensable positive factor to reduce the soil erosion rate. Meanwhile, as a crucial complement to understanding hydrological function of canopy structure, the result of LRM in this study indicated that the higher-grade canopy

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structure was a most important morphological feature to reduce the odds ratio of random soil events in all restoration vegetation types. This result suggests that, the larger canopy diameter would have relatively stronger capacity for disturbing the transmission processes of stochastic signal of rainfall on the soil surface than that of other morphological properties. From the perspective of erosion stochasticity, the higher-grade canopy structure could finally attribute to the lower probability of R and S event generation. Therefore, the diversity of canopy structures in different vegetation types could act a key role on both reducing the intensity and probability of soil erosion generation.

Secondly, many studies have also discovered that the denser root system distributing in soil matrix could improve the reinfiltration of the overland (Gyssels et al., 2005). This reinfiltration process is an effective way to recharge soil water stores when the overland flow started to occur in hillslopes, which was also an indispensable contributing factor to reduce the unit area runoff production (Moreno-de las Heras et al., 2009;Moreno-de las Heras et al., 2010). In this study, the potential reinfilitration capacity of soil matrix could be positively affected by the saturated hydraulic soil conductivity (SHC) index. Figure 7 further indicated the distribution patterns of root system in three restoration vegetation types. Meanwhile, the result of LRM also implied that the grade of SHC could negatively affect the odds ratio of stochastic erosion event, which improved the understanding of the hydrological function of root distribution of plant from the view of erosion randomness. It may suggest that the denser root system could create more macropores in the subsurface to provide more probability of reinfiltration of overland flow. This disturbance of overland flow by SHC could reduce the probability of erosion event generation.

Thirdly, the litter layer was proved to act multiple roles on conserving the rainfall, improving infiltration of throughfall, as well as cushioning the splashing of raindrop (Gyssels et al., 2005; Munoz-Robles et al., 2011;Geißler et al., 2012). Therefore, the thicker litter layer in T2 (figure 7) probably has stronger capacity for conserving and infiltrating throughfall, as well as inhibiting splash erosion than that of other restoration vegetation types (Woods and Balfour, 2010). Although the result of LRM indicated that there was no significant correlation between the litter layer thickness (TLL) and the odds ratio of soil erosion (table 6), the interactive effect of TLL and CRO significantly affect the odds ratio of stochastic erosion events (table 7). The interaction result implied that, under the relative low-grade CRO condition, the higher-grade TLL could have stronger disturbance on the transmission of stochastic signals of rainfall to improve the throughfall absorption to reduce the probability of splash or sheet erosion occurrence.

Additionally, table 7 explored more interactive effects of the soil and plant properties on odds ratio of random runoff and sediment event. These explorations suggested that the interactions between soil and vegetation properties formed more complex hydrological functions to affect the stochastic soil erosion event during whole ecohydrological processes in semi-arid environment (Ludwig et al., 2005).

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Although the hydrological-trait of vegetation acted as core roles on reducing the soil erosion depending on the mechanical properties of their morphological structures (Zhu et al., 2015), the LRM analysis in this study further illuminated that these hydrological-trait morphological structure of vegetation may also play an important role on affecting the stochasticity of soil erosion. Actually, the different stochasticity of soil erosion in three restoration vegetation types reflected the different extents of disturbance of vegetation types on the transmission of stochastic signals of rainfall into soil-plant systems. Therefore, the relative smaller canopy structure, thinner litter layer, and shallower root system in T3 have relatively weaker capacity to disturbing the stochastic signal of rainfall than that of T1 and T2 with obvious hydrological-trait morphological structures (figure 7). The effect of diverse morphological structures on stochasticity of soil erosion was a meaningful complement to studying on the hydrological functions of restoration vegetation types in semi-arid environment.

Figure 7

Table 6

Table 7

5.3 The implication of integrated probabilistic assessment

The integrated probabilistic assessment (IPA) could be an important complement to expand on the understanding of hydrological function existing in vegetation types. The hydrological-trait of morphological structures owned by different plants is closely related to the function of vegetation-driven in affecting the intensity of erosion events. The vegetation-driven-spatial-heterogeneity (VDSH) theory (Puigdefábregas, 2005) could be regarded as a clear concise summary to emphasize the dominant role of vegetation in restructuring soil erosion processes. It reflected the effect of spatial distribution patterns of vegetation on their corresponding hydrological functions on controlling erosion rate in patch, stand, and even regional scales. Therefore, VDSH theory has provided an innovative view to investigating the soil erosion and other ecohydrological phenomena affected by vegetation (Sanchez and Puigdefábregas, 1994;Puigdefábregas, 1998;Boer and Puigdefábregas, 2005). In the study, depending on the long-term experimental data and fundamental probability theories, the IPA concentrated on the hydrological function of vegetation-driven in affecting the randomness of erosion events rather than the erosion rate. It could enrich the comprehension of hydrological function of vegetation morphological structure on soil erosion phenomena, and also be effective complement for application of VDSH theory on interpreting the stochastic erosion events.

Additionally, in our study, the IPA could also provide a new framework for practitioners to develop restoration strategies which focused on controlling the risk of erosion generation rather than only on reducing erosion rate. The framework contains three stages including construction of stochastic environment, description of random erosion events, and evaluation of probabilistic attribution (figure 8).

The first stage in the framework aims to build a unified platform to describe the stochasticity of different hydrological phenomena closely related to the erosion event. This stage generally investigates the stochastic background under which soil erosion generation, which is also an indispensable precondition for quantifying the probability of R and S in stage II. The second stage is designed to construct a phased adjustment of monitoring processes based on the principle of Bayes theory as well as on the parameter analysis of Binomial and Poisson models. In this phased-adjustment monitoring, the Bayes, Binomial and Poisson models were applied on simulating the randomness of erosion events in short-term, mid-term and long-term monitoring stages, respectively. This model-driven monitoring approach could be regarded as a more reasonable method to explore the complexity of stochastic erosion events in larger temporal scales, but also provide a new perspective for researchers to more effectively evaluate the stochasticity of erosion events in stage III. The objective of stage III is to assess the probabilistic attribution of rainfall, soil and vegetation properties on erosion events generation. This probabilistic attribution evaluation by LRM, could develop the restoration strategies for more effectively selecting vegetation types with stronger capacity for reducing the erosion risk, and finally improve the management of soil and water conservation in a semi-arid environment.

As a result, this stochasticity-based restoration strategy was developed by a combination of experimental data with multiple probabilistic theories to deal with the soil erosion randomness under complex stochastic environment. It is different from the trait-based restoration scheme derived from the functional diversity of vegetation community to reduce the soil erosion rate (Zhu et al., 2015; Baetas et al., 2009). Meanwhile, with the increase of monitoring duration, more stochastic information of erosion events could be added into the IPA framework. This addition could finally fulfil the self-renewal and self-adjustment of the IPA to improve the restoration strategy for selecting more reasonable vegetation types with stronger capacity for controlling erosion risk in long term. Therefore, the IPA framework containing three stages could translate the event-driven erosion stochasticity into restoration strategies concentrating on erosion randomness, which may be a meaningful complement for restoration management in a semi-arid environment.

Figure 8

6. Conclusion

In this study, we applied an integrated probabilistic assessment (IPA) to describe, simulate and evaluate the stochasticity of soil erosion in three restoration vegetation types in the Loess Plateau of China, and draw the following conclusions:

(1) In the IPA, the OCIRS was an innovative event-driven system to standardize the definition of hydrological random events, which is also a foundation for quantifying the stochasticity of soil erosion events under complex environment conditions.

(2) Both of binomial and Poisson PMFs in the IPA could simulate the probability distribution of the numbers of runoff and sediment events in all restoration vegetation types. However, Poisson PFM could more effectively simulate the stochasticity of soil erosion at larger temporal scales.

(3) The difference of morphological structures in restoration vegetation types is the main source of different stochasticity of soil erosion from T1 to T3 under same rainfall condition. Larger canopy, thicker litter layer and denser root distribution could more effectively affect the transmission of stochastic signal of rainfall into soil erosion.

The IPA is an important complement to developing restoration strategies to improve the understanding of stochasticity of erosion generation rather than only of the intensity of erosion event. It could also be meaningful to researchers and practitioners to evaluate the efficacy of soil control practices in a semi-arid environment.

4.2 Disturbances of vegetation on erosion stochasticity

The different stochasticity of soil erosion in three restoration vegetation types reflects the different extents of disturbance of vegetation types on the transmission of stochastic signals of rainfall into soil-plant systems. These disturbances is closely related to the variety of morphological structure with complex ecohydrological functions affecting the whole process of runoff production and sediment yield (Jost et al., 2012; Wang et al., 2012; Woods and Balfour, 2010). Specifically, the morphological structures including canopy, litter layer and root distribution could have obvious hydrological function to control soil erosion. Firstly, the largest crown diameters of T1 could have stronger interception capacity than that of T2 and T3. Because many studies have proved that canopy structure could have specific capacities for precipitation retention, and prevent rainfall from directly forming overland flow or splashing soil surface particles (Liu, 2001;Mohammad and Adam, 2010;Morgan, 2001), For this reason, the canopy structure of T1 could have stronger capacity to reduce the transmission of stochastic signal of amount and energy of rainfall directly on soil surface, which finally attributed to the relative lower probability of R and S event in T1. This could also probable explained the decreased vegetation coverage significantly correlated with the increased probability of S event in table 4.

Secondly, there was abundant litter material covering on the soil surface of T2 (figure 7), which formed a significant largest average thickness of the litter layer. Many studies also proved that litter layer structure acts multiple roles on conserving the rainfall, improving infiltration of throughfall, as well as cushioning the splashing of raindrop (Gyssels et al., 2005;Johns, 1983;Munoz-Robles et al., 2011;Geißler et al., 2012). For these reasons, the litter layer structure of T2 also have stronger disturbance on the transmission of stochastic signals of rainfall through improving the throughfall absorption to reduce the probability of R event as well as inhibiting the splash or sheet erosion occurrence.

The distribution of root system could be the third important morphological structure to disturb the stochastic signal of rainfall transmitting on soil-plant system. More macropores formed by root system of vegetation types distributing in the soil matrix was proved to improve the reinfiltration of the overland (Gyssels et al., 2005). The reinfiltration process is an important way to recharge soil water stores when the overland flow occurred in hillslopes, but also an indispensable contributing factor to reduce the unit area runoff (Moreno-de las Heras et al., 2009;Moreno-de las Heras et al., 2010). Consequently, showed in figure 7, denser root system distributing the underground of T2 could create more macropores in the subsurface than that of T1 and T3. It reduce the transmission of stochastic signal of rainfall by means of supplying more opportunity to reinfiltrate the potential overland flow into a deep soil layer, and finally decreased the probability of soil erosion in T2.

The interactions between plant and soil erosion in semi-arid environment is a complex ecohydrological processes (Ludwig et al., 2005), which also reflects in the complexity of stochasticity of soil erosion in different restoration vegetation types. However, due to the mechanical characteristics of morphological structures of vegetation having strong negative correlation with soil erosion in this study region (Zhu et al., 2015), these hydrological trait morphological structures of vegetation could be key factors to affect the randomness of soil erosion. Just as in this study, the limited hydrological trait morphological structures – such as relative smaller canopy structure, thinner thickness of litter layer, and shallower root system distribution in soil layer of T3 – more significantly restricted its hydrological functions on intercepting rainfall as well as on conserving overland flow than that of T1 and T2 with obvious canopy structure and thicker litter respectively. As a result, these differences of morphological structures finally lead to the different stochasticity of runoff and sediment in T1 to T3.

Figure 7

Table 4

4.3 Assessment of stochasticity prediction modellings

PMFs of binomial and Poisson are effective probabilistic modellings to predict the stochasticity of soil erosion in restoration vegetation types in semi-arid environment. The binomial and Poisson distribution functions were extensively applied on analyzing the stochastic hydrological phenomenon in natural condition Eagleson (1978). In the

OCIRS-Bayes analysis framework, R and S events were both subsets of sample space composed by I events, therefore, the stochasticity of R and S have close connection with the stochastic signals of I events. In this study, the PMFs of binomial and Poisson indicates relative good predication about probabilistic distribution of soil erosion in all restoration vegetation types over five rainy seasons, however, with the ongoing experiment (supposing the monitoring of soil erosion last for 10 rainy seasons' for instance), whether these two PMFs would still have stable and consistent well-prediction about the stochasticity of soil erosion in T1 to T3, which could be an interesting and important assessment of the two PMFs. Based on above assumption, we compared the temporal effects of prediction in the two PMFs, and employed MLE and UMVUE (Robert et al., 2013) which are most important point estimation methods to make parameter analysis on PMFs of binomial and Poisson. The parameters p_R, p_S, λ_S and λ_R are deduced from experimental data, and contain all stochasticity information about R and S occurring in different restoration vegetation types. Specifically, take the stochasticity of R event for instance, we defined X_i as the number of times of R event occurrence in a specific restoration vegetation in i th rainy season. Therefore, in this study, five independent and identical (*iid*) random variables have the same and mutually independent PMFs of binomial or Poisson, which are simply expressed as follow:

$$X_1, X_2, \dots, X_5 \xrightarrow{iid} binomial\ (p_R) \text{ or } X_1, X_2, \dots, X_5 \xrightarrow{iid} Poisson\ (\lambda_R) \qquad (16)$$

Supposing the monitoring of soil erosion are continued to be conducted infinitely, then the numbers of corresponding I events (n) and rainy seasons (i) would approach infinity ($n, i \to \infty$). (16) would be transformed as follow:

$$X_1, X_2, \ldots, X_i \xrightarrow{iid} binomial\ (p) \quad \text{or} \quad X_1, X_2, \ldots, X_i \xrightarrow{iid} Poisson\ (\lambda) \qquad (17)$$

In the (17), p and λ are two population parameters representing the whole randomness information of R events under longer monitoring period with i rainy seasons. The real p or λ is unknown, but, theoretically, they can be estimated by searching for the best reasonable population estimators $\hat{p}$ or $\hat{\lambda}$ through MLE and UMVUE methods. During the estimator searching processes, appendix B proved that the best estimator $\hat{p}$ in Binomial PMF is the unbiasedness and consistency of the MLE of p. And appendix C, however, proved that the best estimator $\hat{\lambda}$ in Poisson PMF is not only the unbiasedness and consistency of the MLE of λ, but also the UMVUE of MLE. Consequently, comparing the two appendices, the best estimator $\hat{\lambda}$ implies that the Poisson PMF would be more beneficial for predicting the stochasticity of R and S events in different restoration vegetation types over long-term observation periods than that of Binomial PMF.

Besides having better prediction about stochasticity of soil erosion at larger temporal scale, the Poisson PMF could also be fit for predicting the stochasticity of S event in the closed design plot system. As Boix-Fayos et al, (2006) mentioned, the closed runoff plot was not fit for long term soil erosion monitoring, because it forms an obstruction to prevent the transportable material from entering the close monitoring system. With the ongoing monitoring at longer temporal scale, the transport-limited erosion pattern could gradually transform into detachment-limited pattern in the closed-plot (Boix-Fayos et al., 2007;Cammerraat, 2002). This probably leads to the sediment transformation becoming more and more difficult to generate, and finally reduces the probability of S events under the same precipitation condition. And fortunately, the parameters in Poisson PMF at larger temporal scale could successfully express the decreasing of probability of S event in closed plot system. Because, in order to satisfying the fact that $\lambda = np$ in Poisson PMF is an unknown constant, when the numbers of times of I events (n) approach infinity, the probability (p) of R or S events generation have to approach to zero, Actually, above inference coincides with the assuming situation for sediment transformation in closed plot system at long temporal scale (Boix-Fayos et al., 2006), which further proves that Poisson PMF could be a reliable prediction model for soil erosion. However, affected by the globe climate change, the occurring frequency of extreme weather condition probably increase. Under that background, the stochastic signals of increasing extreme I events could inevitably be transmitted into the stochasticity of soil erosion in the further. Consequently, it is necessary to furtherly focus on the disturbance of rare event with extreme amount or energy on the soil-plant systems under a changing environment.

5. Conclusion

In this study, we applied stochastic approach to analyze the effects of restoration vegetation types on the stochasticity of runoff and sediment in the Loess Plateau of China, and draw the following conclusions:

(1) OCIRS-Bayes framework is an innovative analysis system which not only quantify the stochasticity of environment in terms of random event pattern, but also characterize the interactive relationship between rainfall and soil erosion by means

of probability theory.

(2) The difference of morphological structures in restoration vegetation types is the source of different stochasticity of soil erosion in T1 to T3 under same rainfall condition. Larger canopy, thicker litter layer and denser root distribution could more effectively affect the transmission of stochastic signal of rainfall into soil erosion.

(3) Both of binomial and Poisson PMFs could well predict the probability distribution of numbers of times runoff and sediment events in T1 to T3, however, Poisson PFM could be more fit for predicting stochasticity of soil erosion at larger temporal scales

This study provide a new analysis framework to describe the soil erosion property, which could be meaningful to researchers and policy makers to evaluate the efficacy of soil control practices and their ecosystem service in a semi-arid environment.

Appendix A. The transformation from binominal to Poisson PMF

Let $p = \frac{\lambda}{n}$, then:

$$pmf_{Xbin}(x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} \cdot \left(\frac{\lambda}{n}\right)^x \cdot \left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\lambda!}{x!} \cdot \frac{n(n-1)(n-2)\cdots 1}{(n-x)(n-x-1)\cdots 1} \cdot \frac{1}{n^x} \cdot \left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{\lambda!}{x!} \cdot 1 \cdot \left(1-\frac{1}{n}\right) \cdot \left(1-\frac{2}{n}\right) \cdots \left(1-\frac{x-1}{n}\right) \cdot \left(1+\frac{-\lambda}{n}\right)^n \cdot \left(1-\frac{\lambda}{n}\right)^{-x} \quad \text{(A1)}$$

In equation (A1), when $n \to \infty$, and x, λ is finite and constant, then

$$\lim_{n\to\infty}(1-\frac{1}{n}) = \cdots = \lim_{n\to\infty}\left(1-\frac{x-1}{n}\right) = \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-x} = 1 \quad \text{(A2)}$$

And

$$\lim_{n\to\infty}\left(1+\frac{-\lambda}{n}\right)^{n}=e^{-\lambda} \tag{A3}$$

And according to equation (A2) and (A3), the equation (A1) can be transformed as:

$$\lim_{n\to\infty}\left[\frac{n!}{x!\,(n-x)!}\cdot\left(\frac{\lambda}{n}\right)^{x}\cdot\left(1-\frac{\lambda}{n}\right)^{n-x}\right]=\frac{\lambda^{x}e^{-\lambda}}{x!}\quad x=0,1,2,\ldots \tag{A4}$$

or

$$pmf_{Xbin}(x)\xrightarrow{n\to\infty}\frac{\lambda^{x}e^{-\lambda}}{x!}=pmf_{Xpoi}(x) \tag{A5}$$

Appendix B. Parameter estimation of p in Poisson PMF

(1) Derivatization of the MLE $\hat{p}$

Let the random sample $\mathrm{X}_1,\mathrm{X}_2,\ldots,\mathrm{X}_i \overset{iid}{\rightarrow} pmf_{Xbin}(p)$ and assume the binomial distribution as:

$$P(\mathrm{X}=x_i)=\binom{m}{x_i}p^{x_i}(1-p)^{m-x_i} \tag{B1}$$

The likelihood function $L(p)$ is joint binomial PDF with parameter p as follow:

$$L(p)=f_X(\mathrm{X}_1,\ldots,\mathrm{X}_n,p)=\prod_{i=1}^{n}\binom{m}{x_i}p^{\sum_{i=1}^{n}\mathrm{X}_i}(1-p)^{(mn-\sum_{i=1}^{n}\mathrm{X}_i)} \tag{B2}$$

By taking logs on both side of equation (B2):

$$lnL(p)=ln\left(\prod_{i=1}^{n}\binom{m}{x_i}\right)+\sum_{i=1}^{n}\mathrm{X}_i\,ln\,p+\left(mn-\sum_{i=1}^{n}\mathrm{X}_i\right)ln(1-p) \tag{B3}$$

And differentiating with respect to p in $lnL(P)$ and let the result be zero:

$$\frac{\partial lnL(p)}{\partial p}=\frac{\sum_{i=1}^{n}\mathrm{X}_i}{p}-\frac{(mn-\sum_{i=1}^{n}\mathrm{X}_i)}{(1-p)}=0 \tag{B4}$$

Solution $\hat{p}=\frac{\sum_{i=1}^{n}\mathrm{X}_i}{mn}$, let $m=n,\Longrightarrow\ \hat{p}=\frac{\bar{\mathrm{X}}}{n}$

Therefore, $\hat{p}=\frac{\bar{\mathrm{X}}}{n}$ is the MLE of population parameter p in binomial PMF model.

(2) Discussion of the unbiasedness and consistency of $\hat{\boldsymbol{p}}$

Let $E_p(\hat{p})$ be the expectation of M.L.E $\hat{p}$ when population parameter p is true in random sample which is $\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_i \overset{iid}{\rightarrow} pmf_{Xbin}(p)$, then

$$E_p(\hat{p}) = E_P(\overline{\mathrm{X}}/n) = \frac{1}{n^2}\sum_{i=1}^{n} E_P(\mathrm{X}_i) = \frac{1}{n^2} n^2 p = p \tag{B5}$$

Which proved that MLE $\hat{p} = \frac{\overline{\mathrm{X}}}{n}$ is a unbiased estimator for p. And furthermore then let $Var_p(\hat{p})$ be the variance of $\hat{p}$ when population p is true.

$$Var_p(\hat{p}) = Var_p\left(\sum_{i=1}^{n} \mathrm{X}_i/n^2\right) = \frac{1}{n^4}\sum_{i=1}^{n} Var_p(\mathrm{X}_i) = \frac{p(1-p)}{n^2} \tag{B6}$$

As the *n* approaches to infinite:

$$\lim_{n\to\infty} Var_p(\hat{p}) = \lim_{n\to\infty}\left(\frac{p(1-p)}{n^2}\right) = 0 \tag{B7}$$

Equation (B5)~(B7) satisfied the theme of weak law of larger number, which lead the $\hat{p} = \frac{\overline{\mathrm{X}}}{n}$ is probabilistic converge to population parameter p:

$$\lim_{n\to\infty} P(|\hat{p} - p| \geq \varepsilon) = 0, \text{for all } \varepsilon > 0 \tag{B8}$$

Consequently, the unbiased MLE $\hat{p} = \frac{\overline{\mathrm{X}}}{n}$ is consistent for p.

Appendix C. Parameter estimation of $\boldsymbol{\lambda}$ in Poisson PMF

(1) Derivatization of the MLE $\hat{\boldsymbol{\lambda}}$

Let the random sample $\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_i \overset{iid}{\rightarrow} pmf_{Xpoi}(\lambda)$, and assume the poisson distribution as:

$$pmf_{\mathrm{X}poi}(x_i) = \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \tag{C1}$$

The likelihood function $L(\lambda)$ is joint PDF with parameter λ as follow:

$$L(\lambda) = f_X(\mathrm{X}_1, \dots, \mathrm{X}_n, \lambda) = f(\mathrm{X}_1, \lambda) \times \cdots \times f(\mathrm{X}_n, \lambda) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \quad \text{(C2)}$$

Taking logs on $L(\lambda)$ in equation (B4) and differentiating logarithm function with respect to λ:

$$\frac{\partial lnL(\lambda)}{\partial \lambda} = \frac{\partial (\prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!})}{\partial \lambda} = -n \frac{\lambda^{\sum_{i=1}^{n} \mathrm{X}_i}}{(x_1 x_2 \cdots x_n)!} e^{-n\lambda} + \frac{\sum_{i=1}^{n} \mathrm{X}_i \lambda^{(-1+\sum_{i=1}^{n} \mathrm{X}_i)}}{(x_1 x_2 \cdots x_n)!} \quad \text{(C3)}$$

Let the equation (C3) equal to zero, and has solution:

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} \mathrm{X}_i = \overline{\mathrm{X}} \quad \text{(C4)}$$

Therefore, $\hat{\lambda} = \overline{\mathrm{X}}$ is the MLE of population parameter λ in Poisson PMF model.

(2) Discussion of the unbiasedness and consistency of $\hat{\lambda}$

Let $E_\lambda(\hat{\lambda})$ be the expectation of MLE $\hat{\lambda}$ when population parameter λ is true in random sample $\mathrm{X}_1, \mathrm{X}_2, \dots, \mathrm{X}_i \xrightarrow{iid} pmf_{\mathrm{X}poi}(\lambda)$, then:

$$E_\lambda(\hat{\lambda}) = E_\lambda(\overline{\mathrm{X}}) = \frac{1}{n^2} \sum_{i=1}^{n} E_\lambda(\mathrm{X}_i) = \frac{1}{n} n\lambda = \lambda \quad \text{(C5)}$$

which proved that MLE $\hat{\lambda} = \overline{\mathrm{X}}$ is a unbiased estimator for λ. Meanwhile, let $Var_\lambda(\hat{\lambda})$ be the variance of MLE $\hat{\lambda}$ when population parameter λ is true

$$Var_\lambda(\hat{\lambda}) = Var_\lambda(\overline{\mathrm{X}}) = Var_\lambda\left(\sum_{i=1}^{n} \mathrm{X}_i / n^2\right) = \frac{1}{n^4} \sum_{i=1}^{n} Var_\lambda(\mathrm{X}_i) = \frac{\lambda}{n} \quad \text{(C6)}$$

And

$$\lim_{n\to\infty} Var_\lambda(\hat{\lambda}) = \lim_{n\to\infty} \left(\frac{\lambda}{n}\right) = 0 \quad \text{(C7)}$$

According to the weak law of large number theme, equation (B7, B8, C1) lead that unbiased MLE $\hat{\lambda} = \overline{\mathrm{X}}$ is probabilistic converge to λ:

$$\lim_{n\to\infty} P(|\hat{\lambda} - \lambda| \geq \varepsilon) = 0, \text{for all } \varepsilon > 0 \quad \text{(C8)}$$

Therefore, MLE $\hat{\lambda} = \overline{\mathrm{X}}$ is consistent for population parameter λ.

(3) Determination of UMVUE $\hat{\lambda}$ of population parameter

Firstly, MLE $\hat{\lambda} = \overline{\mathrm{X}}$ is an unbiased estimator of parameter λ which is the precondition of UMVUE determination. Secondly, by using Cramer-Rao lower bound to check whether the unbiased MLE was UMVUE or not. Then we have:

$$ln f_X(X,\lambda) = -ln x! + x ln\,\lambda - \lambda \tag{C9}$$

$$\frac{\partial(ln f_X(X,\lambda))}{\partial\lambda} = \frac{x}{\lambda} - 1 \tag{C10}$$

And

$$\frac{\partial^2 ln f_X(X,\lambda)}{\partial\lambda^2} = \frac{\partial(\frac{x}{\lambda} - 1)}{\lambda} = -\frac{x}{\lambda^2} \tag{C11}$$

Accordingly the expectation of equation (C11) when the population parameter λ is true:

$$E_\lambda\left[\frac{\partial^2 ln f_X(X,\lambda)}{\partial\lambda^2}\right] = E_\lambda\left(-\frac{X}{\lambda^2}\right) = -\frac{1}{\lambda^2}E_\lambda(X) = -\frac{\lambda}{\lambda^2} = -\frac{1}{\lambda} \tag{C12}$$

So the Cramer-Rao lower bound (CRLB) is

$$\mathrm{CRLB} = \frac{1}{-nE_\lambda\left[\frac{\partial^2 ln f_X(X,\lambda)}{\partial\lambda^2}\right]} = \frac{1}{-n\cdot(-\frac{1}{\lambda})} = \frac{\lambda}{n} = Var_\lambda(\hat{\lambda}) = Var_\lambda(\overline{\mathrm{X}}) \tag{C13}$$

Consequently, MLE $\hat{\lambda} = \overline{\mathrm{X}}$ is UMVUE of population parameter λ.

Figure captions

Figure 1 The construction of OCIRS system : (a) a flow chart to determine all random event types in OCIRS framework; (b) the different combining patterns of rainfall and non-rainfall events in three consecutive days to form ten observed random event sequences on five rainy seasons; (c) Venn diagram to reveal the relationship among all random events types in OCIRS framework.

Figure 2 Study area and experimental design: (a) location of the Yangjuangou Catchment; (b) three restoration vegetation types including *Armeniaca sibirica* (T1), *Spiraea pubescens* (T2), and *Artemisia copria* (T3); (c) the dynamic measurement of soil moisture and data collection to provide the information about average antecedent soil moisture; (d) the measurement of field saturated hydraulic conductivity to determine the average infiltration capability; (e): the investigation of morphological properties of restoration vegetation by setting quadrats

Figure 3 The probability distribution of different random rainfall event types (Iw, Is, Il, and Ie) and random non-rainfall event types (Ch and Cd) at monthly and seasonal scales from rainy season of 2008 to 2012.

Figure 4 The probability distribution of random runoff and sediment events generating in three restoration vegetation types at monthly and seasonal scales from rainy season of 2008 to 2012, the Arabic numbers and letter "T" on the abscissa indicate the month and season respectively, the same as follow figures

Figure 5 The comparison between simulation of stochasticity of runoff and sediment events by Binomial and Poisson PMFs and the observed frequencies of numbers of times of soil erosion events in three restoration vegetation type, Exp_B and Exp_P indicates the simulated values in Binomial and Poisson PMF respectively, and the histogram represents the observed values.

Figure 6 The distribution of probabilistic contribution of four random rainfall event types on anyone runoff or sediment event stochastically generating in three restoration vegetation types at monthly and seasonal scales from rainy season of 2008 to 2012

Figure 7 Morphological properties of three restoration vegetation types including the thickness of litter layer, the distribution of root system. The dashed lines indicates the diameter and depth of soil samples with approximating 10 cm and 30 cm respectively.

Figure 8 The framework of integrated probabilistic assessment for soil erosion monitoring and restoration strategies

~~Figure captions~~

~~Figure 1~~

Description of the study area, (a) Location of the Yangjuangou Catchment; (b) restoration vegetation types at the runoff plot scale, from left to right: *Armeniaca sibirica* (T1), *Spiraea pubescens* (T2), and *Artemisia copria* (T3); (c) field saturated conductivity measurement using Model 2800 K1 Guelph Permeameter; (d) a 1 m^2 quadrat to measure vegetation coverage

Figure 2

Construction process of OCIRS-Bayes analysis framework, (a) flow chart of confirming process of all elements in OCIRS-Bayes system; (b) Venn diagram of the relationships of all elements in OCIRS-Bayes system; (c) result of hierarchical cluster analysis of 130 individual rainfall events

Figure 3

The probability distributions of four rainfall event types at month and seasonal scales over five rainy seasons

Figure 4

The probability distributions of soil erosion in three restoration vegetation types at month and seasonal scales over five rainy seasons, the Arabic numbers and letter "T" on the abscissa in each plot represent the month and total reason respectively, the same as follow figures

Figure 5

The distribution of probabilistic contribution of four rainfall event types on one stochastic soil erosion in three restoration vegetation types at month and seasonal scales over five rainy seasons

Figure 6

The comparison the prediction of stochasticity of soil erosion by binomial and Poisson PMFs and observed frequency of numbers of times of soil erosion event in three restoration vegetation types, Exp_B and Exp_P means the expected values in binomial and Poisson PMF respectively, and histogram represents observed value.

Figure 7

Morphological structure properties of thee restoration vegetation types including litter layer, root system distribution. The diameter and depth of samples which were indicted by the dashed line are approximately 10 cm and 30 cm respectively

Acknowledgment

This work was funded by the National Natural Science Foundation of China (No. 41390464) and the National Key Research and Development Program (No. 2016YFC0501602). We specially thank for associated editor, and two reviewers whose suggestions and advices improve the quality of this study, we also thank professor Chen Lin-An with National Chiao Tung University (NCTU) for his great help on the mathematical statistical inference in this manuscript, and thank Liu Yu, Liu Jianbo and Wang Jian for their support for soil erosion monitoring.

~~This work was funded by the National Natural Science Foundation of China (No. 41390464 and 41471094). We also thank Prof. Chen Lin-An with National Chiao Tung University (NCTU) for his great help on the mathematical statistical inference in manuscript.~~

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Tables

Table 1 The summary of main researches on the stochasticity of soil erosion rate and the stochasticity of factors to affect the soil erosion rate

[a]Stochasticity (Uncertainty)	[b]Approach or method	[c]Driven types	Main Hydrological behaviors	Main Influencing factors	Spatiotemporal Scale	Reference
			Stochasticity of soil erosion rate			
Runoff connectivity	Probabilistic model Conceptual model	(1)Data-Mapping (2)Theory	Infiltration processes Precipitation	Topography Soil depth	Hillslope scale in USA	Janzen, D., and McDonnell, J 2015
Runoff processes	Probabilistic model Conceptual model	(1)Simulation (2)Theory	Infiltration processes Precipitation	Topography		Janzen, D., and McDonnell, J 2015
Runoff production	Probabilistic model Conceptual model	(1)Theory (2)Simulation	Runoff absorption Water storage Infiltration capacity	Soil moisture Evaporation Recharge	Point and basin scale	Moore, 2007
Flood prediction and runoff	Probabilistic model Multivariate analysis	(1)Simulation (2)Data-Calibration	Stochastic rainfall process	Parameters in rainfall-runoff model	Multiple catchment scales in Iran	Yazdi, J. et al., 2014
Rainfall and runoff processes	Probabilistic model hydrological mechanism	(1)Simulation (2)Random event (3)Theory	Soil storage	Given climate regime hydraulic conductivity landform development	Hillslope scale	Freeze, 1980
Erosion rate	Probabilistic model Mechanical mechanism	(1)Data-Calculation (2)stochastic forcing		Bed shear stress Critical shear stress	Laboratory scales in Netherlands	Prooijen and Winterwerp, 2010

Erosion rate	Physical model Probabilistic model Conceptual model	(1)Theory (2)Simulation	Simulated near-bed flow	Soil structure Oscillating flow		Sidorchuk, 2005
Erosion risk	Empirical model Geo-statistics	(1)Data-Mapping	Erosive precipitation	Factors in RUSLE	Annual and Reginal scales in China	Jiang et al., 2012
Uncertainty of soil loss	Empirical model Geo-statistics Error analysis	(1)Simulation (2)Data-calibration	Erosive precipitation Runoff and sediment	Spatiotemporal Rainfall erosivity distribution	Annual time and catchment scale in USA	Wang et al., 2002
Uncertainty and variability of erosion rate	Empirical model	(1)Hypotheses (2)Data-calculation	Total rainfall volume and 30-minute rainfall intensity	Stochastic environment conditions Scale effect		Kim et al., 2016
Stochasticity of factors to affect soil erosion rate						
Soil moisture related to soil erosion	Probabilistic model Physical model	(1)Hypotheses, (2)Simulation (3)Theory	Precipitation Evapotranspiration	Temporal patterns of rainfall property	Daily time and Hillslope scale in	Ridolfi et al., 2003
Antecedent soil moisture related to soil erosion	Probabilistic model Physical model	(1)Data-Mapping (2)Theory	Runoff response Infiltration processes		Daily time and multiple catchment scales in Spain	Castillo et al., 2003
Stochastic rainfall related to flood and runoff	Probabilistic model Conceptual model	(1)Data-Calibration (2)Random event (3)Hypothesis	Stochastic storm Runoff and flood	Parameters in Peak flow models	Hourly-daily time and multiple catchment scales in Germany	Haberlandt and Radtke, 2014
Stochastic rainfall related to runoff and erosion	Physical model Empirical model	(1)Simulation (2)Data-calibration	Overland/channel flow Erosion transport Precipitation	Spatiotemporal rainfall distribution	Seasonal and annual time catchment scale in USA	Lopes, 1996
Uncertainty of soil	Empirical model	(1)Simulation		Spatiotemporal soil	Regional scales in	Wang et al.,

erodibility	Geo-statistics	(2)Data-Mapping		types, depth and parent material	USA	2001
Stochastic rainfall related to runoff	Probabilistic model Conceptual model Physical model	(1)Data-calibration (2)Theory	Sewer overflows	Rainfall depth and duration, climate conditions	Seasonal and annual time catchment scales in Spain	Andres-Domenech et al., 2010

a: the main contents of different studies focusing on the stochasticity (uncertainty) of soil erosion and its influencing factors

b: the main statistical methods or different types of mathematic and physical models to be employed to describe and analyze the stochasticity of soil erosion

c: the main properties of analyzing framework in the different studies and the characteristics of data application on the evaluation of stochasticity of soil erosion

Table 2 Definition and explanation of all random events in OCIRS

symbol	Physical meaning of random event types	Probabilistic meaning of random event types	Influencing factors and implication
O	observation events with time step ranging from 0 to 72 hours, including non-rainfall and rainfall events	random events composing the sample space of OCIRS system. The probability $P(O) = 1$	indicating the general stochastic weather conditions over rainy seasons
C	non-rainfall events with time step ranging from 0 to 24 hours, including sunny or cloudy weather condition at hour or day scales	random events, the probability of C events is the ratio of numbers of C events to O events $C \subset O, 0 \le P(C) \le P(O) = 1$	implying the extent of evaporation or potential evapotranspiration in weather condition.
Cd	non-rainfall events with time step being 24 hours, including observed sunny or cloudy at day scale	random events composing the subset of C events, $Cd \subseteq C, 0 \le P(Cd) \le P(C)$	implying the duration of evaporation or evapotranspiration at day scale
Ch	non-rainfall events with time step being less than 24hours, including observed sunny or cloudy at hour scales which intercepted by rainfall events within a day	random events composing the subset of C events, the intersection of Ch and Cd is null, $Ch \subseteq C, Cd \cup Ch = C, Cd \cap Ch = \emptyset, 0 \le P(Ch) \le P(C)$	influenced by the frequency of rainfall events generation, and implying the alternation of sunny and rainy in a day

I	an individual rainfall event with different precipitation, intensity and duration ranging from 0 to 72 hours, the time interval between two I events is more than 6 hours	random events, the probability of I event is ratio of numbers of I events to O events over observation $I \subset O, I \cup C = O, I \cap C = \emptyset, 0 \leq P(I) \leq P(O) = 1$	a driven force of soil erosion, which could be intercepted by vegetation and transformed into throughfall
Ie	an extreme longest individual rainfall event whose average precipitation, intensity and duration were 96.6 mm, 0.022 mm/min, and 73 hours, respectively.	random events composing the subset of I events, $Ie \subseteq I, 0 \leq P(Ie) \leq P(I)$	rainfall events with low intensity and longest duration, inclining to infiltration-excess runoff generation
Il	a second longest individual rainfall events types whose average precipitation, intensity and duration were 47.3 mm, 0.027 mm/min, and 30 hours, respectively.	random events composing the subset of I events, the intersection of Il and Ie is null, $Il \subseteq I,\ Il \cap Ie = \emptyset, 0 \leq P(Il) \leq P(I)$	rainfall events with low intensity and long duration, inclining to infiltration-excess runoff generation
Is	A rainfall event type spanning two days whose average precipitation, intensity and duration were 22.7 mm, 0.042 mm/min, and 10 hours, respectively	random events composing the subset of I events, $Is \subseteq I,\ Is \cap Il \cap Ie = \emptyset, 0 \leq P(Il) \leq P(I)$	rainfall events with strongest rainfall intensity in middle duration, inclining to runoff and sediment generation
Iw	a rainfall event type generating within a day whose average precipitation, intensity and duration were 9.8 mm, 0.045 mm/min, and 5 hours, respectively. it usually generates several times within one day.	random events composing the subset of I events, $Iw \subseteq I,\ Iw \cap Is \cap Il \cap Ie = \emptyset,\ Iw \cup Is \cup Il \cup Ie = I, 0 \leq P(Iw) \leq P(I)$	rainfall events with fewest and shortest precipitation and duration, which is different to trigger soil erosion
R	runoff event type generating on vegetation land types, it occurs on rainfall processes, and its duration is negligible	random events responding to I events, $R \subset I, R \cap C = \emptyset, 0 \leq P(R) < P(I)$	influenced by rainfall and vegetation properties.
S	sediment event occurring on vegetation land types, it occurs on runoff processes, and its duration is negligible	random events responding to R events, $S \subset R \subset I, S \cap C = \emptyset, 0 \leq P(S) \leq P(R) < P(I)$	driven by R events, and affected by rainfall and vegetation properties.

Table 3 Main characteristics of four types of random rainfall event over five rainy seasons

Rainy season	Rainfall event types	Average precipitation (mm)	Average intensity (mm/min)	Average duration (hour)
2008	Iw	16.7	0.122	2.3
	Is	19.2	0.066	4.8
	Il	53.2	0.032	27.7
	Ie	96.6	0.022	73.2
2009	Iw	9.0	0.027	5.6
	Is	35.4	0.059	10.0
	Il	47.9	0.032	24.9
	Ie	×	×	×
2010	Iw	9.0	0.018	8.3
	Is	7.6	0.012	10.6
	Il	×	×	×
	Ie	×	×	×
2011	Iw	3.3	0.031	1.8
	Is	21.5	0.040	9.0
	Il	42.5	0.020	35.4
	Ie	×	×	×
2012	Iw	10.8	0.028	6.4
	Is	30.0	0.031	16.1
	Il	45.5	0.023	33.0
	Ie	×	×	×
Average	Iw	9.8	0.045	4.9
	Is	22.7	0.042	10.1
	Il	47.3	0.027	30.3
	Ie	96.6	0.022	73.2

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Table 4 Basic properties of soil, vegetation and erosion in different restoration vegetation types

Basic properties of different vegetation types	[h]N	Restoration vegetation types		
		Armeniaca sibirica Type 1 (T1)	*Spiraea pubescens* Type 2 (T2)	*Artemisia copria* Type3 (T3)
Topography property				
Slope aspect	9	Southwest	Southwest	Southwest
Slope gradation (%)	9	≈26.8	≈26.8	≈26.8
Slope size for each (m)	9	3×10	3×10	3×10
Soil property				
[a]DBD (g cm^{-3})	30	1.28±0.08	1.16±0.12	1.23±0.10
Clay (%)	30	11.07±2.43	11.98±3.05	9.54±1.48
Silt (%)	30	26.11±1.50	25.24±3.84	26.72±2.87
Sand (%)	30	62.82±0.94	62.78±4.51	63.74±3.24
[b]Texture type		Sandy loam	Sandy loam	Sandy loam
[c]SHC (cm min^{-1})	20	0.46±0.82(a)	2.22±0.66(b)	0.50±0.60(a)
[d]SOM (%)	30	1.28±0.63(a)	0.98±0.15(b)	0.90±0.09(b)
Vegetation property				
Restoration years	9	20	20	20
Crown diameters (cm)	27	211.6±15.4(c)	80.5±4.5(b)	64.1±6.3(a)
Litter layer (cm)	30	1.2±0.3(a)	3.4±1.8(b)	1.8±0.5(a)
Height (cm)	27	256.3±11.1(c)	128.3±8.3(b)	61.8±1.1(a)
LAI	27	×	2.31	1.78
[e]Ave. Coverage (%)	27	85	90	90
Rainfall/Erosion property				
Times of rainfall events			130	
Times of runoff events		30/30/30	45/45/45	45/45/45
Times of sediment events		13/13/13	19/19/19	32/32/32
[f]Ave. runoff depth (cm)		0.012(a)	0.014(a)	0.083(b)
[g]Ave. sediment amount (g)		5.8(a)	6.8(a)	25.7(b)

a: dry bulk density; b: texture type is determined by textural triangle method based on USDA; c: field saturated hydraulic conductivity, and all the values with same letter in each row indicates non-significant difference at α=0.05 which is the same as follow rows; d: soil organic matter; e: average coverage of three restoration vegetation types over five rainy seasons; f: average runoff depth in restoration types over rainy seasons; g: average sediment yield in restoration types over rainy seasons; h: sample number.

Table 5 The definition and classification of properties of rainfall soil and plant ordinal variables

Ordinal variable	Physical meaning of classified influencing factors	Standard of influencing factor classification			
		Low (L)	Middle (M)	High (H)	Extreme (E)
PREC	classified precipitation variable of a single random rainfall event	0~15 mm	15~30 mm	30~60 mm	>60 mm
INT	classified intensity variable of a single random rainfall event	0~0.025 mm/min	0.025~0.05 mm/min	0.05~0.1 mm/min	>0.1 mm/min
ASM	classified variable of the antecedent soil moisture	0~5 %	5~10 %	10~20 %	>20 %
SHC	classified variable of the filed saturated hydraulic conductivity	0~1 cm/min	×	>1 cm/min	×
CRO	classified variable of the average crown width in vegetation types	0~60 cm	60~80 cm	>80 cm	×
TLL	classified variable of the average thickness of litter layers	0～2 cm	×	>2 cm	×
Y_R	dichotomous dependent variable to indicate whether a random runoff event has generation or not	If Y_R =1, it means that a random runoff event has generated; If Y_R =0, it means that a random runoff event has not generated			
Y_S	dichotomous dependent variable to indicate whether a random sediment event has generation or not	If Y_S =1, it means that a random sediment event has generated; If Y_S =0, it means that a random sediment event has not generated			

Table 6 Logistic regression model to analysis the single effect of rainfall, plant and soil ordinal variable on the erosion events presence/absence in all restoration vegetation types

Grade levels	PREC (Low)	INT (Low)	ASM (Low)	SHC (Low)	CRO (Low)	TLL (Low)
Odds ratio of all random runoff events						
Extreme	[a] $\times^{NS}$	[b]90.91***	[c]2.19*	Null	Null	Null
High	$\times^{NS}$	32.26***	2.01*	[d]0.85*	[e]7.53×10^{-3}**	[f] $\times^{NS}$
Middle	$\times^{NS}$	2.09*	1.59*	Null	7.17×10^{-2}**	Null
Odds ratio of all random sediment events						
Extreme	142.85***	166.67***	15.40*	Null	Null	Null
High	16.95**	125.00***	13.79**	0.78*	6.27×10^{-3}**	$\times^{NS}$
Middle	6.09**	34.48***	6.36*	Null	2.55×10^{-2}**	Null

a: making the low-grade of PREC ordinal variable as reference, the odds ratio of all random runoff event in extreme-grade of PREC is not significantly larger than that of low-grade of PREC; b: making the low-grade of INT ordinal variable as reference, the odds ratio of all random runoff events in extreme-grade of INT is 90.91 times significantly larger than that of low-grade of INT, under the controlled PREC condition with P≤0.001; c: making the low-grade of ASM ordinal variable as reference, the odds ratio of all random runoff events in extreme-grade of ASM is 2.19 times significantly larger than that of low-grade of ASM, under the controlled PREC and INT condition with P≤0.1; d: making the low-grade of SHC ordinal variable as reference, the odds ratio of all random runoff events in high-grade of SHC is 0.85 times significantly larger than that of low-grade of SHC, under the controlled PREC, INT and ASM condition with P≤0.1; e: making the low-grade of CRO ordinal variable as reference, the odds ratio of all random runoff events in high-grade of CRO is 7.53×10^{-3} larger than that of low-grade of CRO, under the controlled PREC, INT, ASM and SHC condition with P≤0.01; f: making the low-grade of TLL ordinal variable as reference, the odds ratio of all random runoff event in high-grade of TLL is not significantly larger than that of low-grade of TLL, under the controlled PREC, INT, ASM, SHC and CRO condition. (Wald test statistic is applied to test the significant of odds ratio *** P≤0.001, ** P≤0.01, * P≤0.1, NS: not significant, $\times^{NS}$: the nonsignificant value cannot be estimated)

Table 7 Logistic regression model to analysis the interactive effect of rainfall, plant and soil ordinal variables on the erosion events presence/absence in all restoration vegetation types

Grade levels	Reference of grade levels	Soil_ASM: ASM (low)	Soil_ASM: ASM (middle)	Soil_ASM: ASM (high)	Soil_ASM: ASM (extreme)	Plant_TLL: TLL (low)	Plant_TLL: TLL (high)
		Odds ratio of all random runoff events					
Soil_SHC	SHC (low)	Ref.	[a]2.23^{NS}	3.19^{NS}	7.02*	Null	Null
Plant_TLL	TLL (Low)	Ref.	2.23^{NS}	3.19^{NS}	7.02*	Null	Null
Plant_CRO	CRO (low)	Ref.	[b]64.34*	70.77*	486.43**	Ref.	[c]0.12***
	CRO(middle)	Ref.	$\times^{NS}$	2.32^{NS}	22.49*	Null	Null
	CRO (high)	Ref	Null	Null	Null	Null	Null
		Odds ratio of all sediment runoff events					
Soil_SHC	SHC (low)	Ref.	$\times^{NS}$	1.22^{NS}	1.82^{NS}	Null	Null
Plant_TLL	TLL (Low)	Ref.	$\times^{NS}$	1.22^{NS}	1.82^{NS}	Null	Null
Plant_CRO	CRO (low)	Ref.	$\times^{NS}$	$\times^{NS}$	$\times^{NS}$	Ref.	0.33**
	CRO(middle)	Ref.	$\times^{NS}$	$\times^{NS}$	$\times^{NS}$	Null	Null
	CRO (high)	Ref	Null	Null	Null	Null	Null

a: making the interactive effect of low-grade of SHC and low-grade of ASM as reference, the odds ratio of all random runoff events affected by the interactive effect of low-grade of SHC and middle-grade of ASM is 2.23 times larger than that interactive effect of low-grade SHC and low-grade of ASM under controlled rainfall conditions; b: making the interactive effect of low-grade of CRO and low-grade of ASM as reference, the odds ratio of all random runoff events affected by the interactive effect of low-grade of CRO and middle-grade of ASM is 64.34 times significantly larger than that interactive effect of low-grade of CRO and low-grade of ASM under controlled rainfall conditions, with $P \leq 0.1$; c: making the interactive effect of low-grade of CRO and low-grade of TLL as reference, the odds ratio of all random runoff events affected by the interactive effect of low-grade of CRO and high-grade of TLL is 0.12 times significantly larger than that interactive effect of low-grade of CRO and low-grade of TLL, with $P \leq 0.001$
(Wald test statistic is applied to test the significant of odds ratio *** $P \leq 0.001$, ** $P \leq 0.01$, * $P \leq 0.1$, NS: not significant, $\times^{NS}$: the nonsignificant value cannot be estimated)

Tables

Table 1 Basic properties of soil, vegetation and erosion in different restoration vegetation types

Basic properties of different vegetation types	[h]N	Restoration vegetation types		
		Armeniaca sibirica Type 1	*Spiraea pubescens* Type 2	*Artemisia copria* Type3
Topography property				
Slope aspect	9	southwest	southwest	Southwest
Slope gradation (%)	9	≈26.8	≈26.8	≈26.8
Slope size for each (m)	9	3×10	3×10	3×10
Soil property				
[a]DBD (g cm^{-3})	30	1.28±0.08	1.16±0.12	1.23±0.10
Clay (%)	30	11.07±2.43	11.98±3.05	9.54±1.48
Silt (%)	30	26.11±1.50	25.24±3.84	26.72±2.87
Sand (%)	30	62.82±0.94	62.78±4.51	63.74±3.24
[b]Texture type		Sandy loam	Sandy loam	Sandy loam
[c]Kfc (cm min^{-1})	20	0.46±0.82(a)	2.22±0.66(b)	0.50±0.60(a)
[d]SOM (%)	30	1.28±0.63(a)	0.98±0.15(b)	0.90±0.09(b)
Vegetation property				
Restoration years	9	20	20	20
Crown diameters (cm)	27	211.6±15.4(c)	80.5±4.5(b)	64.1±6.3(a)
Litter layer (cm)	30	1.2±0.3(a)	3.4±1.8(b)	1.8±0.5(a)
Height (cm)	27	256.3±11.1(c)	128.3±8.3(b)	61.8±1.1(a)
LAI	27	×	2.31	1.78
[e]Ave. Coverage (%)	27	85	90	90
Rainfall/Erosion property				
Times of rainfall events			130	

Times of runoff events	30/30/30	45/45/45	45/45/45
Times of sediment events	13/13/13	19/19/19	32/32/32
[f]Ave. runoff depth (cm)	0.012(a)	0.014(a)	0.083(b)
[g]Ave. sediment amount (g)	5.8(a)	6.8(a)	25.7(b)

a: dry bulk density; b: texture type is determined by textural triangle method based on USDA; c: field saturated hydraulic conductivity, and all the values with same letter in each row indicates non-significant difference at α=0.05 which is the same as follow rows; d: soil organic matter; e: average coverage of three restoration vegetation types over five rainy seasons; f: average runoff depth in restoration types over rainy seasons; g: average sediment yield in restoration types over rainy seasons; h: sample number.

Table 2 Definition and explanation of all elements in OCIRS systems based on rainfall-erosion stochasticity framework

Type	Physical characteristic	Probabilistic characteristics	Reoccurrences and implication
O	observation event including non-rainfall and rainfall events	random events composing the sample space of OCIRS system, the probability $P(O) = 1$	indicating general stochasticity of weather conditions over rainy seasons
C	non-rainfall events including sunny or cloudy weather conditions	random events, the probability of C events is the ratio of times of C events to O events over observation, $C \subset O, 0 \le P(C) \le P(O) = 1$	implying the extent of potential evapotranspiration in weather condition
I	an individual rainfall event with different precipitation, intensity and duration ranging from 0 to 72 hours, the time interval between two I events is more than 6 hours	random events, the probability of I event is ratio of times of I events to O events over observation $I \subset O, I \cup C = O, I \cap C = \emptyset, 0 \le P(I) \le P(O) = 1$	a driven force of erosion, which could be intercepted by vegetation and have high reoccurrences in rainy season
I_A	a classified rainfall event with average precipitation and intensity being 5 mm and 0.015 mm/min respectively	random events composing the subset of I events, $I_A \subseteq I, 0 \le P(I_A) \le P(I)$	having lowest rainfall erosivity nearly triggering no soil erosion events, and highest reoccurrences in all I events
I_B	a classified rainfall event with average precipitation and intensity being 27.6 mm and 0.072 mm/min respectively	random events composing the subset of I events $I_B \subseteq I, I_B \cap I_A = \emptyset, 0 \le P(I_B) \le P(I_A) \le P(I)$	having middle rainfall erosivity generally triggering runoff events, and middle reoccurrences in all I events
I_C	a classified rainfall event with average precipitation and intensity being 70 mm and 0.062 mm/min respectively	random events composing the subset of I events, $I_C \subseteq I, I_C \cap I_B \cap I_A = \emptyset, 0 \le P(I_C) \le P(I_B) \le P(I_A) \le P(I)$	having high rainfall erosivity almost driving runoff and sediment events, and low reoccurrences in all I events
I_D	an extreme rainfall event with precipitation and intensity being 4.6 mm and 0.78 mm/min respectively	random events composing the subset of I events, $I_D \subseteq I, I_D \cap I_C \cap I_B \cap I_A = \emptyset, 0 \le P(I_D) \le P(I_C) \le P(I_B) \le P(I_A) \le P(I)$	having extreme rainfall erosivity to soil erosion, lowest reoccurrences in all I events
R	runoff event generating on restoration vegetation types, it occurs on rainfall processes, its duration is negligible	random events responding to I events, $R \subset I, R \cap C = \emptyset, 0 \le P(R) < P(I)$	having different reoccurrences depending on rainfall and vegetation
S	sediment event occurring on vegetation types, it occurs on runoff processes, duration is negligible	random events responding to R events, $S \subset R \subset I, S \cap C = \emptyset, 0 \le P(S) \le P(R) < P(I)$	having different reoccurrences depending on rainfall and vegetation

Table 3 Basic properties of classified types of rainfall and their effect on runoff and sediment events distribution in different restoration vegetation types

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Rainfall events	Events number	Precipitation (mm)		Intensity (mm/min)		Duration (min)		Runoff events distribution			Sediment events distribution		
		Mean	[a]SD	Mean	SD	Mean	SD	[c]T1	T2	T3	T1	T2	T3
I_A	94	5.0	5.5	0.015	0.016	365.4	313.0	1	9	10	0	0	6
I_B	26	27.6	12.5	0.072	0.050	668.5	629.9	19	26	25	5	10	18
I_C	9	69.0	11.7	0.062	0.033	1597.8	1214.3	9	9	9	7	8	8
I_D	1	4.6	[b]NA	0.779	NA	5.9	NA	1	1	1	1	1	1
Total	130							30	45	45	13	19	33

a: Standard deviation; b: not applicable, the same below; c:vegetation type

Table 4 Correlation analysis between vegetation coverage and stochasticity of runoff and sediment events

[a]Vegetation types	Runoff Events			Sediment Events		
	Probability	Expectation	Variation	Probability	Expectation	Variation
			I_A Type			
Type 1	NA	NA	NA	NA	NA	NA
Type 2	-0.61	-0.57	-0.63	NA	NA	NA
Type 3	-0.32	-0.50	-0.18	NA	NA	NA
			I_B Type			
Type 1	-0.74*	-0.48	-0.82*	NA	NA	NA
Type 2	-0.51	-0.94*	-0.78*	-0.70*	-0.60	-0.54
Type 3	-0.88*	-0.80*	0.20	-0.81*	-0.63	-0.41
			I_C Type			
Type 1	NA	NA	NA	NA	NA	NA
Type 2	NA	NA	NA	NA	NA	NA
Type 3	NA	NA	NA	NA	NA	NA
			All Types			
Type 1	-0.28	-0.32	-0.36	NA	NA	NA
Type 2	-0.13	-0.61	[b]-0.77*	-0.33	-0.58	-0.42
Type 3	-0.09	-0.36	-0.23	-0.36	-0.69	-0.33

a: vegetation coverage; b: * means significant at α=0.05

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Figures

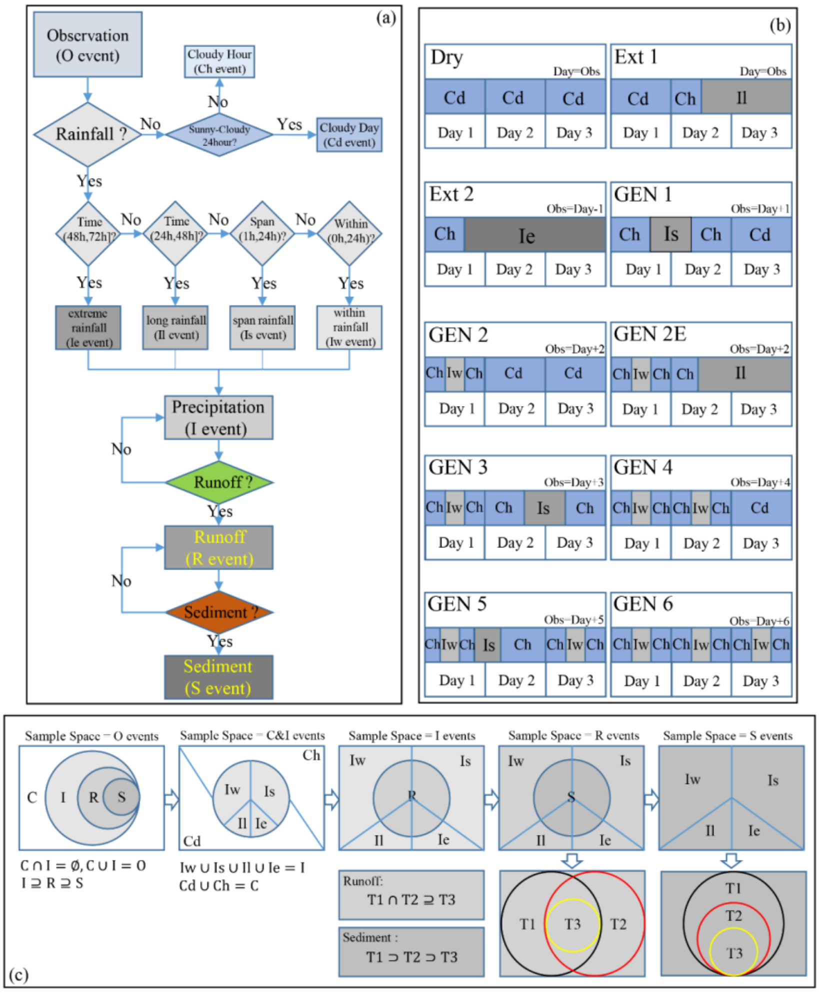


Figure 1 The construction of OCIRS system : (a) a flow chart to determine all random event types in OCIRS framework; (b) the different combining patterns of rainfall and non-rainfall events in three consecutive days to form ten observed random event sequences on five rainy seasons; (c) Venn diagram to reveal the relationship among all random events types in OCIRS framework.

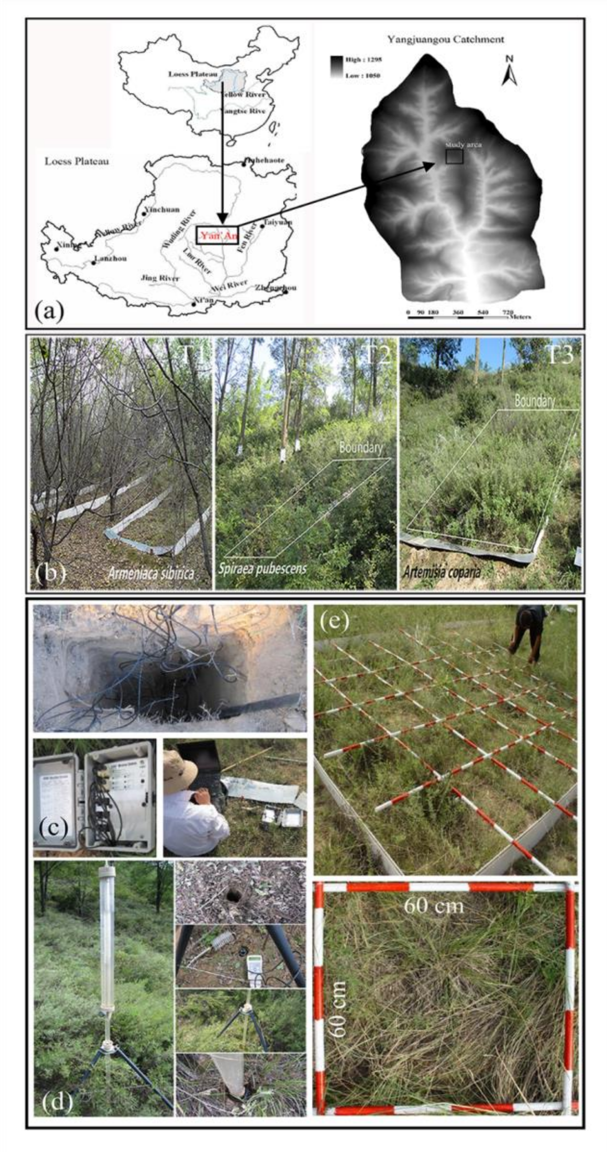


Figure 2 Study area and experimental design: (a) location of the Yangjuangou Catchment; (b) three restoration vegetation types including *Armeniaca sibirica* (T1), *Spiraea pubescens* (T2), and *Artemisia copria* (T3); (c) the dynamic measurement of soil moisture and data collection to provide the information about average antecedent soil moisture; (d) the measurement of field saturated hydraulic conductivity to determine the average infiltration capability; (e): the investigation of morphological properties of restoration vegetation by setting quadrats

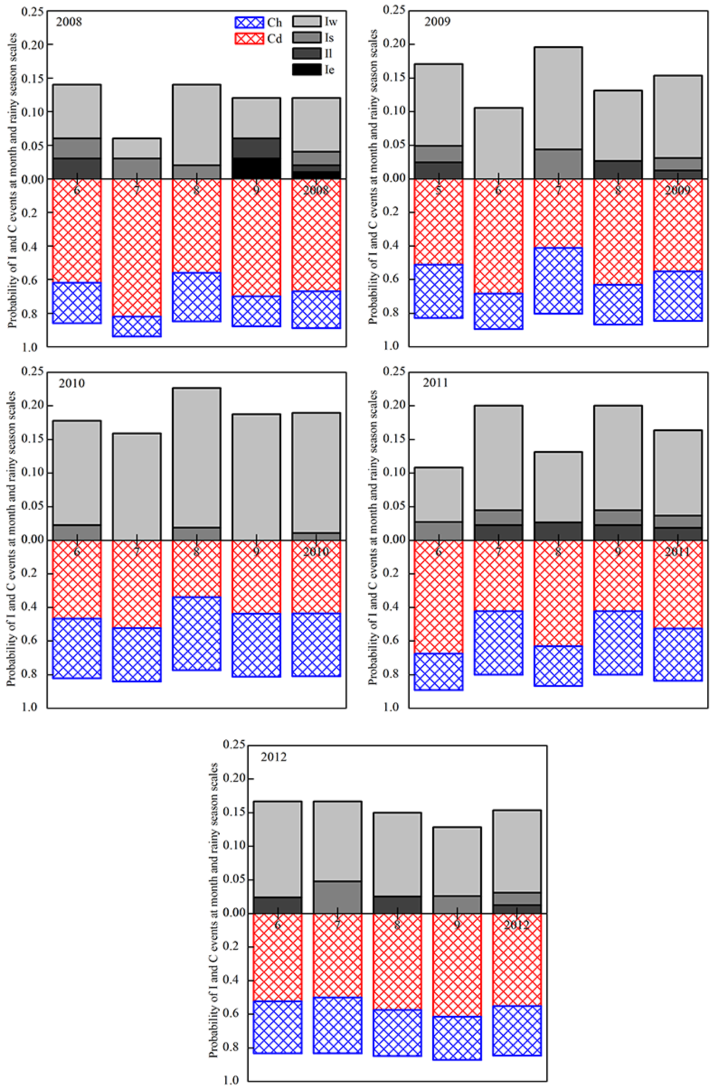


Figure 3 The probability distribution of different random rainfall event types (Iw, Is, Il, and Ie) and random non-rainfall event types (Ch and Cd) at monthly and seasonal scales from rainy season of 2008 to 2012.

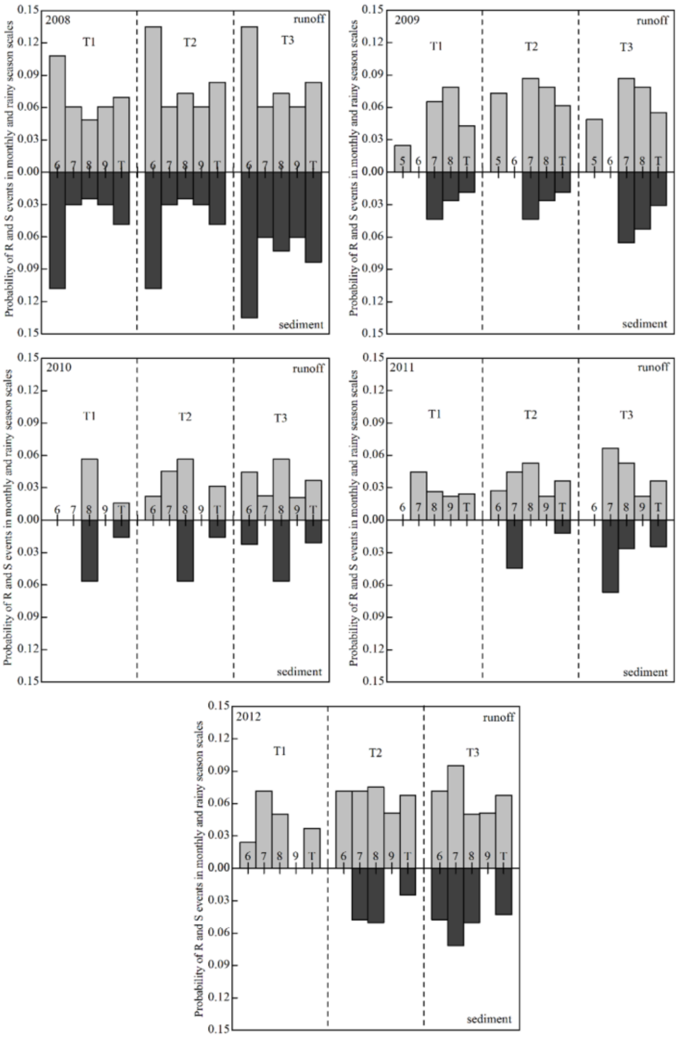


Figure 4 The probability distribution of random runoff and sediment events generating in three restoration vegetation types at monthly and seasonal scales from rainy season of 2008 to 2012, the Arabic numbers and letter "T" on the abscissa indicate the month and season respectively, the same as follow figures

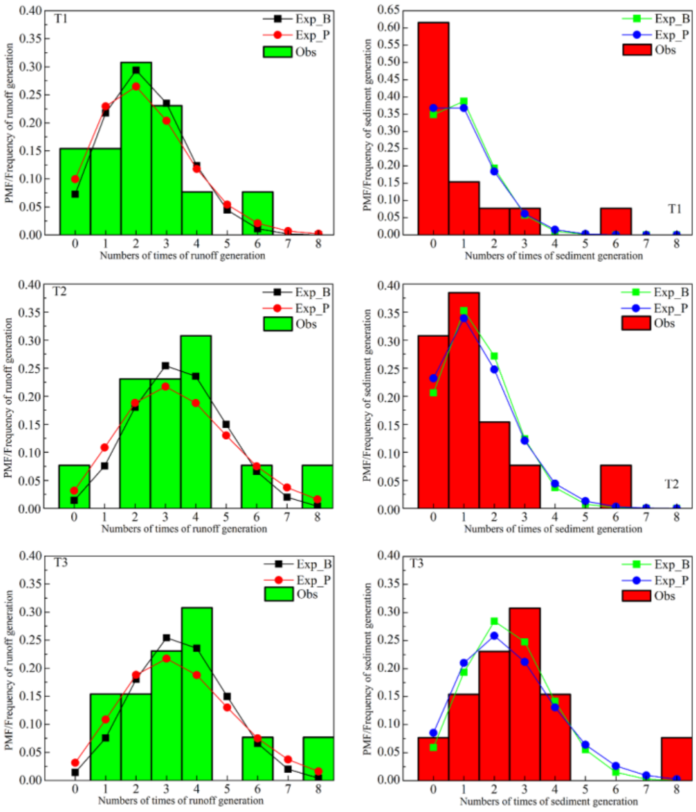


Figure 5 The comparison between simulation of stochasticity of runoff and sediment events by Binomial and Poisson PMFs and the observed frequencies of numbers of times of soil erosion events in three restoration vegetation type, Exp_B and Exp_P indicates the simulated values in Binomial and Poisson PMF respectively, and the histogram represents the observed values.

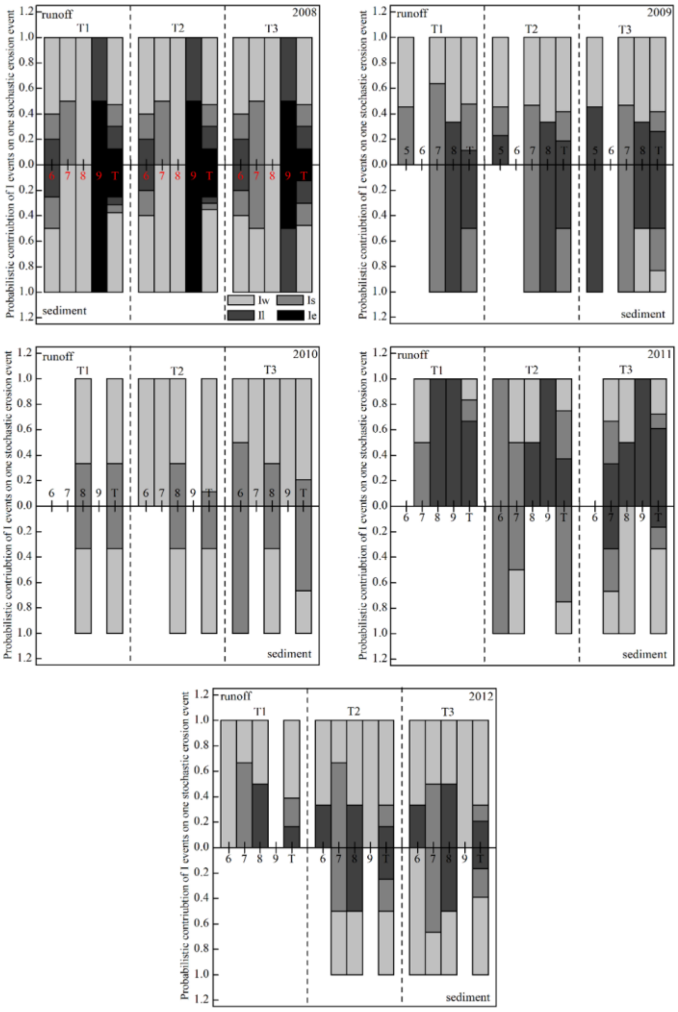


Figure 6 The distribution of probabilistic contribution of four random rainfall event types on anyone runoff or sediment event stochastically generating in three restoration vegetation types at monthly and seasonal scales from rainy season of 2008 to 2012

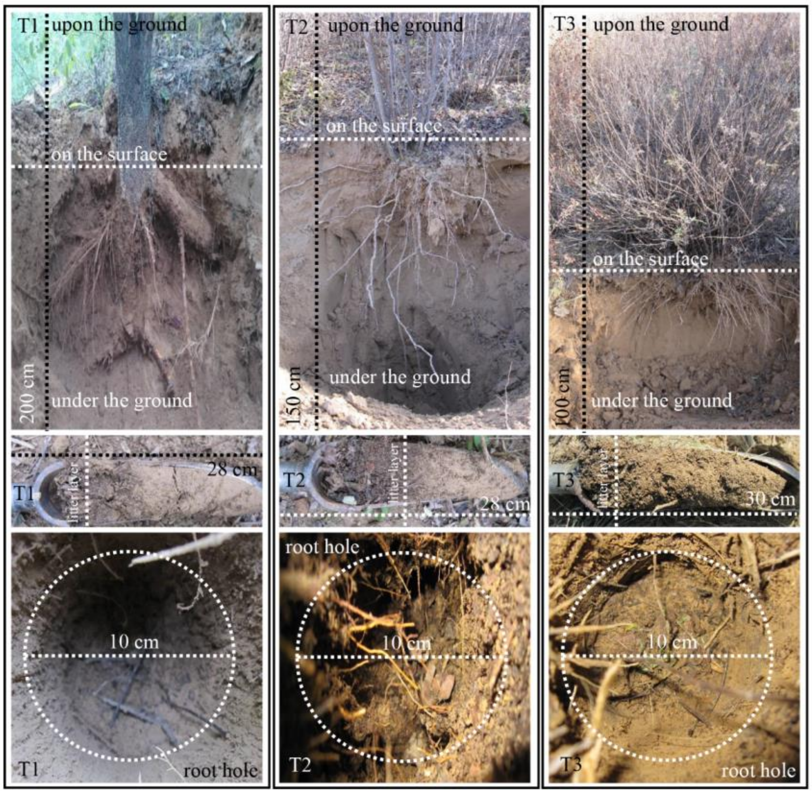


Figure 7 Morphological properties of three restoration vegetation types including the thickness of litter layer, the distribution of root system. The dashed lines indicates the diameter and depth of soil samples with approximating 10 cm and 30 cm respectively.

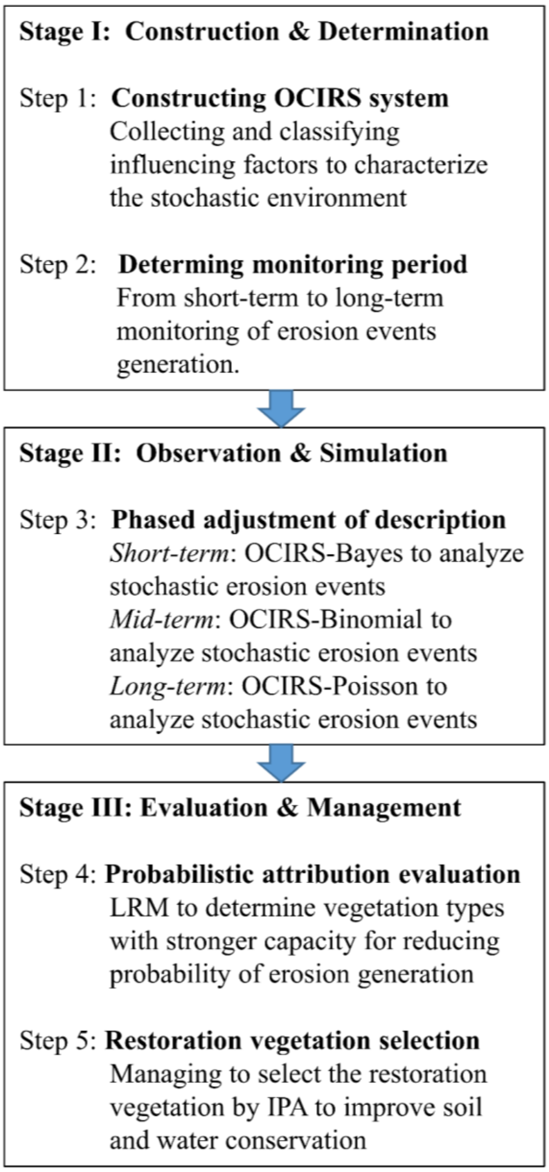


Figure 8 The framework of integrated probabilistic assessment for soil erosion monitoring and restoration strategies

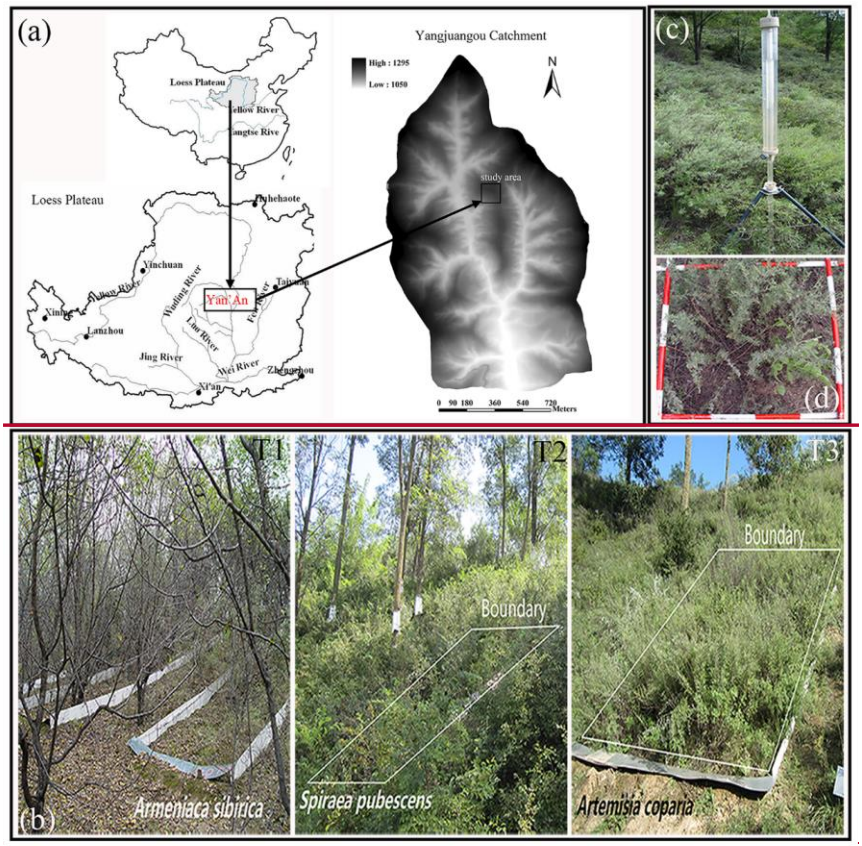


Figure 1 Description of the study area, (a) Location of the Yangjuangou Catchment; (b) restoration vegetation types at the runoff-plot scale, from left to right: *Armeniaca sibirica* (T1), *Spiraea pubescens* (T2), and *Artemisia copria* (T3); (c) field saturated conductivity measurement using Model 2800 K1 Guelph Permeameter; (d) a 1 m^2 quadrat to measure vegetation coverage

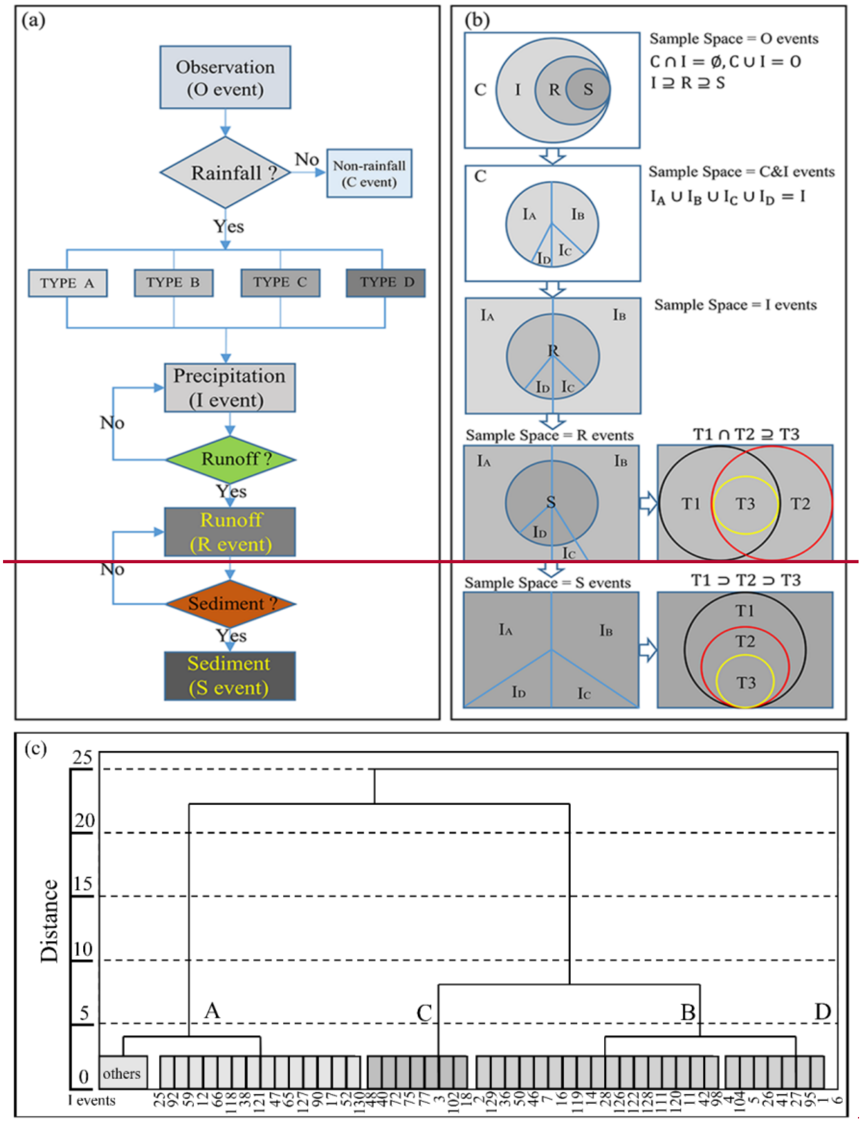


Figure 2 Construction process of OCIRS-Bayes analysis framework, (a) flow chart of confirming process of all elements in OCIRS-Bayes system; (b) Venn diagram of the relationships of all elements in OCIRS-Bayes system; (c) result of hierarchical cluster analysis of 130 individual rainfall events

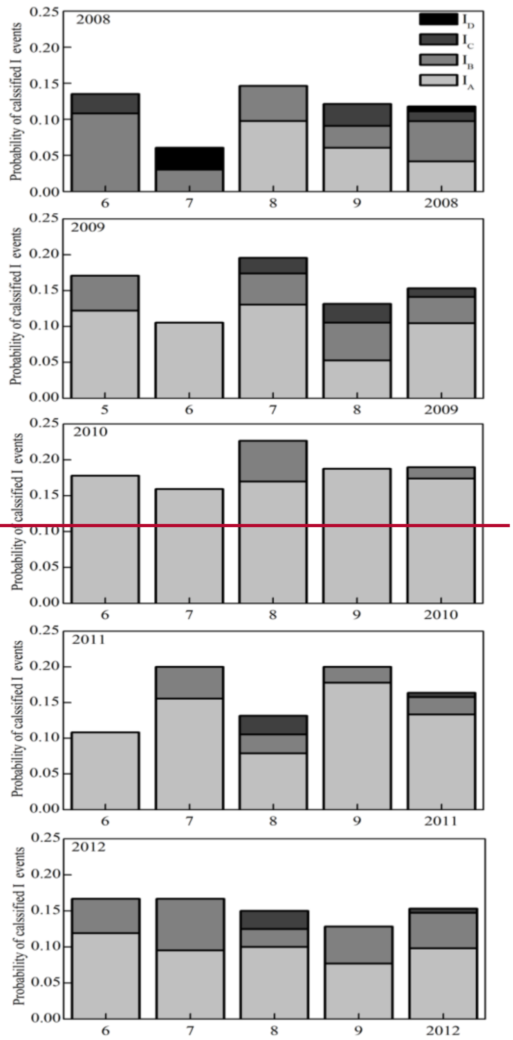


Figure 3 The probability distributions of four rainfall event types at month and seasonal scales over five rainy seasons

带格式的：左侧： 3.17 厘米，右侧： 3.17 厘米，顶端： 2.54 厘米，底端： 2.54 厘米，宽度： 21 厘米，高度： 29.7 厘米，无网格

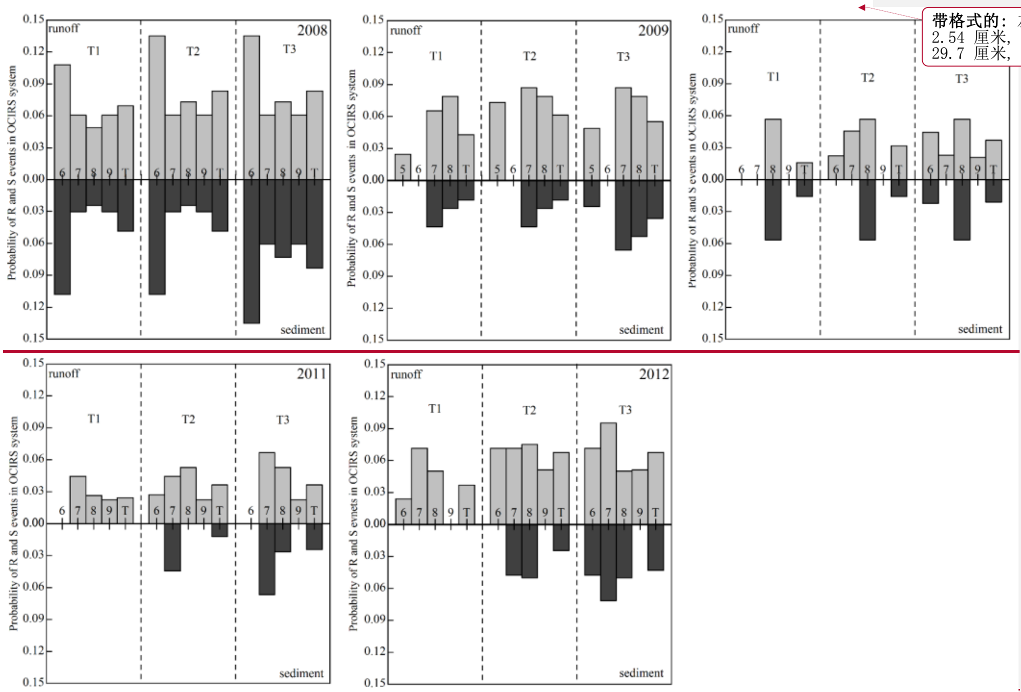


Figure 4 The probability distributions of soil erosion in three restoration vegetation types at month

and seasonal scales over five rainy seasons, the Arabic numbers and letter "T" on the abscissa in each

plot represent the month and total reason respectively, the same as follow figures

Figure 5

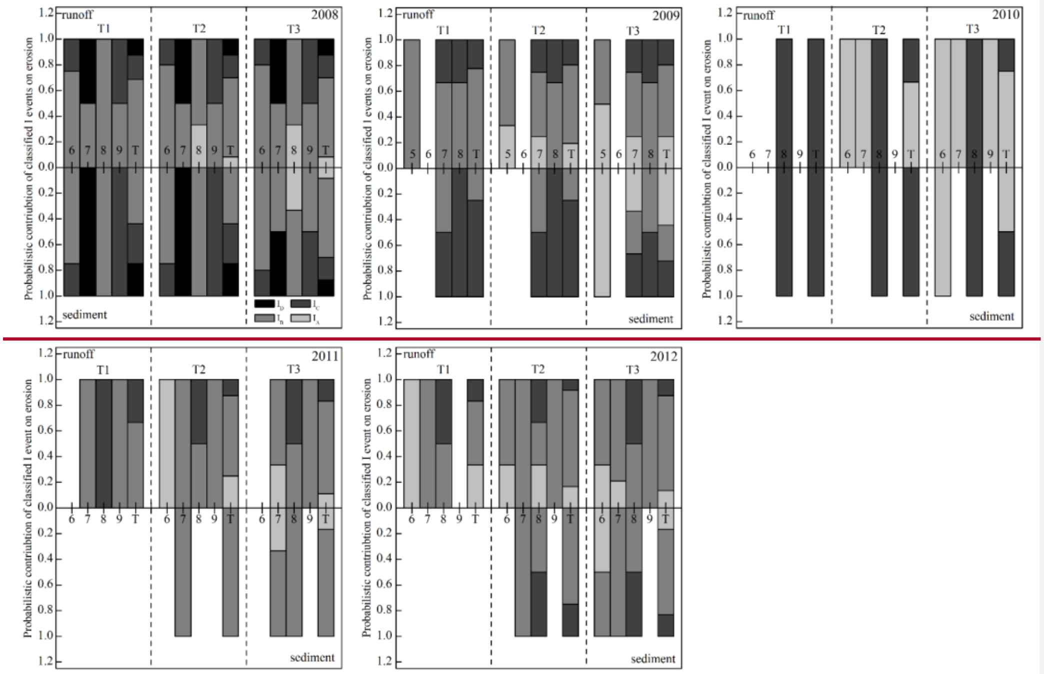


Figure 5 The distribution of probabilistic contribution of four rainfall event types on one stochastic soil erosion in three restoration vegetation types at month and seasonal scales over five rainy seasons

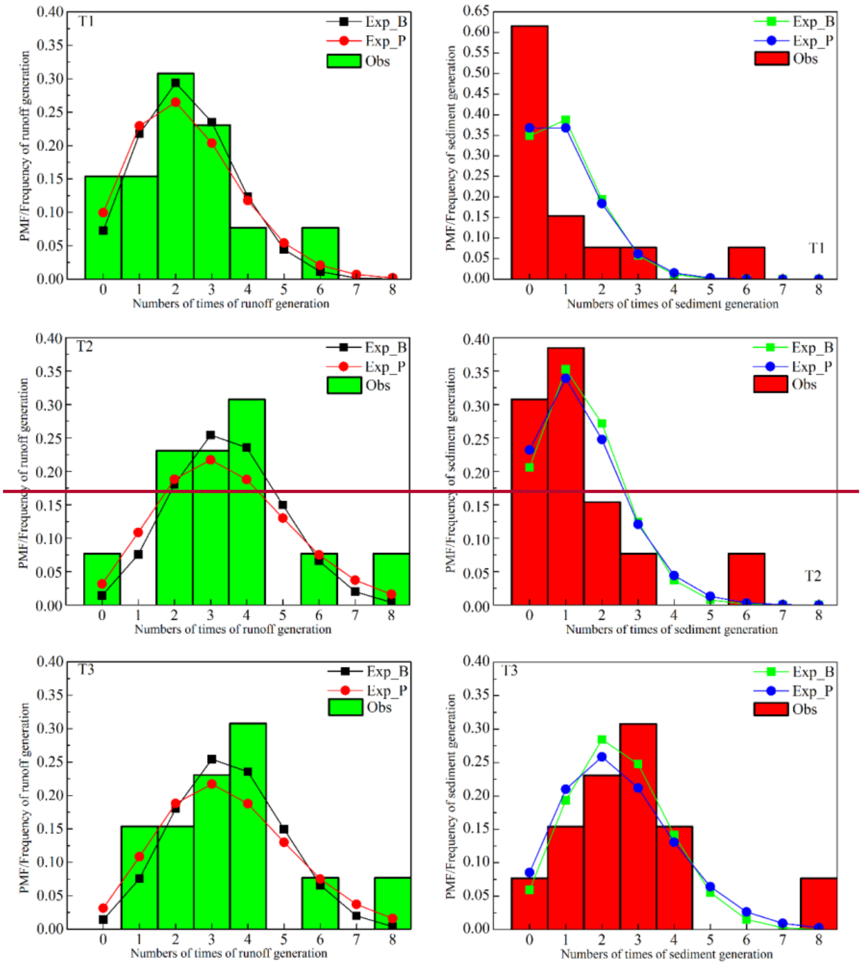
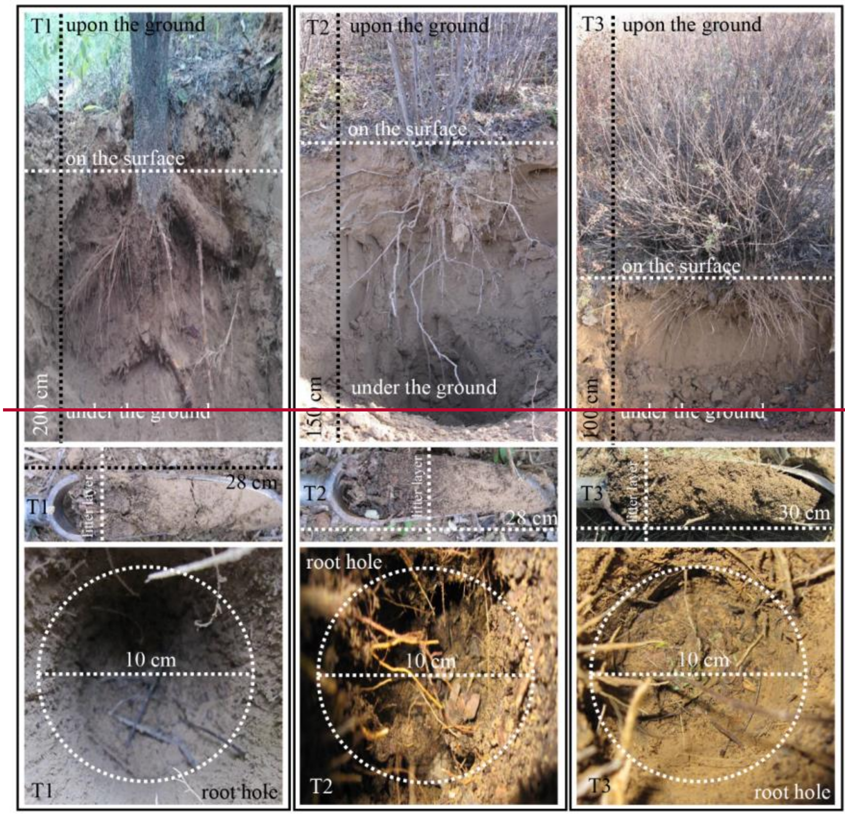


Figure 6 The comparison the prediction of stochasticity of soil erosion by binomial and Poisson PMFs and observed frequency of numbers of times of soil erosion event in three restoration vegetation types, Exp_B and Exp_P means the expected values in binomial and Poisson PMF respectively, and histogram represents observed value.



~~Figure 7 Morphological structure properties of thee restoration vegetation types including litter layer, root system distribution. The diameter and depth of samples which were indicted by the dashed line are approximately 10 cm and 30 cm respectively~~

Authors’ Response:

Response to Anonymous Referee #1

We very thank for the referee’s positive comments on our original manuscript, and also appreciate these detailed comments which can undoubtedly improve the quality of this paper. We have carefully read all the comments and make some brief responses by point to point. In the revised manuscript, we have rewritten and restructured the original manuscript on the basis of the referee’s useful suggestions and comments.

Comment 1:
In the abstract L32. Change the “erosion random events” into “random erosion events”

Response 1:
In the revised manuscript, we have changed the incorrect expression. We also have carefully checked the similar incorrect expression.

Comment 2:
In the introduction section L93-95. Deleting this sentence or putting it on the end of introduction

Response 2:
We have rewritten the introduction section in the revised manuscript, and have deleted this sentence.

Comment 3:
At the end of introduction (L117-126). Restructuring this part, or adding L93-95 and L111-115 into this part.

Response 3:
We have adjusted this part and rewritten the aim and meaning of this study in the line 168-181 in the introduction section of revised manuscript.

Comment 4:
In the method and material section (L178-206). Modification of the terminology about random event expression.

Response 4:
There are some incorrected terminological expressions about random events from L178 to L206. We have rewritten and supplemented this part in revised manuscript from line 187 to line 219.

Comment 5:
In the result section (L314-351). Explanation of the reason for using two probabilistic approaches, and the difference between the two approaches

Response 5:
In the result section, in order to quantitatively describe the stochasticity of environment affecting the generation of runoff and sediment. Firstly, we introduce the OCIRS system to calculate the probability of random runoff and sediment events occurring in different plot types based on probability theory in the study area. Actually, the OCIRS system could be regarded as an event-driven conceptual model indicating the relationship between all observed different weather conditions and erosion events based on the exploration of stochastic information. Because, OCIRS provide a platform to compare the risk of erosion generation through probability values. Therefore the causal effects of erosive rainfall events on the randomness of runoff and sediment could be constructed on the basis of the application of OCIRS system.

On the other hand, as a second method, Bayes model could be regard as an “inverse” application of OCIRS on describing the stochastic relationship between environment and erosion events. Because, in this study, the Bayes model could be considered as a feedback of a random erosion events on four different rainfall event types. Just like the referee's mention, the Bayes model supply more stochastic information about erosion properties, it implies how much contribution of rainfall events to any random erosion. Especially, under information deficiency conditions, Bayes model is an important supplement for assessing the randomness of erosive events occurring in different vegetation types.

We have follow the referee's suggestion, and systematical discussed the meaning and implication of the two probabilistic models in the line 527-597 in the revised manuscript.

Comment 6:
In the result section (L356-380). Explanation of the reason for using binomial and Poisson distribution function, rather than using other probabilistic distribution functions?

Response 6:
In the study, we generally hypothesize that the stochastic information or signal of rainfall is one of most important and indispensable factor to be transmitted into erosion phenomenon. Reported by former literatures, especially depending on the relevant research by Eagleson, Binomial and Poisson distribution method were applied to describe the probabilistic distribution of rainfall events, which moreover have good predictive effect on annual rainfall events. Therefore, the closed causal relationship between stochasticity of rainfall and erosion generation is our first reason for selecting binomial and Poisson distribution to describe and predict the probability distribution of runoff and sediment events.

The second reason for choosing binomial and Poisson is that the phenomenon of soil erosion can be simplified to a series random variable satisfying the theoretical hypothesis of binomial distribution. These characteristics of random variables also more satisfy with the fundamental and premise of binomial distribution application than other probabilistic models. As to the response of the referee's comment, we have added the relevant explanation in revised manuscript in discussion section.

Comment 7:
In the discussion section (L385-422). Making clearer explanation of the reason for designing OCIRS-Bayes framework

Response 7:
This comment is similar with comment 5 and 6. We have followed the referee's suggestion, and added the interpretation in discussion section of revised manuscript.

Comment 8
Checking the clerical error in figure caption and simplifying the content of tables

Response 8
The "total reason" in L944 is an obvious clerical error, we are very sorry, and we have changed it in revised manuscript. We have carefully checked other clerical errors in original manuscript and invited a native English speaker to polish the language.

Response to Prof. Puigdefábregas' comments and suggestion

Dear Prof. Puigdefábregas:

We very thank for your suggestion to our manuscript. Your comments and suggestion give us great inspiration and help to improve the quality of this paper greatly.

We also appreciate and admire the accomplishments you and your colleagues have achieved in the soil erosion science. Especially, the vegetation-driven-spatial-heterogeneity (VDSH) theory proposed by you in 2005, give us deep impression for studying the relationship between soil erosion and vegetation patterns. Because we believe this theory provides a new perspective for exploring the role of vegetation acting on the erosion processes in water-limited environment. Moreover, some of your other studies conducting in Spain also enlighten our study focusing on the soil erosion in the Loess Plateau. It is a great honor for us to receive your guidance and suggestion for our erosion study.

We have carefully read all the comments and suggestions, and also have gathered together to discussed some of suggestions very carefully. According to your and another anonymous referee suggestions, we have rewritten and restructured the original manuscript. At first, we make some brief responses by point to point to your suggestion and comments as follows:

Comment 1:

Their measure of stochasticity is a measure of probability of extreme values or of the classes of frequency values, and ignore memory of the system, which lacking characterizes true stochasticity.

Response 1:

In the original manuscript, the probability of soil erosion was measured by the frequency values of runoff and sediment events generating over five rainy seasons depending on the observational data.

The frequency value could be regarded as some of properties of erosion stochasticity, because all the erosion events were triggered by stochastic rainfall events, and to some extent, the generation of soil erosion could be regarded as a result of how the random signals of rainfall to be transmitted into the soil system and finally generate erosion events.

Prof. Puigdefábregas mentioned the ignorance of memory of the system in this paper, which gave us a very important suggestion to improve our original manuscript.

According to our field observation, we believed that, besides the randomness of rainfall events, the properties of plants and soil could also be the main factors to impact on the probability of soil erosion, and further affect the memory of the system, therefore, we have modified the original manuscript from the following aspects:

1. Make clear clarification of the stochasticity of soil erosion in the revised manuscript.
2. Reclassify and redefined all the observed rainfall events types to highlight their roles playing on the quantifying the probability of soil erosion in the revised manuscript. (more explanation is in supplementary)
3. Highlight the effects of the properties of plant and soil on the randomness of runoff and sediment events generating in different vegetation types by using logistic regression model in the revised manuscript.

Comment 2:

The approach lacks explicit of any theory and totally relies on empirical ad hoc information from small plots. And the method is designed to be used in restorations, and as such it should deal with different vegetation, topographies and soil attributes. How to deal with this issue should be commented by the authors

Response 2:

In the revised manuscript, we have supplemented the logistic regression method to analyze the effect of vegetation and soil hydrological properties on the probability of soil erosion.

In the discussion section of revised manuscript, we have highlighted that why the theory of Binomial and Poisson distribution functions could be used to describe the randomness of soil erosion, and what the difference is between binomial and Poisson distribution applied on the calculation of erosion stochasticity.

Actually, in this paper nearly all the empirical ad hoc information from small plots were quantified by the probability theories from Bayes theories to binomial-Poisson theories as well as to a series of point estimation theories.

In revised manuscript, we have tried to explore a method to systematically describe the probability of soil erosion by using Binomial-Poisson method, as well as to make attribution-analysis of randomness of erosion phenomenon by using Bayes and logistic regression method. Consequently, the combination of probability theories and model could form an integrated probabilistic assessment to analyze the erosion randomness in different vegetation types.

Secondly, we admitted the limitation of the experiment design in the study. Just as Prof. Puigdefábregas' mention, the increasing of vegetation, topographies and soil attributes will increase the numbers of small plots as well as increase the cost of operation, we have commented in the discussion section of the revised manuscript.

According to Prof. Puigdefábregas' suggestion, in the next step of erosion stochasticity study, we will try to construct some bare plots in the study area to collected more random information of soil erosion to enrich the understanding of stochastic property of erosion in different land covers.

Comment 3:
The parameter used in the transfer probability functions comes from the plots, where the application is performed. This seems incurring in circularity. The authors should clarify that in the interpretation of results.

Response 3:
The circularity of argument could probably related to our unclear expression in paper. When we received this comment of Prof. Puigdefábregas, we came together and carefully discussed the meaning of application of binomial and Poisson distribution function in original manuscript, and finally concluded that:

1. The application of binomial and Poisson probability function could act as an important role on detailing or simulating the stochastic information of soil erosion in different restoration vegetation types under month scale, rather than on predicting randomness of soil erosion mentioned by the original manuscript. Therefore in the revised manuscript, we have modified former expression.

2. The purpose of application binomial and Poisson probability function is to select more appropriate method to describe the stochastic property of erosion in detail. According to the point estimation depending on the maximum likelihood estimator and uniformly minimum variance unbiased estimator, Poisson probability function was found to be more appropriate for describing the probability of erosion generation in long-term monitoring period.

Consequently, we re-establish the whole logical structure in revised manuscript as follows:

(1) Proposing hypothesis: Randomness of soil erosion is one of important properties of erosion phenomenon, how to systematically describe the stochasticity of erosion depending on long-term field observations? And how the rainfall, vegetation and soil properties affects the stochasticity of erosion?

(2) Testing hypothesis: First, take the conditional probability to describe the probability of runoff and sediment events under rainy season scales; secondly, apply binomial and Poisson

probability function to simulate the randomness of soil erosion in detail on month scale, and compare the observed frequency distribution with simulated probability distribution; Thirdly, analyze the effect of properties of rainfall, vegetation and soil saturated hydraulic conductivity on the random runoff and sediment events by using logistic regression models; finally propose that the multiple-probability models could be regarded as an integrated probabilistic assessment to analyze stochasticity of soil erosion.

(3) Discussing hypothesis: First, make the parameter estimation to compare the appropriative of application of Binomial and Poisson probability distribution on stochasticity description. Secondly, explain the role of vegetation and soil properties acting on affecting the probability of soil erosion in different restoration vegetation types. Thirdly, mention the meaning and implication of the integrated probabilistic assessment on soil erosion study

Consequently, the adjusted logical structure in revised manuscript may be avoid the circularity in whole argument processes.

Comment 4:
The author don not mention the spatial stochasticity of rainfall and of the land attributes. The references almost lack mentioning the efforts done since the eighties in the same direction by combining temporal and spatial stochasticity.

Response 4:
Thanks for Prof. Puigdefábregas' suggestion. We have supplemented the contribution and efforts of temporal and spatial stochasticity in introduction section of revised manuscript. As Prof. Puigdefábregas' mention, there exist spatial stochasticity of rainfall and of the land attributes, however, we mainly focused on the plot scale, and to same extent, assume precipitation and soil characteristics in plot scale are continuous. The properties of different soil saturated hydraulic conductivity in the three vegetation types could probably affect the stochasticity of soil erosion, which have discussed by using logistic regression method in the revised manuscript.

Finally, we thank again for Prof. Puigdefábregas' great help and guidance for improving our study on soil erosion.