Supplement: Description frameworks of random events

Each observed stochastic weather condition with different durations in field monitoring period was defined as a random experiment. All possible outcomes of a random experiment constituted a sample space (Ω) defined as a random observational event (short for O event). Two mutually exclusive random event types—random rainfall event (I event) and random non-rainfall event (C event)—constituted the O event. Precipitation is a necessary condition of runoff production, therefore, the random runoff production event (R event) is a subset of I event. Similarly, R event is also a necessary condition of random sediment migration event (S event), causing S is contained in R. As a result, O, C, I, R, and S events constituted a random events framework (OCIRS) to describe the stochasticity of environment.

The random event duration in OCIRS is an important property of stochastic weather conditions. In particular, the duration property of I event was closely related to the transmission of stochastic signals of rainfall into the R and S events. According to the rainfall duration patterns in China (Wei et al., 2007; Yin et al., 2014), the time interval between two adjacent individual I events is set to be more than 6 hours, forming the criteria for individual rainfall classification. Therefore, we summarized duration property of all I events and classified them into four mutually exclusive I event types. They were a random extreme long rainfall event type (Ie event), a random general long duration rainfall event type (Il event), a random spanning rainfall event type (Is event) whose duration spans two consecutive days, and a random within rainfall event type (Iw event) generated in a day. Additionally, the C event can also be divided into two types at daily scale. They are random non-rainfall event type lasting a whole day (Cd event), and random non-rainfall event type whose duration is less than 24 hours (Ch event) which is interrupted by I event. Table 1 summarized the physical, probabilistic properties and implication of all random event types in OCIRS. The determining process of all random event types in OCIRS was sketched by Figure 1a, and Venn diagrams in Figure 1c explored the relationships of all random event types in OCIRS.

In fact, various combinations of I and C events formed different random event sequences, constituting the observed stochastic weather condition over field monitoring period. Considering the observed longest duration of Ie event approximating 72 hours (Table 1), we defined a random event sequence unit (RESU) as a combing pattern of I and C events in three consecutive days, and summarized ten observed RESUs in five rainy seasons (Figure 1b).

Reference:

Wei, W., C. L., B. Fu, Z. Huang, D. Wu, and L. Gui (2007), The effect of land uses and rainfall regimes on runoff and soil erosion in the semi-arid loess hilly area, China, *Journal of Hydrology*, *335*(3-4), 247-258.

Yin, S., Y. Wang, Y. Xie, and A. Liu (2014), Characteristics of intra-storm temporal pattern over China, *advances in Water Science (Chinese)*, *25*(5), 617-624.

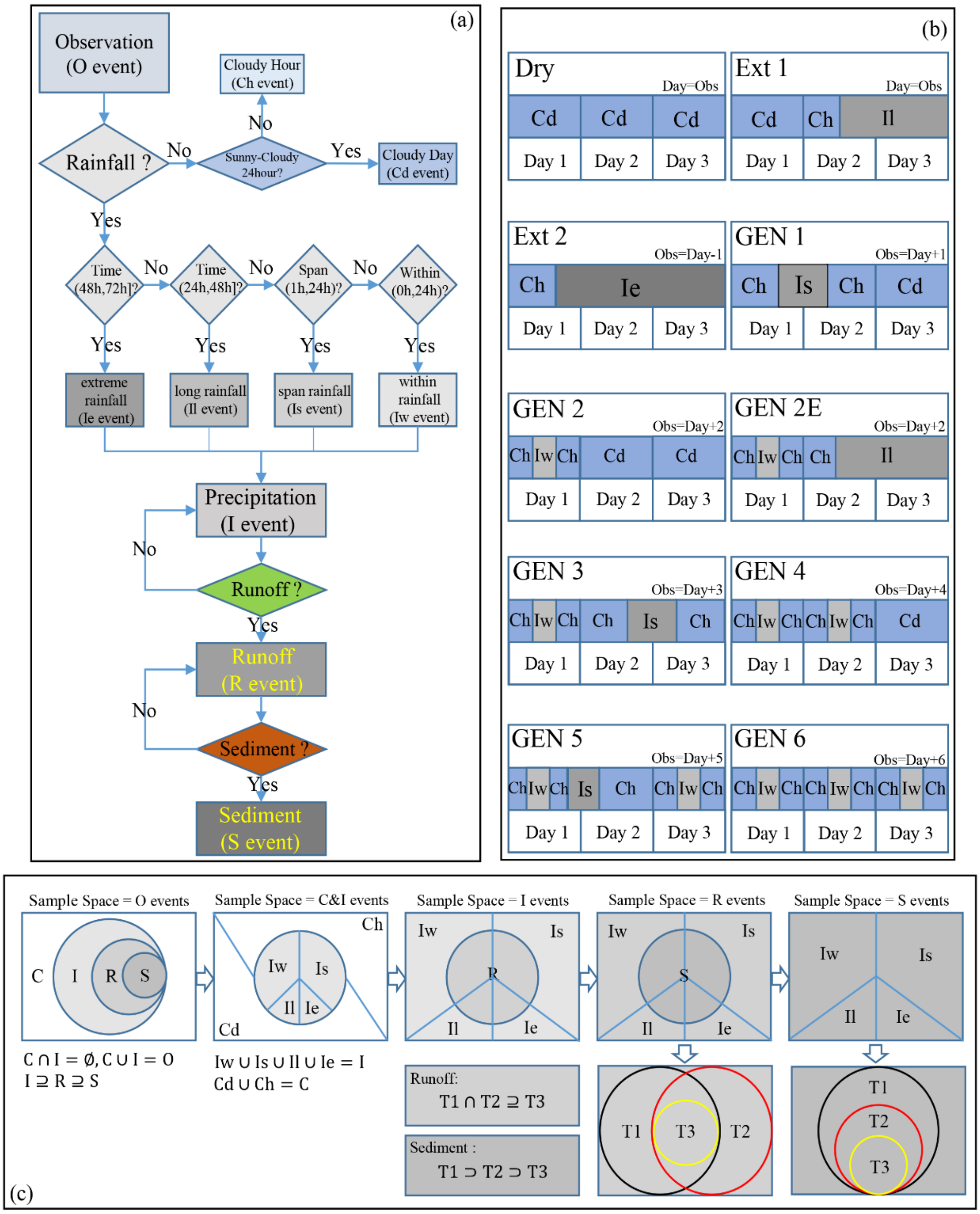

Figure 1. The OCIRS-RESU system: (a) a flow chart to determine all random event types in OCIRS framework; (b) the different combining patterns of rainfall and non-rainfall events in three consecutive days to form ten observed RESUs on five rainy seasons; (c) Venn diagram to reveal the relationship among all random events types in OCIRS framework.

Table 1. Physical and probabilistic meanings of all random event types in OCIRS

symbol	Physical meaning of random event types	Probabilistic meaning of random event types	Influencing factors and implication
O	observation events with time step ranging from 0 to 72 hours, including non-rainfall and rainfall events	random events composing the sample space of OCIRS system. The probability $P(O) = 1$	indicating the general stochastic weather conditions over rainy seasons
C	non-rainfall events with time step ranging from 0 to 24 hours, including sunny or cloudy weather condition at hour or day scales	random events, the probability of C events is the ratio of numbers of C events to O events $C \subset O, 0 \leq P(C) \leq P(O) = 1$	implying the extent of evaporation or potential evapotranspiration in weather condition.
Cd	non-rainfall events with time step being 24 hours, including observed sunny or cloudy at day scale	random events composing the subset of C events, $Cd \subseteq C, 0 \leq P(Cd) \leq P(C)$	implying the duration of evaporation or evapotranspiration at day scale
Ch	non-rainfall events with time step being less than 24hours, including observed sunny or cloudy at hour scales which intercepted by rainfall events within a day	random events composing the subset of C events, the intersection of Ch and Cd is null, $Ch \subseteq C, Cd \cup Ch = C, Cd \cap Ch = \emptyset, 0 \leq P(Ch) \leq P(C)$	influenced by the frequency of rainfall events generation, and implying the alternation of sunny and rainy in a day
I	an individual rainfall event with different precipitation, intensity and duration ranging from 0 to 72 hours, the time interval between two I events is more than 6 hours	random events, the probability of I event is ratio of numbers of I events to O events over observation $I \subset O, I \cup C = O, I \cap C = \emptyset, 0 \leq P(I) \leq P(O) = 1$	a driven force of soil erosion, which could be intercepted by vegetation and transformed into throughfall
Ie	an extreme longest individual rainfall event whose precipitation, intensity and duration were 96.6 mm, 0.022 mm/min, and 71 hours, respectively.	random events composing the subset of I events, $Ie \subseteq I, 0 \leq P(Ie) \leq P(I)$	rainfall events with low intensity and longest duration, inclining to infiltration-excess runoff generation
Il	a second longest individual rainfall events types whose average precipitation, intensity and duration were 46.8 mm, 0.026 mm/min, and 31.17 hours, respectively.	random events composing the subset of I events, the intersection of Il and Ie is null, $Il \subseteq I,\ Il \cap Ie = \emptyset, 0 \leq P(Il) \leq P(I)$	rainfall events with low intensity and long duration, inclining to infiltration-excess runoff generation
Is	A rainfall event type spanning two days whose average precipitation, intensity and duration were 28 mm, 0.049 mm/min, and 10.7 hours, respectively	random events composing the subset of I events, $Is \subseteq I,\ Is \cap Il \cap Ie = \emptyset, 0 \leq P(Il) \leq P(I)$	rainfall events with strongest rainfall intensity in middle duration, inclining to runoff and sediment generation
Iw	a rainfall event type generating within a day whose average precipitation, intensity and duration were 8.4	random events composing the subset of I events, $Iw \subseteq I,\ Iw \cap Is \cap Il \cap Ie = \emptyset,\ Iw \cup Is \cup Il \cup Ie =$	rainfall events with fewest and shortest precipitation and duration,

	mm, 0.035 mm/min, and 5.6 hours, respectively. it usually generates several times within one day.	$I, 0 \leq P(Iw) \leq P(I)$	which is different to trigger soil erosion
R	runoff event type generating on vegetation land types, it occurs on rainfall processes, and its duration is negligible	random events responding to I events, $R \subset I, R \cap C = \emptyset, 0 \leq P(R) < P(I)$	influenced by rainfall and vegetation properties.
S	sediment event occurring on vegetation land types, it occurs on runoff processes, and its duration is negligible	random events responding to R events, $S \subset R \subset I, S \cap C = \emptyset, 0 \leq P(S) \leq P(R) < P(I)$	driven by R events, and affected by rainfall and vegetation properties.