

# 1 Rainfall and streamflow sensor network design: a review of 2 applications, classification, and a proposed framework

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7  
8 **Abstract.** Sensors and sensor networks play an important role in decision-making related to water quality,  
9 operational streamflow forecasting, flood early warning systems and other areas. In this paper we review a number  
10 of existing applications and analyse a variety of evaluation and design procedures for sensor networks with respect  
11 to various criteria. Most of the existing approaches focus on maximising the observability and information content  
12 of a variable of interest. From the context of hydrological modelling only a few studies use the performance of the  
13 hydrological simulation in terms of output discharge as a design criteria. In addition to the review, we propose a  
14 framework for classifying the existing design methods, and a generalised procedure for an optimal network design  
15 in the context of rainfall-runoff hydrological modelling.

16  
17 **Keywords:** Sensor network design, Surface hydrological modelling, Precipitation, Discharge, Review,  
18 Geostatistics, Information Theory, Expert Recommendations

## 19 1 Introduction

20 Optimal design of sensor networks is a key procedure for improved water management as it provides information  
21 about the states of water systems. As the processes taking place in catchments are complex and the measurements  
22 are limited, the design of sensor networks is (and has been) a relevant topic since the beginning of the International  
23 Hydrological decade (1965 – 1974, TNO 1986) until today (Pham and Tsai 2016). During this period, the scientific  
24 community has not yet arrived to an agreement about a unified methodology for sensor network design due to the  
25 diversity of cases, criteria, assumptions, and limitations. This is evident from the range of existing reviews on  
26 hydrometric network design, such as those presented by WMO (1972), TNO (1986), Nemec and Askew (1986),  
27 Knapp and Marcus (2003), Pryce (2004), NRC (2004) and Mishra and Coulibaly (2009).

28  
29 The design of rainfall and streamflow sensor networks depends to a large extent on the scale of the processes to  
30 be monitored and the objectives to address (TNO 1986, Loucks et al. 2005). Therefore, the temporal and spatial  
31 resolution of measurements are driven by the measurement objectives. For example, information for long-term  
32 planning does not require the same level of temporal resolution as for operational hydrology (WMO 2009,  
33 Dent 2012). On the global and country scale, sensor networks are commonly used for climate studies and trend  
34 detection (Cihlar et al. 2000, Grabs and Thomas 2002, WMO 2009, Environment Canada 2010, Marsh 2010,  
35 Whitfield et al. 2012), and denoted as National Climate Reference Networks (WMO 2009). On a regional or  
36 catchment-scale, applications require careful selection of monitoring stations, since water resources planning and

37 management decisions, such as operational hydrology and water allocation, require high temporal and spatial  
38 resolution data (Dent 2012).

39  
40 This paper presents a review of methods for optimal design and evaluation of precipitation and discharge sensor  
41 networks at catchment scale, proposes a framework for classifying the design methods, and suggests a generalised  
42 framework for optimal network design for surface hydrological modelling. It is possible to extend this framework  
43 to other variables in the hydrological cycle, since optimal sensor location problems are similar. The framework  
44 here introduced is part of the results of the FP7 WeSenseIt project ([www.wesenseit.eu](http://www.wesenseit.eu)), and the validation of the  
45 proposed methodology will be presented in subsequent publications. This review does not consider in-situ  
46 installation requirements or recommendations, so the reader is referred to WMO (2008a) for the relevant and  
47 widely accepted guidelines, and to Dent (2012) for current issues in practice.

48  
49 The structure of this paper is as follows: first, a classification of sensor network design approaches according to  
50 the explicit use of measurements and models is presented, including a review of existing studies. Next, a second  
51 way of classification is suggested, which is based on the classes of methods for sensor network analysis, including  
52 statistics, Information Theory, case-specific recommendations and others. Then, based on the reviewed literature,  
53 an aggregation of approaches and classes is presented, identifying potential opportunities for improvement.  
54 Finally, a general procedure for the optimal design of sensor networks is proposed, followed by conclusions and  
55 recommendations.

## 56 **1.1 Main principles of network design**

57 The design of a sensor network use the same concepts as experimental design (Kiefer and Wolfowitz 1959, Fisher  
58 1974). The design should ensure that the data is sufficient and representative, and can be used to derive the  
59 conclusions required from the measurements. (EPA 2002), or to assess the water status of a river system (EC 2000).  
60 In the context of rainfall-runoff hydrological modelling, provide the sufficient data for accurate simulation and  
61 forecasting of discharge and water levels, at stations of interest.

62  
63 The objectives of the sensor network design have been categorised into two groups, the optimality alphabet  
64 (Fedorov 1972, Box 1982, Fedorov and Hackl 1997, Pukelsheim 2006, Montgomery 2012), which uses different  
65 letters to name different design criteria, and the Bayesian framework (Chaloner en Verdinelli 1995, DasGupta  
66 1996). The alphabetic design is based on the linearization of models, optimising particular criteria of the  
67 information matrix (Fedorov and Hackl 1997). Bayesian methods are centred on principles of decision making  
68 under uncertainty, in which it seeks to maximise the gain in Information (Shannon 1948) between the prior and  
69 posterior distributions of parameters, inputs or outputs (Lindley 1956, Chaloner and Verdinelli 1995). Among the  
70 most used alphabetic objectives are the D-optimal, which minimises the area of the uncertainty ellipsoids around  
71 the model parameters; and G-optimal, which minimises the variance of the predicted variable, which can also be  
72 used as objective functions in the Bayesian design.

73  
74 These general objectives are indirectly addressed in the literature of optimisation of hydrometric sensor networks,  
75 achieved by the use of several functional alternatives. These approaches do not consider block experimental design

76 (Kirk 2009), due to the incapacity to replicate initial conditions in a non-controlled environment, such as natural  
77 processes.

78  
79 On the practical end, the design of a sensor network should start with the institutional setup, purposes, objectives  
80 and priorities of the network (Loucks et al. 2005, WMO 2008b). From the technical point of view, an optimal  
81 measurement strategy requires the identification of the process, for which data is required (Casman et al. 1988,  
82 Dent 2012). Considering that neither the information objectives are unique and consistent, nor the characterisation  
83 of the processes is complete, the re-evaluation of the sensor network design should occur on a regular basis.  
84 Therefore, the sensor network should be re-evaluated when either the studied process, information needs,  
85 information use, or the modelling objectives change. Consequently, regulations regarding monitoring activities are  
86 not often strict in terms of station density, but in the suitability of data to provide information about the status of  
87 the water system (EC 2000, EPA 2002).

88  
89 The design of meteorological and hydrometric sensor networks should consider at least three aspects. First, it  
90 should meet various objectives that are sometimes conflicting (Loucks et al. 2005, Kollat et al. 2011). Second, it  
91 should be robust under the events of failure of one or more measurement stations (Kotecha et al. 2008). Third, it  
92 must take into account different purposes and users with different temporal and spatial scales (Singh et al. 1986).  
93 Therefore, the design of an optimal sensor network is a multi-objective problem (Alfonso et al. 2010b).

94  
95 The sensor network design can also be seen from an economic perspective (Loucks et al. 2005). In most cases, the  
96 main limitation in the deployment of sensor networks is related to costs, being sometimes the main driver of  
97 decisions related to reduction of the monitoring networks. The valuation between the costs of the sensor networks  
98 and the cost of having insufficient information is not usually considered, because the assessment of the  
99 consequences of decisions is made *a-posteriori* (Loucks et al. 2005, Alfonso et al. 2016). In most studies, it is seen  
100 that the improvement of information content metrics (e.g., entropy, uncertainty reduction, among others) is  
101 marginal as the number of extra sensors increases (Pardo-Iguzquiza 1998, Dong et al. 2006, Ridolfi et al. 2011),  
102 and thus the selection of the adequate number of sensors can be based on a threshold in the rate of increment in  
103 the objective function. However, in many practical applications the number of available sensors may be defined  
104 by budget limitations. Therefore, the optimal number of sensors in a network is strictly case-specific (WMO 2008).

## 105 **1.2 Scenarios for sensor network design: Augmentation, relocation and reduction**

106 Scenarios for designing of sensor networks may be categorised into three groups: augmentation, relocation and  
107 reduction (NRC 2004, Mishra and Coulibaly 2009, Barca et al. 2015). *Augmentation* refers to the deployment of  
108 at least one additional sensor in the network, whereas *Reduction* refers to the opposite case, where at least one  
109 sensor is removed from the original network. *Relocation* is about repositioning the existing network nodes.

110  
111 The lack of data usually drives the sensor network augmentation, whereas economic limitations usually push for  
112 reduction. These costs of the sensor network usually relate to the deployment of physical sensors in the field,  
113 transmission, maintenance and continuous validation of data (WMO 2008).

115 Augmentation and relocation problems are fundamentally similar, as they require estimation of the measured  
116 variable at ungauged locations. For this purpose, statistical models of the measured variable are often employed.  
117 For example, Rodriguez-Iturbe and Mejia (1974) described rainfall regarding its correlation structure in time and  
118 space; Pardo-Igúzquiza (1998) expressed areal averages of rainfall events with ordinary Kriging estimation;  
119 Chacón-Hurtado et al. (2009) represented rainfall fields using block Kriging. In contrast, for network reduction,  
120 the analysis is driven by what-if scenarios, as the measurements become available. Dong et al. (2005) employ this  
121 approach to re-evaluate the efficiency of a river basin network based on the results of hydrological modelling.  
122

123 In principle, augmentation and relocation aim to increase the performance of the network (Pardo-Igúzquiza 1998,  
124 Nowak et al. 2010). In reduction, on the contrary, network performance is usually decreased. The driver for these  
125 decisions is usually related to factors such as operation and maintenance costs (Moss et al. 1982, Dong et al. 2005).

### 126 **1.3 Role of measurements in rainfall-runoff modelling**

127 The typical data flow for hydrological rainfall-runoff modelling can be summarised as in Fig. 1. For discharge  
128 simulation, precipitation and evapotranspiration are the most common data requirements (WMO 2008, Beven  
129 2012), while discharge data is commonly employed for model calibration, correction and update (Sun et al. 2015).  
130 Data-driven hydrological models may use measured discharge as input variables as well (e.g., Solomatine and Xue  
131 2004, Shrestha and Solomatine 2006). Methods for updating of hydrological models have been widely used in  
132 discharge forecasting as data assimilation, using the model error to update the model states. In this way, more  
133 accurate discharge estimates can be obtained (Liu et al. 2012, Lahoz and Schneider 2014). In real-time error  
134 correction schemes, typically, a data-driven model of the error is employed which may require as input any of the  
135 mentioned variables (Xiong and O'Connor 2002, Solomatine and Ostfeld 2008).  
136

137 In a conceptual way, we can express the quantification of discharge at a given station as (Solomatine and Wagener  
138 2011):  
139

$$Q = \hat{Q}(x, \theta) + \varepsilon \quad (1)$$

140  
141 Where  $Q$  is the recorded discharge,  $\hat{Q}(x, \theta)$  represents a hydrological model, which is function of measured  
142 variables (mainly precipitation and discharge,  $x$ ) and the model parameters ( $\theta$ ).  $\varepsilon$  is the simulation error, which is  
143 ideally independent of the model, but in practice is conditioned by it. Considering that neither the measurements  
144 are perfect, nor the model unbiased, the variance of the estimates is proportional to the uncertainty in the model  
145 inputs,  $\sigma^2(x)$ , and the uncertainty in model parameters,  $\sigma^2(\theta)$ :  
146

$$\sigma^2(\hat{Q}(x, \theta)) \propto \sigma^2(x), \sigma^2(\theta) \quad (2)$$

147 **2 Classification of approaches for sensor network evaluation**

148 There is a variety of approaches for the evaluation of sensor networks, ranging from theoretically sound to more  
149 pragmatic. In this section, we provide a general classification of these approaches, and more details of each method  
150 are given in the next section.

151  
152 Although most of the approaches for the design of sensor networks make use of data, some rely solely on  
153 experience and recommendations. Therefore, a first tier in the proposed classification consists of recognising both  
154 measurement-based and measurement-free approaches (Fig. 2). The former make use of the measured data to  
155 evaluate the performance of the network (Tarboton et al. 1987, Anctil et al. 2006), while the latter use other data  
156 sources (Moss and Tasker 1991), such as topography and land use.

157 **2.1 Measurement-based evaluation**

158 The measurement-based approach can be furtherly subdivided into model-free and model-based approaches  
159 (Fig. 2), depending on the use of modelling results in the performance metric.

160 **2.1.1 Model-free performance evaluation**

161 In model-free approaches, water systems and the external processes that drive their behaviour are observed through  
162 existing measurements, without the use of catchment models. Then, metrics about amount and quality of  
163 information in space and time are evaluated with regards to the management objectives and the decisions to be  
164 made in the system. Some performance metrics in this category are joint entropy (Krstanovic and Singh 1992),  
165 Information Transfer (Yang and Burn 1994), interpolation variance (Pardo-Igúzquiza 1998, Cheng et al. 2007)  
166 and autocorrelation (Moss and Karlinger 1974), among others. Fig. 3 presents the flowchart for the case when  
167 precipitation and discharge, as main drivers of catchment hydrology (WMO 2008) are considered, in model-free  
168 network evaluation.

169  
170 Fundamentally, the model-free approach aims to minimise the variance of the measured variable, therefore, (and  
171 in theory) minimising the variance in the estimation (equation 3). However, a design that is optimal for estimation  
172 is not necessarily also optimal for prediction (Chaloner and Verdinelli 1995).

173

$$\min \sigma^2(\hat{Q}(x, \theta)) \propto \min(\sigma^2(x)) \quad (3)$$

174  
175 Application of model-free approaches can be found in Krstanovic and Singh (1992), Nowak et al. (2010), Li et al.  
176 (2012). Model-free evaluations are suitable for sensor network design aiming mainly to water resources planning,  
177 in which diverse water interests must be balanced. Due to the lack of a quantitative performance metric that relates  
178 simulated discharge, this kind of evaluations do not necessarily improve rainfall-runoff simulations.

179 **2.1.2 Model-based performance evaluation**

180 In the model-based approach, the performance of sensor networks is carried out using a catchment model (Dong  
181 et al. 2005, Xu et al. 2013). In this case, measurements of precipitation are used to simulate discharge, which is

182 compared to the discharge measurements at specific locations. Therefore, any metric of the modelling error could  
183 be used to evaluate the performance of the network. Fig. 4 presents a generic model-based approach for evaluating  
184 sensor networks.

185  
186 In the model-based design of sensor networks, it is assumed that the model structure and parameters are adequate.  
187 Therefore, it is possible to identify a set of measurements ( $x$ ) which minimise the modelling error as.

188

$$\min \sigma^2(\epsilon) \alpha \min(|Q - \hat{Q}(x, \theta)|) \quad (4)$$

189  
190 The need for the catchment model and possible high computational efforts for multiple model runs are some  
191 disadvantages of this approach. The computational load is especially critical in case of complex distributed models.  
192 It is worth mentioning particular model error metrics (Nash and Sutcliffe 1970, Gupta et al. 2009) may qualify the  
193 network by its ability to capture certain hydrological processes (Bennet et al. 2013), affecting the network  
194 evaluation.

195 **2.2 Measurement-free evaluation**

196 As it is seen from its name, this approach does not require the previous collection of data of the measured variable  
197 to evaluate the sensor network performance. The evaluation of sensor networks is based on either experience or  
198 physical characteristics of the area such as land use, slope or geology. In this group of methods, the following can  
199 be mentioned: case-specific recommendations (Bleasdale 1965, Wahl and Crippen 1984, Karasseff 1986, WMO  
200 2008a) and physiographic components (Tasker 1986, Laize 2004). This approach is the first step towards any  
201 sensor network development (Bleasdale 1965, Moss et al. 1982, Nemec and Askew 1986, Karasseff 1986).

202 **3 Classification of methods for sensor network evaluation**

203 In this section, we classify the methods used to quantify the performance of the sensor networks based on the  
204 mathematical apparatus used to evaluate the network performance. These methods can be broadly categorised in  
205 statistics-based, information theory-based, expert recommendations, and others.

206 **3.1 Statistics-based methods**

207 Statistics-based methods refer to methods where the performance of the network is evaluated with statistical  
208 uncertainty metrics of the measured or simulated variable. These methods aim to minimise either interpolation  
209 variance (Rodriguez-Iturbe and Mejia 1974, Bastin et al. 1984, Bastin and Gevers 1985, Pardo-Iguzquiza 1998,  
210 Bonacorso 2003), cross-correlation (Maddock 1974, Moss and Karlinger 1974, Tasker 1986), or model error  
211 (Dong et al. 2005, Xu et al. 2015).

212 **3.1.1 Interpolation variance (geostatistical)**

213 Methods to evaluate sensor networks considering a reduction in the interpolation variance assume that for a  
214 network to be optimal, the measured variable should be as certain as possible in the domain of the problem. To  
215 achieve this, a stochastic interpolation model that provides uncertainty metrics is required. Geostatistical methods

such as Kriging (Journel and Huijbregts 1978, Cressie 1993), or Copula interpolation (Bárdossy 2006) have an explicit estimation of the interpolation error. This characteristic makes it suitable to identify areas with expected poor interpolation results, (Bastin et al. 1984, Pardo-Igúzquiza 1998, Grimes et al. 1999, Bonaccorso et al. 2003, Cheng et al. 2007, Nowak et al. 2009, 2010, Shafiei et al. 2013).

In the case of Kriging, the optimal estimation of a variable at ungauged locations is assumed to be a linear combination of the measurements, with a Gaussian distributed probability distribution function. Under the ordinary Kriging formulation, the variance in the estimation ( $\sigma^2$ ) of a variable at location ( $u$ ) over a catchment is:

$$\sigma^2(u) = C_0 - \sum_{\alpha=1}^n \lambda_\alpha(u) - C(u_\alpha - u) \quad (5)$$

Where  $C_0$  refers to the variance of the random field,  $\lambda_\alpha$  are the Kriging weights for the station  $\alpha$  at the ungauged location  $u$ .  $C(u_\alpha - u)$  is the covariance between the station  $\alpha$  at the location  $u_\alpha$  and the interpolation target at the location  $u$ .  $n$  represents the total number of stations in the neighbourhood of  $u$  and used in the interpolation.

Therefore, as an objective function the optimal sensor network is such that the total Kriging variance (TKV) is minimum:

$$TKV = \sum_{u=1}^U \sigma^2(u) \quad (6)$$

Where  $U$  is the total number of discrete interpolation targets in the catchment or domain of the problem.

Bastin and Gevers (1984) optimised a precipitation sensor network at pre-defined locations to estimate the average precipitation for a given catchment. Their selection of the optimal sensor location consisted of minimising the normalised uncertainty by reducing the network. The main drawback of their approach is that the network can only be reduced and not augmented. Similar approaches have also been used by Rodriguez-Iturbe and Mejia (1974), Bogárdi et al. 1985, and Morrissey et al. (1995). Pardo-Igúzquiza (1998) advanced this formulation by removing the pre-defined set of locations (allowing augmentation). Instead, rain gauges were allowed to be placed anywhere in the catchment and its surroundings. A simulated annealing algorithm is used to search for the find the optimal set of sensors to minimise the interpolation uncertainty.

Copula interpolation is a geostatistical alternative to Kriging for the modelling of spatially distributed processes (Bárdossy 2006, Bárdossy and Li 2008, Bárdossy and Pegram 2009). As a geostatistical model, the copula provides metrics of the interpolation uncertainty, considering not only the location of the stations and the model parameterisation but also the value of the observations. Li et al. (2011) use the concept of copula to provide a framework for the design of a monitoring network for groundwater parameter estimation, using a utility function, related to the cost of a given decision with the available information.

252 In the case of copula, the full conditional probability distribution function of the variable is interpolated. As such,  
253 the interpolation uncertainty depends on the confidence interval, measured values, parameterisation of the copula  
254 and the relative position of the sensors in the domain of the catchment. More details on the formulation of copula-  
255 based design can be found in Bárdossy and Li (2008).

256  
257 Cheng et al. (2007), as well as Shafiei et al. (2013), recognised that the temporal resolution of the measurements  
258 affects the definition of optimality in minimum interpolation variance methods. This change in the spatial  
259 correlation structure occurs due to more correlated precipitation data between stations in coarser sampling  
260 resolutions (Ciach and Krajewski 2006). For this purpose, the sensor network has to be split into two parts, a base  
261 network and non-base sensors. The former should remain in the same position for long periods, to characterise  
262 longer fluctuation phenomena, based on the definition of a minimum threshold for an area with acceptable  
263 accuracy. The latter is relocated to improve the accuracy of the whole system, and should be relocated as they do  
264 not provide a significant contribution to the monitoring objective.

265  
266 Recent efforts have used minimum interpolation variance approaches to consider the non-stationarity assumption  
267 of most geostatistical applications in sensor network design (Chacon-Hurtado et al. 2014). To this end, changes in  
268 the precipitation pattern and its effect on the uncertainty estimation were considered during the development of a  
269 rainfall event.

270

### 271 **3.1.2 Cross-correlation**

272 The objective of minimum cross-correlation methods is to avoid placing sensors at sites that may produce  
273 redundant information. Cross-correlation was suggested by Maddock (1974) for sensor network reduction, as a  
274 way to identify redundant sensors. In this scope, the objective function can be written as:

275

$$\rho(X_i, X_j) = \sum_{i=1}^n \sum_{j=i+1}^n \frac{cov(x_i, x_j)}{\sigma(x_i)\sigma(x_j)} \quad (7)$$

276  
277 Where  $cov$  is the covariance function between a pair of stations  $(i, j)$ , and  $\sigma$  is the standard deviation of the  
278 observations.

279  
280 Stedinger and Tasker (1985) introduced the method called Network Analysis Using Generalized Least Squares  
281 (NAUGLS), which assesses the parameters of a regression model for daily discharge simulation based on the  
282 physiographic characteristics of a catchment (Stedinger and Tasker 1985, Tasker 1986, Moss and Tasker 1991).  
283 The method builds a Generalised-Least-Square (GLS) covariance matrix of regression errors to correlate flow  
284 records and to consider flow records of different length, so the sampling mean squared error can be expressed as:  
285

$$SMSE = \frac{1}{n} \sum_{i=1}^j X_i^T (X^T \Lambda^{-1} X)^{-1} X_i \quad (8)$$

286

287 Where  $X[k, w]$  is the matrix of the ( $k$ ) basin characteristics in a window of size  $w$  at discharge measuring site  $i$ .  $\Lambda$   
288 is the GLS Weighting matrix, using a set of  $n$  gauges (Tasker 1986)

290 A comparable method was proposed by Burn and Goulter (1991), who used a correlation metric to cluster similar  
291 stations. Vivekanandan and Jagtap (2012) proposed an alternative for the location of discharge sensors in a  
292 recurrent approach, in which the most redundant stations were removed, and the most informative stations  
293 remained using the Cooks' D metrics, a measure of how the spatial regression model at a particular site is affected  
294 by removing another station. The result of these type of sensors is sparse, as the redundancy of two sensors  
295 increases with the inverse of the distance between them (Mishra and Coulibaly 2009).

296 **3.1.3 Model output error**

297 These methods assume that the optimal sensor network configuration is such that satisfy a particular modelling  
298 purpose, e.g. a minimum error in simulated discharge. Considering this, the design of a sensor network should be  
299 such that minimises the difference between the simulated and recorded variable:

$$300 \min f(|Q - \hat{Q}(x, \theta)|) \quad (9)$$

301  
302 Where  $f$  is a metric that summarises the vector error such as Bias, Root Mean Squared Error (RMSE), or Nash-  
303 Sutcliffe Efficiency (NSE);  $Q$  is the measurements of the simulated variable, and  $\hat{Q}$  is the simulation results using  
304 inputs  $x$ , and parameters  $\theta$ . Bias measures the mean deviation of the results between the observations ( $Q$ ) and  
305 simulation results ( $\hat{Q}$ ) for  $t$  pairs of observations and simulation results:

$$306 \text{Bias} = \frac{1}{n} \sum_{i=1}^t (\hat{Q}_i - Q_i) \quad (10)$$

307  
308 This metric theoretically varies from minus infinity to infinity, and its optimal value is equal to zero. The root  
309 mean square error (RMSE) measures the standard deviation of the residuals as:

$$310 \text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^t (\hat{Q}_i - Q_i)^2} \quad (11)$$

311  
312 The RMSE can vary then from zero to infinity, where zero represents a perfect fit between model results and  
313 observations. As RMSE is a statistical moment of the residuals, the result is a magnitude rather than a score.  
314 Therefore, benchmarking between different case studies is not trivial. To overcome this issue, Nash and Sutcliffe  
315 (1970) proposed a score (also known as coefficient of determination) based on the ratio of the model results in  
316 variance over the observation variance as:

317

$$NSE = 1 - \frac{\sum_{i=1}^t (\hat{Q}_i - Q_i)^2}{\sum_{i=1}^t (Q_i - \bar{Q}_i)^2} \quad (12)$$

318

319 In which  $Q$  are the measurements,  $\hat{Q}$  are the model results and  $\bar{Q}$  is the average of the recorded series.

320

321 Theoretically, this score varies from minus infinity to one. However, its practical range lies between zero and one.  
 322 On the one hand, an NSE equal to zero indicates that the model has the same explanatory capabilities that the mean  
 323 of the observations. On the other end, a value of one represents a perfect fit between model results and observations.  
 324 Model output error formulations have been used to identify the most convenient set of sensors that provide the  
 325 best model performance (Tarboton et al. 1987) to propose measurement strategies regarding the number of gauges  
 326 and sampling frequency.

327

328 Another application is provided by Dong et al. (2005) who proposed to evaluate the rainfall network using a  
 329 lumped HBV model. They found that the model performance does not necessarily improve when extra rain gauges  
 330 are placed. A similar approach was presented by Xu et al. (2013) who evaluated the effect of diverse rain gauge  
 331 locations on runoff simulation using a similar hydrological model. It was found that rain gauge locations could  
 332 have a significant impact and suggest that a gauge density less than 0.4 stations per 1000 km<sup>2</sup> can negatively affect  
 333 the model performance.

334

335 Anctil et al. (2006) aimed at improving lumped neural network rainfall-runoff forecasting models through mean  
 336 areal rainfall optimisation, and concluded that different combinations of sensors lead to noticeable streamflow  
 337 forecasting improvements. Studies in other fields have also used this method. For example, Melles et al. (2009,  
 338 2011), obtained optimal monitoring designs for radiation monitoring networks, which minimise the prediction  
 339 error of mean annual background radiation. The main drawback of this approach is that multiple error metrics are  
 340 considered, as specific objectives relate to different processes

341

342 **3.2 Information Theory-based methods**

343 The use of Information Theory (Shannon 1948) in the design of sensor networks for environmental monitoring is  
 344 based on Communication Theory, which studies the problem of transmitting signals from a source to a receiver  
 345 throughout a noisy medium. Information Theory provides the possibility of estimating probability distribution  
 346 functions in the presence of partial information with the less biased estimation (Jaynes 1957). Some of its concepts  
 347 are analogous to statistics concepts, and therefore similarities between entropy and uncertainty, as mutual  
 348 information and correlation, etc., can be found (Cover and Thomas 2005, Alfonso 2010, Singh 2013).

349

350 Information Theory-based methods for designing sensor networks mainly consider the maximisation of  
 351 information content that sensors can provide, in combination with the minimisation of redundancy among them  
 352 (Krstanovic and Singh 1992, Mogheir and Singh 2002, Alfonso et al. 2010a,b, Alfonso 2010, Alfonso et al. 2013,  
 353 Singh 2013). Redundancy can be measured by using either Mutual Information (Singh 2000, Steuer et al. 2002),

354 Directional Information Transfer (Yang and Burn 1994), Total Correlation (Alfonso et al. 2010a,b, Fahle et al.  
355 2015), among others.

356 **3.2.1 Entropy**

357 The Principle of Maximum Entropy (POME) is based on the premise that probability distribution with the largest  
358 remaining uncertainty (i.e., the maximum entropy) is the one that best represent the current stage of knowledge.  
359 POME has been used as a criterion for the design of sensor networks, by allowing the identification of the set of  
360 sensors that maximises the joint entropy among measurements (Krstanovic and Singh 1992). In other words, to  
361 provide as much information content, from the Information Theory perspective, as possible (Jaynes 1988).

362

363 In the design of sensor networks, the objective is to maximise the joint entropy ( $H$ ) of the sensor network as:

364

$$H(X_1, X_2, \dots, X_n) = - \sum_{i=1}^k \dots \sum_{j=1}^m p(x_{i1}, \dots x_{jm}) \log p(x_{i1}, \dots x_{jm}) \quad (13)$$

365

366 Where  $p(X)$  is the probability of the random variable  $X$  to take a discrete value  $x_m$ . As in many applications,  $X$  is a  
367 continuous variable which has to be discretised (quantised) into intervals ( $k, m$ ) to calculate its entropy. The  
368 probabilities are calculated following frequency analysis, such that the probability of a variable  $X$  to take a value  
369 in the interval  $i, \dots, j$  which is defined by the number of times in which this value appear, divided by the complete  
370 length of the dataset. When calculating the entropy of more than one variable simultaneously (joint entropy), joint  
371 probabilities are used.

372

373 Krstanovich and Singh (1992) presented a concise work on rainfall network evaluation using entropy. They used  
374 POME to obtain multivariate distributions to associate different dependencies between sensors, such as joint  
375 information and shared information, which was used later either reduce the network (in the case of high  
376 redundancy) or expand it (in the case of lack of common information).

377

378 Fuentes et al. (2007) proposed an entropy-utility criterion for environmental sampling, particularly suited for air-  
379 pollution monitoring. This approach considers Bayesian optimal sub-networks using an entropy framework,  
380 relying on the spatial correlation model. An interesting contribution of this work is the assumption of non-  
381 stationarity, contrary to traditional atmospheric studies, and relevant in the design of precipitation sensor networks.

382

383 The use of hydraulic 1D models and metrics of entropy have been used to select the adequate spacing between  
384 sensors for water level in canals and polder systems (Alfonso et al. 2010a,b). This approach is based on the current  
385 conditions of the system, which makes it useful for operational purposes, but it does not necessarily support the  
386 modifications in the water system conditions or changes in the operation rules. Studies on the design of sensor  
387 networks using these methods are on the rise in the last years (Alfonso 2010, Alfonso et al. 2013, Ridolfi et al.  
388 2013, Banik et al 2017).

389

390 Benefits of POME include the robustness of the description of the posterior probability distribution since it aims  
 391 to define the less biased outcome. This is because neither the models nor the measurements are completely certain.  
 392 Li et al. (2012) presented, as part of a multi-objective framework for sensor network optimisation, the criteria of  
 393 maximum (joint) entropy, as one of the objectives. Other studies in this direction have been presented by Lindley  
 394 (1956), Caselton and Zidek (1984), Guttorp et al. (1993), Zidek et al. (2000), Yeh et al. (2011) and Kang et al.  
 395 (2014).

396  
 397 More recently, Samuel et al. (2013) and Coulibaly and Samuel (2014), proposed a mixed method involving  
 398 regionalisation and dual entropy multi-objective optimisation (CRDEMO), which is a step forward if compared to  
 399 single-objective optimisation for sensor network design.

400 **3.2.2 Mutual information (trans-information)**

401 Mutual information is a measurement of the amount of information that a variable contains about another. This is  
 402 measured as the *relative entropy between the joint distribution and the product distribution* (Cover and Thomas  
 403 2005). In the simplest expression (two variables), the mutual information can be defined as:

$$I(X_1, X_2) = H(X_1) + H(X_2) - H(X_1, X_2) \quad (14)$$

404  
 405 where  $H(X_1)$  and  $H(X_2)$  is the entropy of each of the variables, and  $H(X_1, X_2)$  is the joint entropy between them.  
 406 The extension of the mutual information for more than two variables should not only consider the joint entropy  
 407 between them, but also the joint entropy between pairs of variables, leading to a significantly complex expression  
 408 for the multivariate mutual information. Regarding this issue, the multivariate mutual information can be addressed  
 409 as a nested problem, such that:  
 410

$$I(X_1, X_2, \dots, X_n) = I(X_1, X_2, \dots, X_{n-1}) - I(X_1, X_2, \dots, X_{n-1} | X_n) \quad (15)$$

411  
 412 Where  $I(X_1, X_2, \dots, X_n)$  is the multivariate mutual information among  $n$  variables, and  $I(X_1, X_2, \dots, X_{n-1} | X_n)$  is the  
 413 conditional information of  $n-1$  variables with respect to the  $n^{\text{th}}$  variable. The conditional mutual information can  
 414 be understood as the amount of information that a set of variables share with another variable (or variables). The  
 415 conditional mutual information of two variables ( $X_1$  and  $X_2$ ) with respect to a third one ( $X_3$ ) can be quantified as:  
 416

$$I(X_1, X_2 | X_3) = H(X_1 | X_3) - H(X_1 | X_2, X_3) \quad (16)$$

417  
 418 Where  $H(X_1 | X_3)$  is the conditional entropy of  $X_1$  to  $X_3$  and  $H(X_1 | X_2, X_3)$  is the conditional entropy of  $X_1$  with  
 419 respect to  $X_2$  and  $X_3$  simultaneously. The conditional entropy can be understood as the amount of information that a  
 420 variable does not share with another. The joint entropy between two variables can be quantified as:  
 421

$$H(X_1 | X_2) = \sum_{i=1}^k \sum_{j=1}^m p(X_{1i}, X_{2j}) \log \frac{p(X_{1i})}{p(X_{1i}, X_{2j})} \quad (17)$$

423 where  $p(X_1, X_2)$  is the joint probability, for  $k$  and  $m$  discrete values, of  $X_1$  and  $X_2$ .

424  
425  
426 An optimal sensor network should avoid collecting repetitive or redundant information, in other words, it should  
427 be such that reduces the mutual (shared) information between sensors in the network. Alternatively, it should  
428 maximise the transferred information from a measured to a modelled variable at a point of interest (Amoroco and  
429 Espildora 1973). Following this idea, Husain (1987) suggested an optimisation scheme for the reduction of a rain  
430 sensor network. His objective was to minimise the trans-information between pairs of stations. However,  
431 assumptions of the probability and joint probability distribution functions are strong simplifications of this method.  
432 To overcome these assumptions, the Directional Information Transfer (DIT) index was introduced (Yang and Burn  
433 1994) as the inverse of the coefficient of non-transferred information (NTI) (Harmancioglu and Yevjevich 1985).  
434 Both DIT and NTI are a normalised measure of information transfer between two variables ( $X_1$  and  $X_2$ ).  
435

$$DIT = \frac{I(X_1, X_2)}{H(X_1)} \quad (18)$$

436  
437 Particularly for the design of precipitation sensor networks, Ridolfi et al. (2011) presented a definition of the  
438 maximum achievable information content for designing a dense network of precipitation sensors at different  
439 temporal resolutions. The results of this study show that there exists a linear dependency between the non-  
440 transferred information and the sampling frequency of the observations.

441  
442 Total Correlation ( $C$ ) is an alternative measure of the amount of shared information between two or more variables,  
443 and has also been used as a measure of information redundancy in the design of sensor networks (Alfonso et al.  
444 2010a, b, Leach et al. 2015) as:

$$C(X_1, \dots, X_n) = \sum_{i=1}^n H(X_i) - H(X_1, \dots, X_N) \quad (19)$$

446  
447 Where  $C(X_1, X_2, \dots, X_n)$  is the total correlation among the  $n$  variables,  $H(X_i)$  is the entropy of the variable  $i$ , and  
448  $H(X_1, X_2, \dots, X_n)$  is the joint entropy of the  $n$  variables. Total Correlation can be seen then as a simplification of  
449 the multivariate mutual information, where only the interaction among all the variables is considered. In the design  
450 of sensor networks, it is expected that the mutual information among the different variables is minimum, therefore,  
451 the difference between the total correlation and multivariate mutual information tends to be minimised as well.  
452 The advantage of total correlation is the computational advantage that represents assuming a marginal value for  
453 the interaction among variables.

454  
455 A method to estimate trans-information fields at ungauged locations has been proposed by Su and You (2014),  
456 employing a trans-information-distance relationship. This method accounts for spatial distribution of precipitation,  
457 supporting the augmentation problem in the design of precipitation sensor networks. However, as the relationship

458 between trans-information between sensors and their distance is monotonic, the resulting sensor networks are  
459 generally sparse.

460 **3.3 Methods based on expert recommendations**

461 **3.3.1 Physiographic components**

462 Among the most used planning tools for hydrometric network design are the technical reports presented by the  
463 WMO (2008), in which a minimum density of stations depending on different physiographic units, are suggested  
464 (Table 1). Although these guidelines do not provide an indication about where to place hydrometric sensors, rather  
465 they recommend that their distribution should be as uniform as possible and that network expansion has to be  
466 considered. The document also encourages the use of computationally aided design and evaluation of a more  
467 comprehensive design. For instance, Coulibaly et al. (2013) suggested the use of these guidelines to evaluate the  
468 Canadian national hydrometric network.

469

470 Moss et al. (1982) presented one of the first attempts to use physiographic components in the design of sensor  
471 networks in a method called Network Analysis for Regional Information (NARI). This method is based on relations  
472 of basin characteristics proposed by Benson and Matalas (1967). NARI can be used to formulate the following  
473 objectives for network design: minimum cost network, maximum information and maximum net benefit from the  
474 data-collection program, in a Bayesian framework, which can be approximated as:

475

$$\log \sigma(S(|\hat{Q} - Q|)^\alpha) = a + \frac{b_1}{n} + \frac{b_2}{y} \quad (20)$$

476

477 where the function  $S(|\hat{Q} - Q|)^\alpha$  is the  $\alpha$  percentile of the standard error in the estimation of  $Q$ ,  $a$ ,  $b_1$  and  $b_2$  are the  
478 parameters from the NARI analysis,  $n$  is the number of stations used in the regional analysis, and  $y$  is the harmonic  
479 mean of the records used in the regression.

480

481 Laize (2004) presented an alternative for evaluating precipitation networks based on the use of the Representative  
482 Catchment Index (RCI), a measure to estimate how representative a given station in a catchment is for a given  
483 area, on the stations in the surrounding catchments. The author argues that the method, which uses datasets of land  
484 use and elevation as physiographical components, can help identifying areas with a insufficient number of  
485 representative stations on a catchment.

486

487 **3.3.2 Practical case-specific considerations**

488 Most of the first sensor networks were designed based on expert judgement and practical considerations. Aspects  
489 such as the objective of the measurement, security and accessibility are decisive to select the location of a sensor.  
490 Nemec and Askew (1986) presented a short review of the history and development of the early sensor networks,  
491 where it is highlighted that the use of “basic pragmatic approaches” still had most of the attention, due to its  
492 practicality in the field and its closeness with decision makers.

493

494 Bleasdale (1965) presented a historical review of the early development process of the rainfall sensor networks in  
495 the United Kingdom. In the early stages of the development of precipitation sensor networks, two main  
496 characteristics influencing the location of the sensors were identified: at sites that were conventionally satisfactory  
497 and where good observers were located. However, the necessity of a more structured approach to select the location  
498 of sensors was underlined. As a guide, Bleasdale (1965) presented a series of recommendations on the minimal  
499 density of sensors for operational purposes, summarised in Fig. 5, relating the characteristics of the area to be  
500 monitored and the minimum required a number of rain sensors, as well as its temporal resolution.

501  
502 In a more structured approach, Karasseff (1986) introduced some guidelines for the definition of the optimal sensor  
503 network to measure hydrological variables for operational hydrological forecasting systems. The study specified  
504 the minimum requirements for the density of measurement stations based on the fluctuation scale and the  
505 variability of the measured variable by defining zonal representative areas. This author suggested the following  
506 considerations for selecting the optimal placement of hydrometric stations:

507  
508 • *“in the lower part of inflow and wastewater canals”*  
509 • *“at the heads of irrigation and watering canals taking water from the sources”*  
510 • *“at the beginning of a debris cone before the zone of infiltration, and at its end, where ground-water*  
511 *decrement takes place”*  
512 • *“at the boundaries of irrigated areas and zones of considerable industrial water diversions (towns) ”*  
513 • *“at the sites of hydroelectric power plants and hydro projects”*

514  
515 From a different perspective, Wahl and Crippen (1984), as well as Mades and Oberg (1986) proposed a qualitative  
516 score assessment of different factors related to the use of data and the historical availability of records for the  
517 evaluation of sensor value. Their analyses aimed at identifying candidate sensors to be discontinued, due to their  
518 limited accuracy.

519 **3.3.3 User survey**

520 These approaches aim to identify the information needs of particular groups of users (Sieber 1970), following the  
521 idea that the location of a certain sensor (or group of sensors) should satisfy at least one specific purpose. To this  
522 end, surveys to identify the interests for the measurement of certain variables, considering the location of the  
523 sensor, record length, frequency of the records, methods of transmission, among others, are executed.

524  
525 Singh et al. (1986) applied two questionnaires to evaluate the streamflow network in Illinois: one to identify the  
526 main uses of streamflow data collected at gauging stations, where participants described how data was used and  
527 how they would categorise it in either site-specific management activities, local or regional planning and design,  
528 or determination of long-term trends. The second questionnaire was used to determine present and future needs  
529 for streamflow information. The results showed that the network was reduced due to the limited interest about

530 certain sensors, which allowed for enhancing the existing network using more sophisticated sensors or recording  
531 methods. Additionally, this redirection of resources increased the coverage at specific locations.

532 **3.4 Other methods**

533 There are also other methods that cannot be easily attributed to the previously mentioned categories. Among them,  
534 Value of Information, fractal, and network theory-based methods can be mentioned.

535 **3.4.1 Value of Information**

536 The Value of Information (VOI, Howard 1966, Hirshleifer and Riley 1979) is defined as the value a decision-  
537 maker is willing to pay for extra information before making a decision. This willingness to pay is related to the  
538 reduction of uncertainty about the consequences of making a wrong decision (Alfonso and Price 2012).

539

540 The main feature of this approach is the direct description of the benefits of additional piece of information,  
541 compared with the costs of acquiring that extra piece of information (Black et al. 1999, Walker 2000, Nguyen and  
542 Bagajewicz 2011, Alfonso and Price 2012, Ballari et al. 2012). The main advantage of this method is that provides  
543 a pragmatic framework in which information have a utilitarian value, usually economic, which is especially suited  
544 for budget constraint conditions.

545

546 One of the assumptions of this type of models is that a prior estimation of consequences is needed. If  $a$  is the action  
547 that has been decided to perform,  $m$  is the additional information that comes to make such a decision, and  $s$  is the  
548 state that is actually observed, then the expected utility of any action  $a$  can be expressed as:

549

$$u(a, P_s) = \sum_s P_s u(C_{as}) \quad (21)$$

550

551 where  $P_s$  is the perception, in probabilistic terms, of the occurrence of a particular state ( $s$ ) among a total number  
552 of possible states ( $S$ ), and  $u$  is the utility of the outcome  $C_{as}$  of the actions given the different states. When new  
553 information (i.e., a message  $m$ ) becomes available, and the decision-maker accepts it, his prior belief  $P_s$  will be  
554 subject to a Bayesian update. If  $P(m|s)$  is the likelihood of receiving the message  $m$  given the state  $s$  and  $P_m$  is the  
555 probability of getting a message  $m$  then:

556

$$P_m = \sum_s P_s P(m|s) \quad (22)$$

557

558 The value of a single message  $m$  can be estimated as the difference between the utility,  $u$ , of the action,  $a_m$  that is  
559 chosen given a particular message  $m$  and the utility of the action,  $a_0$ , that would have been chosen without  
560 additional information as:

561

$$\Delta_m = u(a_m, P(s|m)) - u(a_0, P(s|m)) \quad (23)$$

562

563 The Value of Information,  $VOI$ , is the expected utility of the values  $\Delta_m$ :

564

$$VOI = E(\Delta_m) = \sum_M P_m \Delta_m \quad (24)$$

565  
566 Following the same line of ideas, Khader et al. (2013) proposed the use of decision trees to account for the  
567 development of a sensor network for water quality in drinking groundwater applications. VOI is a straightforward  
568 methodology to establish present causes and consequences of scenarios with different types of actions, including  
569 the expected effect of additional information. A recent effort by Alfonso et al. (2016) towards identifying valuable  
570 areas to get information for floodplain planning consists of the generation of VOI maps, where probabilistic flood  
571 maps and the consequences of urbanisation actions are taken into account to identify areas where extra information  
572 may be more critical.

573 **3.4.2 Fractal-based**

574 Fractal-based methods employ the concept of Gaussian self-affinity, where sensor networks show the same spatial  
575 patterns at different scales. This affinity can be measured by its fractal dimension (Mandelbrot 2001). Lovejoy et  
576 al. (1986) proposed the use of fractal-based methods to measure the dimensional deficit between the observations  
577 of a process and its real domain. Consider a set of evenly distributed cells representing the physical space, and the  
578 fractal dimension of the network representing the number of observed cells in the correlation space. The lack of  
579 non-measured cells in the correlation space is known as the fractal deficit of the network. Considering that a large  
580 number of stations have to be available at different scales, the method is suitable for large networks, but less useful  
581 in the deployment of few sensors in a catchment scale.

582  
583 Lovejoy and Mandelbrot (1985) and Lovejoy and Schertzer (1985) introduced the use of fractals to model  
584 precipitation. They argued that the intermittent nature of the atmosphere can be characterised by fractal measures  
585 with fat-tailed probability distributions of the fluctuations, and stated that standard statistical methods are  
586 inappropriate to describe this kind of variability. Mazzarella and Tranfaglia (2000) and Capecchi et al. (2012)  
587 presented two different case studies using this method for the evaluation of a rainfall sensor networks. The former  
588 study concludes that for network augmentation, it is important to select the optimal locations that improve the  
589 coverage, measured by the reduction of the fractal deficit. However, there are no practical recommendations on  
590 how to select such locations. The latter proposes the inspection of seasonal trends as the meteorological processes  
591 of precipitation may have significant effects on the detectability capabilities of the network.

592  
593 A common approach for the quantification of the dimensional deficit is the box-counting method (Song et al. 2007,  
594 Kanevski 2008), mainly used in the fractal characterisation of precipitation sensor networks. The fractal dimension  
595 of the network ( $D$ ) is quantified as the ratio of the logarithm of the number of blocks ( $NB$ ) that have measurements  
596 and the logarithm of the scaling radius ( $R$ ).

597

$$D = \frac{\log(NB(R))}{\log(R)} \quad (25)$$

599 Due to the scarcity of measurements of precipitation type of networks, the quantification of the fractal dimension  
600 may result unstable. An alternative fractal dimension may be calculated using a correlation integral (Mazzarella &  
601 Tranfaglia 2000) instead of the number of blocks, such that:

$$602 \quad CI(R) = \frac{2}{B(B-1)} \sum_{i=1}^B \sum_{j=1}^B \Theta(R - |u_{\alpha i} - u_{\alpha j}|) : \text{for } i \neq j \quad (26)$$

603  
604 In which  $CI$  is the correlation integral,  $R$  is the scaling radius,  $B$  is the total number of blocks at each scaling radius,  
605 and  $U_\alpha$  is the location of station  $\alpha$ .  $\Theta$  is the Heaviside function. A normalisation coefficient is used, as the number  
606 of estimations of the counting of blocks considers each station as a centre.

607  
608 The consequent definition of the fractal dimension of the network is the rate between the logarithm of the  
609 correlation integral and the logarithm of the scaling radius. This ratio is calculated from a regression between  
610 different values of  $R$ , for which the network exhibit fractal behaviour (meaning, a high correlation between  $\log(CI)$   
611 and  $\log(R)$ ).

$$612 \quad D = \frac{\log(CI)}{\log(R)} \quad (27)$$

613  
614 The Maximum potential value for the fractal dimension of a 2-D network (such as for spatially distributed  
615 variables) is two. However, this limit considers that the stations are located on a flat surface, as elevation is  
616 consequence of the topography, and is not a variable that can be controlled in the network deployment.

### 617 3.4.3 Network theory-based

618 Recently, research efforts have been devoted to the use of the so-called network theory to assess the performance  
619 of discharge sensor networks (Sivakumar and Woldemeskel 2014, Halverson and Fleming 2015). These studies  
620 analyse three main features, namely average clustering coefficient, average path length and degree distribution.  
621 Average clustering is a degree of the tendency of stations to form clusters. Average path length is the average of  
622 the shortest paths between every combination of station pairs. Degree distribution is the probability distribution of  
623 network degrees across all the stations, being network degree defined as the number of stations to which a station  
624 is connected. Halverson and Fleming (2015) observed that regular streamflow networks are highly clustered (so  
625 the removal of any randomly chosen node has little impact on the network performance) and have long average  
626 path lengths (so information may not easily be propagated across the network).

627  
628 In hydrometric networks, three metrics are identified (Halverson and Fleming 2015): degree distribution,  
629 clustering coefficient and average path length. The first of these measures is the average node degree, which  
630 corresponds to the probability of a node to be connected to other nodes. The metric is calculated in the adjacency  
631 matrix (a binary matrix in which connected nodes are represented by 1 and the missing links by 0). Therefore, the  
632 degree of the node is defined as:

$$k(\alpha) = \sum_{j=1}^n a_{\alpha,j} \quad (28)$$

634

635 Where  $k(\alpha)$  is the degree of station  $\alpha$ ,  $n$  is the total number of stations, and  $a$  is the adjacency matrix.

636

637 The clustering coefficient is a measure of how much the nodes cluster together. High clustering indicates that  
638 nodes are highly interconnected. The clustering coefficient ( $CC$ ) for a given station is defined as:

639

$$CC(\alpha) = \frac{2}{k(\alpha)(k(\alpha) - 1)} \sum_{j=1}^n a_{\alpha,j} \quad (29)$$

640

641 Additionally, the average path length refers to the mean distance of the interconnected nodes. The length of the  
642 connections in the network, provide some insights in the length of the relationships between the nodes in the  
643 network.

644

$$L = \frac{1}{n(n-1)} \sum_{\alpha=1}^{k(\alpha)} \sum_{j=1}^n d_{\alpha,j} \quad (30)$$

645

646 As can be seen from the formulation, the metrics of the network largely depends on the definition of the network  
647 topology (adjacency matrix). The links are defined from a metric of statistical similitude such as the Pearson  $r$  or  
648 the Spearman rank coefficient. The links are such pair of stations over which statistical similitude is over a certain  
649 threshold.

650

651 According to Halverson and Fleming (2015), an optimal configuration of streamflow networks should consist of  
652 measurements with small membership communities, high-betweenness, and index stations with large numbers of  
653 intracommunity-links. Small communities represent clusters of observations, thus, indicating efficient  
654 measurements. Large numbers of intra-community links ensure that the network has some degree of redundancy,  
655 and thus, resistant to sensor failure. High-betweenness indicates that such stations which have the most inter-  
656 communal links are adequately connected, and thus, able to capture the heterogeneity of the hydrological processes  
657 at a larger scale.

### 658 **3.5 Aggregation of approaches and classes**

659 Table 2 summarises the sensor network design classes and approaches, with the selected references to the relevant  
660 papers in each of the categories for further reference.

661

662 It is of special interest in the review to highlight the lack of model-based information theory methods, as well as  
663 the little amount of publications in network theory-based methods. Also, quantitative studies in the comparison of  
664 different methodologies for the design of sensor networks are limited. It is suggested, therefore, that a pilot

665 catchment is used for the scientific community to test all the available methods for network evaluation, establish  
666 similarities and differences among them.

667

668 Table 3 summarises the main advantages and disadvantages for each of the design and evaluation methods. These  
669 recommendations are general, but take into account the most general points in the design considerations of sensor  
670 networks. Some of the advantages of these methods have been exploited in combined methodologies, such as those  
671 presented by Yeh et al. (2011), Samuel et al. (2013), Barca et al. (2014), Coulibaly and Samuel (2014) and Kang  
672 et al. (2014).

673 **4 General procedure for sensor network design**

674 Based on the presented literature review, in this section an attempt is made to present a first version of a unified,  
675 general procedure for sensor network design. Such procedure logically link in a flowchart various methods,  
676 following the measurement-based approaches (Fig. 6). The flowchart suggests two main loops: one to measure the  
677 network performance (optimisation loop), and a second one to represent the selection in the number of sensors in  
678 either augmentation or reduction scenarios. Most of the measurement-based methods, as well as most of the design  
679 scenarios can be typically seen as particular cases of this generalised algorithmic flowchart.

680

681 The general procedure consists of 11 steps (boxes in Fig. 6). In the first place, physical measurements (1) are  
682 acquired by the sensor network. This data is used to parameterise an estimator (2), which will be used to estimate  
683 the variable at the Candidate Measurement Locations (CML) using, for instance, Kriging (Pardo-Igúzquiza 1998,  
684 Nowak et al. 2009), or 1D hydrodynamic models (Neal et al. 2012, Rafiee 2012, Mazzoleni et al. 2015). The sensor  
685 network reduction does not require such estimator as measurements are already in place.

686

687 The selection of the CML should consider factors such as physical and technical availability, as well as costs  
688 related to maintenance and accessibility of stations, as illustrated by the WMO (2008) recommendations. The  
689 selection of CML can also be based, for example, on expert judgement. These limitations may be presented in the  
690 form of constraints in the optimisation problem.

691

692 Then an optimisation loop starts (Fig. 6), by the estimation of the measured variable at the CML (3), using the  
693 estimator built in (2). Next, the performance of the sensor network at the CML is evaluated (4), using any of the  
694 previously discussed methods. The selection of the method depends on the designer and its information  
695 requirements, which also determines if an optimal solution is found (5). The stopping criteria in the optimisation  
696 problem can be set by a desired accuracy of the network, some non-improved number of solutions or a maximum  
697 number of iterations. As pointed out in the review, these performance metrics can be either model-based or model-  
698 free and should not be confused with the use of a (geostatistical) model of the measured variable.

699

700 In case the optimisation loop is not complete, a new set of CML is selected (6). The use of optimisation algorithms  
701 may drive the search of the new potential CML (Pardo-Igúzquiza 1998, Kollat et al. 2008, Alfonso 2010, Kollat  
702 et al. 2011). The decision about adequate performance should not only consider the expected performance of the  
703 network but also, recognise the effect of a limited number of sensors.

704 Once the performance is optimal, an iteration over the number of sensors is required. If the scenario is for network  
705 augmentation (7), then a possibility of including additional sensors has to be considered (8). The decision to go  
706 for an additional sensor will depend on the constraints of the problem, such as a limitation on the number of sensors  
707 to install, or on the marginal improvement of performance metrics.

709  
710 The network reduction scenario (9) is inverse: due to diverse reasons, mainly of financial nature, networks require  
711 to have fewer sensors. Therefore, the analysis concerns what sensors to remove from the network, within the  
712 problem constraints (10).

713  
714 Finally, the sensor network is selected (11) from the results of the optimisation loop, with the adequate number of  
715 sensors. It is worth mentioning that an extra loop is required, leading to re-evaluation, typically done on a periodical  
716 basis, when objectives of the network may be redefined, new processes need to be monitored, or when information  
717 from other sources is available, and that can potentially modify the definition of optimality.

718  
719 **5 Conclusions and recommendations**

720 This paper summarises some of the methodological criteria for the design of sensor networks in the context of  
721 hydrological modelling, proposed a framework for classifying the approaches in the existing literature and also  
722 proposed a general procedure for sensor network design. The following conclusions can be drawn:

723  
724 Most of the sensor network methodologies aim to minimise the uncertainty of the variable of interest at ungauged  
725 locations and the way this uncertainty is estimated varies between different methods. In statistics-based models,  
726 the objective is usually to minimise the overall uncertainty about precipitation fields or discharge modelling error.  
727 Information theory-based methods aim to find measurements at locations with maximum information content and  
728 minimum redundancy. In network theory-based methods, estimations are generally not accurate, resulting in less  
729 biassed estimations. In methods based on practical case-specific considerations and value of information, the  
730 critical consequences of decisions dictate the network configuration.

731  
732 However, in spite of the underlying resemblances between methods, different formulations of the design problem  
733 can lead to rather different solutions. This gap between methods has not been deeply covered in the literature and  
734 therefore a general agreement on sensor network design procedure is relevant.

735  
736 In particular, for catchment modelling, the driving criteria should also consider model performance. This driving  
737 criterion ensures that the model adequately represents the states and processes of the catchment, reducing model  
738 uncertainty and leading to more informed decisions. Currently, most of the network design methods do not ensure  
739 minimum modelling error, as often it is not the main performance criteria for design.

740  
741 Furthermore, in the last years, the rise of various sensing technologies in operational environments have promoted  
742 the inclusion of additional design considerations towards a unified heterogeneous sensor network. These new

743 sensing technologies include, e.g., passive and active remote sensing using radars and satellites (Thenkabali 2015),  
744 microwave link (Overeem et al. 2011), mobile sensors (Haberlandt and Sester 2010, Dahm et al. 2014),  
745 crowdsourcing and citizen observatories (Huwald et al. 2013, Lanfranchi et al. 2014, Alfonso et al. 2015). These  
746 non-conventional information sources have the potential to complement conventional networks, by exploiting the  
747 synergies between the virtues and reducing limitations of various sensing techniques, and at the same time, require  
748 the new network design methods allowing for handling the heterogeneous dynamic data with varying uncertainty.

749  
750 The proposed classification of the available network design methods was used to develop a general framework for  
751 network design. Different design scenarios, namely relocation, augmentation and reduction of networks are  
752 included, for measurement-based methods. This framework is open and offers “placeholders” for various methods  
753 to be used depending on the problem type.

754  
755 Concerning the further research, from the hydrological modelling perspective, we propose to direct efforts towards  
756 the joint design of precipitation and discharge sensor networks. Hydrological models use precipitation data to  
757 provide discharge estimates, however as these simulations are error-prone, the assimilation of discharge data, or  
758 error correction, reduces the systematic errors in the model results. The joint design of both precipitation and  
759 discharge sensor networks may help to provide more reliable estimates of discharge at specific locations.

760  
761 Another direction of research may include methods for designing dynamic sensor networks, given the increasing  
762 availability of low-cost sensors, as well as the expansion of citizen-based data collection initiatives  
763 (crowdsourcing). These information sources are on the rise in the last years, and one may foresee appearance of  
764 interconnected, multi-sensor heterogeneous sensor networks shortly.

765  
766 The presented review has also shown that limited effort has been devoted to considering changes in long-term  
767 patterns of the measured variable in the sensor network design. This assumption of stationarity has become more  
768 relevant in the last years due to new sensing technologies and increased systemic uncertainties, e.g. due to climate  
769 and land use change and rapidly changing weather patterns. Although this topic has been recognised for quite some  
770 time already (see e.g. Nemec and Askew 1986), the number of publications presenting effective methods to deal  
771 with them is still limited. This problem, and the techniques to solve it, are being addressed in the ongoing research.

772  
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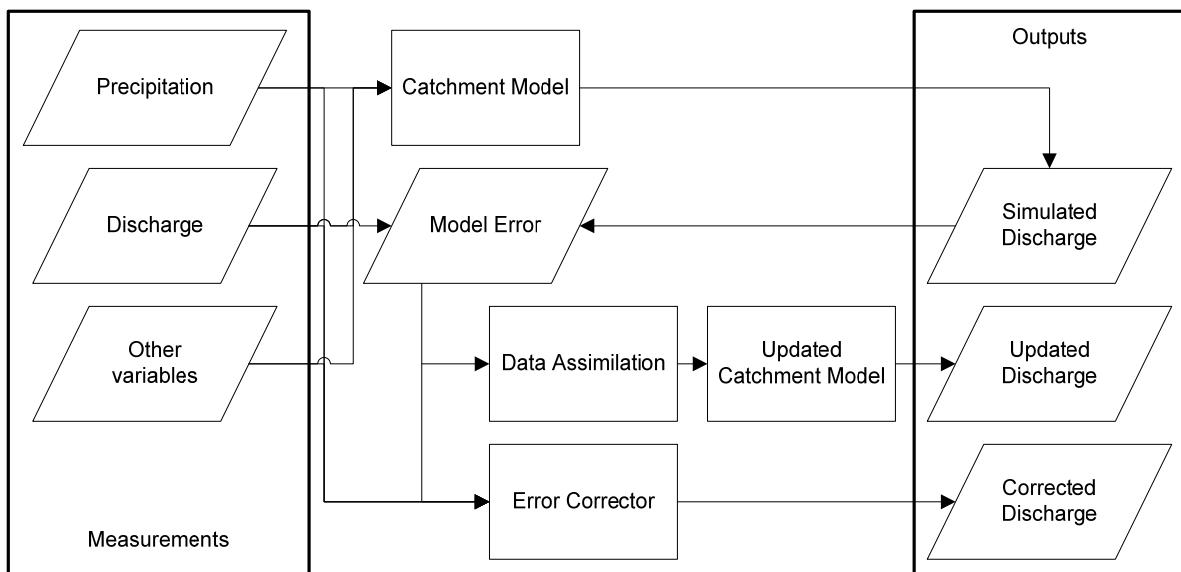
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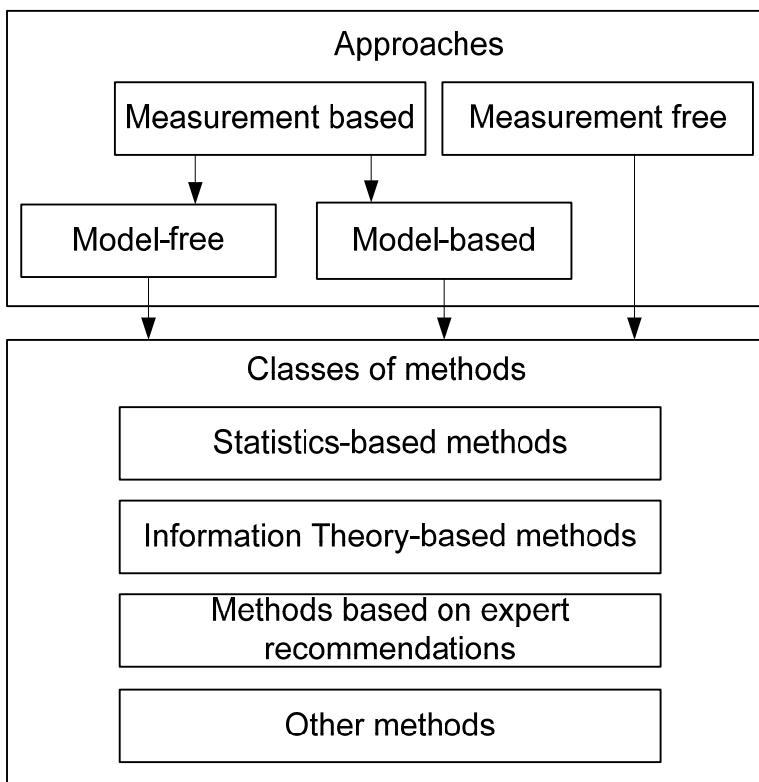
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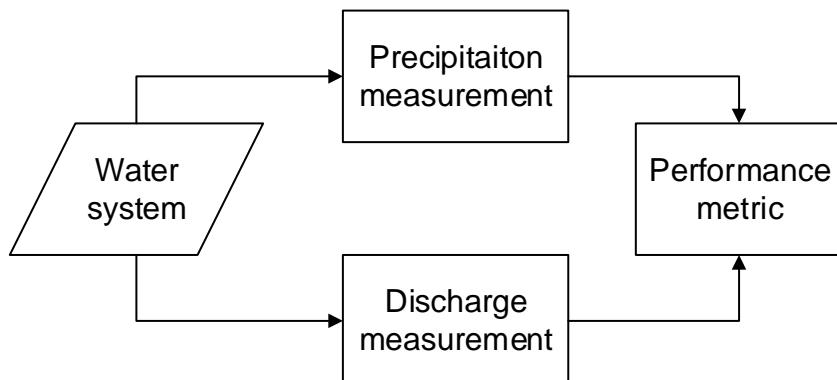
1114 **Figure 1** Typical data flow in discharge simulation with hydrological models

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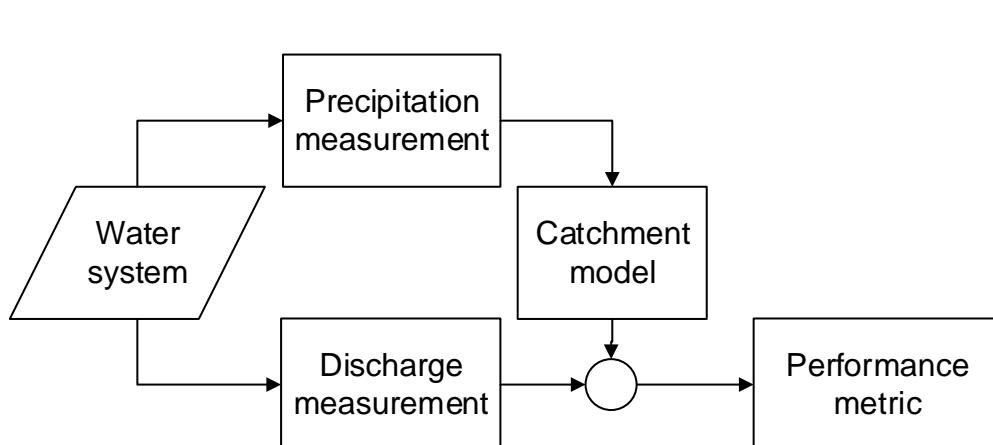


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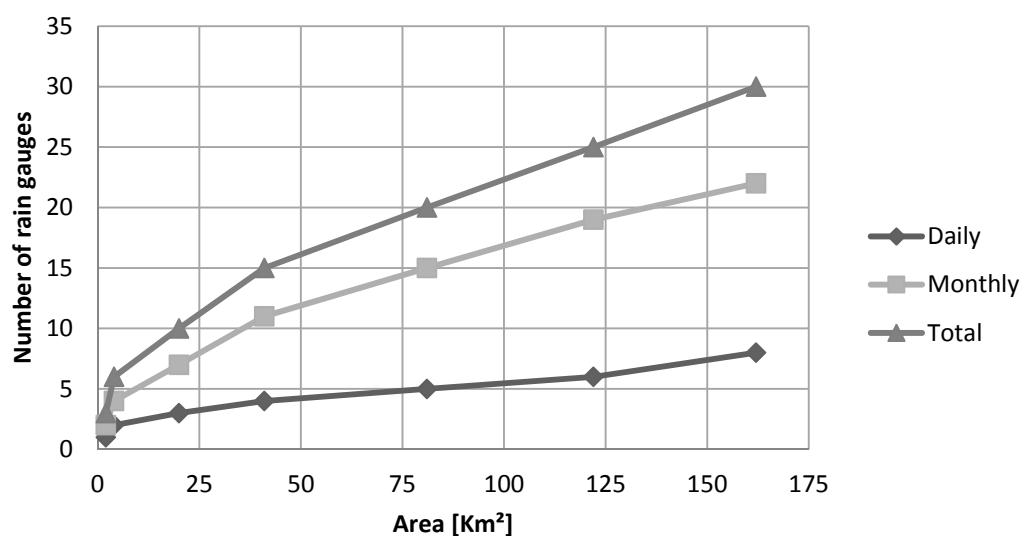
1117 **Figure 2** Proposed classification of methods for sensor network evaluation



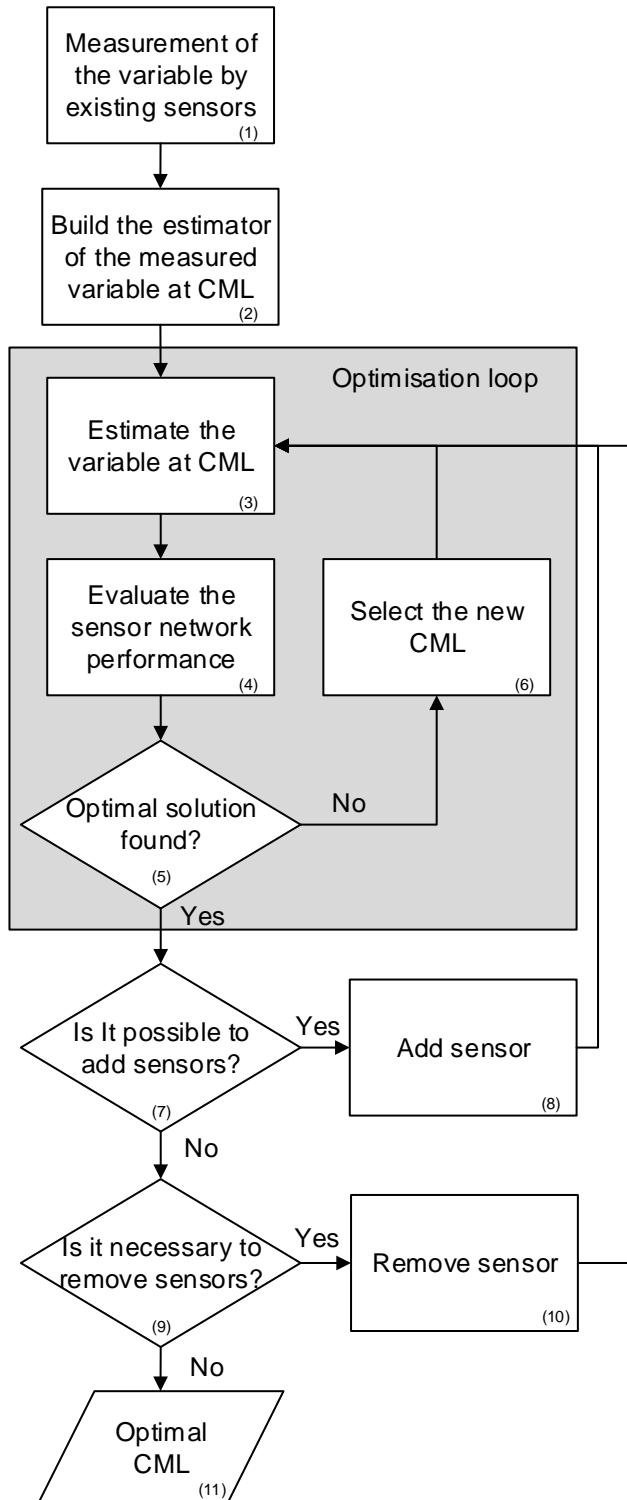
1118  
1119 **Figure 3 General procedure for Model-free sensor network evaluation**



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1122 **Figure 4 General procedure for Model-based sensor network evaluation**



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1125 **Figure 5 Minimum number of rain gauges required in reservoird moorland areas - adapted from: (Bleasdale 1965)**



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1127 **Figure 6 Sensor network (re) design flow chart. (CML=candidate measurement locations)**

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**Table 1 Recommended minimum densities of stations (area in Km<sup>2</sup> per station) – Adopted from WMO [2008]**

Physiographic unit	Precipitation		Evaporation	Streamflow	Sediments	Water Quality
	Non-recording	Recording				
Coastal	900	9,000	50,000	2,750	18,300	55,000
Mountains	250	2,500	50,000	1,000	6,700	20,000
Interior plains	575	5,750	5,000	1,875	12,500	37,500
Hilly/undulating	575	5,750	50,000	1,875	12,500	47,500
Small islands	25	250	50,000	300	2,000	6,000
Urban areas	–	10–20	–	–	–	–
Polar/arid	10,000	10,000	100,000	20,000	200,000	200,000

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1132 **Table 2 Classification of sensor network design criteria including recommended reading**

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		Approaches	
		Measurement-based	Measurement-Free
Classes	Model-free	Model-based	
	Statistics-based		
	Interpolation variance	Pardo-Iguzquiza (1998) Bardossy and Li (2008) Nowak et al. (2010)	
	Cross-correlation	Maddock (1974) Moss and Karlinger (1974)	Vivekanandan and Jagatp (2012)
	Model error		Tarboton et al. (1987) Dong et al. (2005)
	Information Theory		
	Entropy	Krstanovic and Singh (1992) Alfonso et al. (2014)	Pham and Tsai (2016)
	Mutual information	Husain (1987) Alfonso (2010)	Coulibaly and Samuel (2014)
	Expert recommendations		
	Physiographic components	Samuel et al. (2013)	Moss and Karlinger (1974) Moss et al. (1982)
	Practical case-specific considerations		Wahl and Crippen (1984) Nemec and Askew (1986) Karaseff (1986)
	User survey		Sieber (1970) Singh et al. (1986)
	Other methods		
	Value of information	Alfonso and Price (2012)	Black et al. (1999) Alfonso et al. (2016)
	Fractal characterisation		Lovejoy and Mandelbrot (1985) Capecci et al. (2012)
	Network theory	Sivakumar and Woldemeskel (2014) Halverson and Fleming (2015)	

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**Table 3 Advantages and disadvantages of sensor network design methods**

	<b>Advantages</b>	<b>Disadvantages</b>
<b>Statistics-based</b>		
Interpolation variance	Useful to assess data scarce areas No event-driven Minimise uncertainty in spatial distribution of measured variable	Heavily rely on the characterisation of the covariance structure No relationship with final measurement objective
Cross-correlation	Useful for detecting redundant stations Computationally inexpensive	Augmentation not possible without additional assumptions Limited to linear dependency between stations
Model error	Has direct relationship with the measurement objectives	Biased towards current measurement objectives Biased towards model and error metrics
<b>Information Theory</b>		
Entropy	Assess non-linear relationship between variables Unbiased estimation of network performance	Formal form is computationally intensive Quantising (binning) of continuous variables lead to different results Optimal networks are usually sparse Difficult to benchmark Data intensive
Mutual information	Idem	Idem
<b>Expert recommendations</b>		
Physiographic components	Reasonably well understood Functional for heterogeneous catchments with few available measurements Useful at country/continental level	Not useful for homogeneous catchments No quantitative measure of network accuracy
Practical case-specific considerations	No previous measurements are required Useful to observe specific variables	Biased towards expert Collected data does not influence selection Biased towards current data requirements
User survey	Pragmatic Cost-efficient	Extensive user identification Biased towards current data requirements
<b>Other methods</b>		
Value of information	Provides assessment using economics concepts Takes into account decision-maker's prior beliefs in the assessment	Consequences of decisions are difficult to quantify Usually decisions are made with available information Biased towards a rational decision model
Fractal characterisation	Efficient for large networks Does not require data collection	Not suitable for small networks or catchments Does not consider topographic or orographic influence
Network theory	Provides insight in interconnected networks	Not useful for augmentation purposes Data intensive