Response to Interactive comment by Anonymous Referee #2, which was received and published on 28 September 2016, on "Numerical Solution and Application of Time-Space Fractional Governing Equations of One-Dimensional Unsteady Open Channel Flow Process" by Ali Ercan and M. Levent Kavvas

We thank Anonymous Referee #2 for the valuable comments. As copied below, we added a "Discussion" section in the revised manuscript before the "Concluding Remarks" section in order to respond to the comments provided by the referee and to clarify the purpose and contribution of this manuscript. We believe that the manuscript improved significantly after the revision based on the referee's comments/suggestions.

"Discussion

As Yevdjevich (1964) pointed out, the standard Saint Venant equations cannot be solved by analytical methods, except for very special cases. Therefore, various numerical solutions of these equations, differing one from another in the numerical scheme implemented, have already been developed, for example, by utilizing the method of characteristics (e.g., Abbott, 1966), finite-difference methods (e.g., Amein and Fang, 1970; Viessmann et al., 1977; Cunge et al., 1980) and finite element approaches (e.g., Cooley and Moin, 1976; Szymkiewicz, 1991). The kinematic wave, diffusive wave and the dynamic wave forms of the Saint Venant equations have also been widely implemented depending on the governing forces of the flow problem (e.g., Woolhiser and Liggett, 1967; Singh, 1994; Moussa and Bocquillon, 1996).

The purpose of this study is not to propose an alternative way of solving the standard Saint Venant equations, for which numerical solution methodology is already well-developed over the last several decades in terms of computational speed and efficiency. Instead, it is aimed at exploring and discussing the capabilities of the recently proposed time-space fractional governing equations of one-dimensional unsteady open channel flow process (Kavvas and Ercan, 2016) based on the numerical solution of these fractional governing equations. It is important to note that the proposed fractional governing equations of open channel flow process, when orders of the fractional time and space derivatives become one, reduce to standard Saint Venant equations (Kavvas and Ercan, 2016). As demonstrated in the above numerical example, the proposed numerical solution compares well with those by Viessman et al. (1977) and USACE (2016) when the fractional derivative powers become one.

The estimation of fractional derivatives of a flow variable involve considerably more flow information in time and space, compared to that of the integer order derivatives, which involve local information in time and space. Therefore, the proposed numerical solution of the time-space fractional governing equations of one-dimensional unsteady open channel flow process is computationally intensive compared to the numerical solution of the standard Saint Venant equations. On the other hand, the new governing equations in the fractional order differentiation framework have the capability of modeling nonlocal flow processes both in time and in space by taking the global correlations into consideration (see Eqn. 4 and Eqn. 7, which provides the algorithms to estimate time and space fractional derivatives). As was shown in the numerical application section, the proposed time-space fractional governing equations of one-dimensional *unsteady open channel flow process have the capability of modeling flow variables with heavy tails by their ability of modeling nonlocal flow processes. Hence, the proposed generalized flow process may possibly shed light on understanding the theory of the anomalous transport processes and observed heavy tailed distributions of particle displacements in transport processes [for example, as reported in Foufoula-Georgiou and Stark (2010), and the references therein] because the flow process is the main mechanism contributing to the movement of particles in transport. However, this hypothesis needs to be validated by future studies utilizing experimental and/or field data.*

Furthermore, the proposed governing equations and the numerical solution of time-space fractional governing equations of one-dimensional unsteady/non-uniform open channel flow process can be beneficial especially when the physically-known range of Manning's roughness coefficient values [for example as suggested by Chow (1959), and Arcement and Schneider (1989)] are insufficient in the calibration of a 1D unsteady flow model. As Abbott et al. (2001) pointed out, unrealistic values of Manning's roughness coefficient could hide unknown information or physical phenomena not represented in the flow model, and in such case the model would not be predictive, even with an excellent calibration."

In order to respond to the Referee's suggestion on the initial and boundary conditions, sentences below are added after Eq. 8:

"The initial and boundary conditions for the standard Saint Venant equations depends on the flow being subcritical (Froude number<1), supercritical (Froude number>1), or intermediate (partly subcritical and partly supercritical) (Litrico and Fromion, 2009). Because governing equations under consideration in this study are in terms of Caputo fractional derivatives, the traditional initial and boundary conditions similar to those of the standard Saint Venant equations can be applied (Podlubny, 1999)."

As suggested by the referee, authors added Table 1 (in the revised manuscript) to provide comparison with the other numerical solutions of the Saint Venant equations and the proposed numerical solution (when space and time fractional derivative powers are 1). Predictive capability of the proposed numerical solution, when space and time fractional derivative powers are one, is evaluated in comparison to numerical solutions by Viessman et al. (1977) and USACE (2016) in terms of correlation coefficients and Nash–Sutcliffe coefficients. It is found that the flow discharges and water depths at the downstream boundary, estimated by the proposed numerical solution, compare well with those obtained by Viessman et al. (1977) and USACE (2016) for standard Saint Venant equations. Please also see 2nd and 3rd paragraphs of the "Discussion" section.

Table 1. Statistical evaluation of the flow discharges and water depths at the downstream boundary. Results of numerical solutions by Viessman et al. (1977) and USACE (2016) are compared with those of the proposed numerical solution when space and time fractional derivative powers are 1.

	Correlation Coefficient	Nash–Sutcliffe Coefficient
Water depth, y		
Viessman et al. (1977)	0.9999	0.9998
USACE (2016)	0.9968	0.9922
Discharge, Q		
Viessman et al. (1977)	0.9999	0.9998

USACE (2016)	0.9970	0.9940

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