

**Response to Interactive comment by Anonymous Referee #2, which was received and published on 13 September 2016, on “Numerical Solution and Application of Time-Space Fractional Governing Equations of One-Dimensional Unsteady Open Channel Flow Process” by Ali Ercan and M. Levent Kavvas**

We thank Anonymous Referee #2 for his/her comments. For clarity in our responses and to facilitate cross-referencing, we numbered the comments according to the original order provided by Anonymous Referee #2. Our responses are provided in red color below:

1. The Authors present a paper focusing on the fractional time-space fractional governing equations. They propose a Finite Difference numerical solution and provide a simple academic test case

The work focuses solely on the numerical aspects of fractional time-space governing equations methods. In my view, the numerical technique itself is obsolete.

This paper is the only study that reports a numerical algorithm to solve governing equations of unsteady open channel flow in fractional time-space, which was derived by Kavvas and Ercan (2016). Therefore, Referee #2's comment of “numerical technique being obsolete” is baseless. In this paper, numerical algorithms introduced by Murio (2008) and Odibat (2009), which were proposed to solve the Caputo fractional derivative, were coupled in the proposed numerical approach to solve time-space fractional governing equations of one-dimensional unsteady open channel flow process. As such, the presented numerical algorithms for the solution of fractional governing equations of unsteady open channel flow are completely new and original.

Furthermore, this paper does not simply “propose a finite difference numerical solution and provide a simple academic test case”. Authors (Kavvas and Ercan, 2016) presented a detailed derivation of the complete continuity and momentum equations of unsteady open channel flow in fractional time-space, for the first time in hydrology and hydraulics, from the basic mass conservation law and the Newton's second law of motion, and this manuscript provides an approach on the numerical solution of these new equations and insights for the capabilities of these new governing equations by the help of a new numerical solution algorithm and its numerical application.

The derived fractional governing equations under consideration are generalizations of the well-known Saint Venant equations. When powers of the time and space fractional derivatives become one, the proposed governing equations of unsteady/non-uniform open channel flow process in fractional differentiation framework reduce to Saint Venant equations in integer order differentiation framework (Kavvas and Ercan, 2016), and the proposed numerical algorithm reduces to the explicit finite difference scheme similar to the one reported by Viessman, et al. (1977).

Numerical simulations presented here showed that the flow discharge and water depth can exhibit heavier tails in downstream locations as space and time fractional derivative powers decrease from 1. The new governing equations in the fractional order differentiation framework have the capability of modeling nonlocal flow processes both in time and in space by taking the global relations into consideration (see Eqn. 4 and Eqn. 7 in this manuscript, which provides the algorithms to estimate time and space fractional derivatives, and Kavvas and Ercan, 2016). We believe that the generalized flow process that was dealt with in this manuscript may shed light into understanding the theory of the anomalous transport processes and observed heavy tailed distributions of particle displacements because the flow process is the main mechanism contributing to the movement of particles in transport. However, this hypothesis needs to be validated by future studies utilizing experimental and/or field data.

Kavvas, M. and Ercan, A. (2016). "Time-Space Fractional Governing Equations of Unsteady Open Channel Flow." J. Hydrol. Eng., 10.1061/(ASCE)HE.1943-5584.0001460 , 04016052.

2. Considering the type of Journal and their main scopes and covered topics, the paper seems to me out of place.

We do not agree with the Referee's comment. Since the covered topics are relevant with the scopes of the HESS, this paper passed the initial review process and was published as a discussion paper already. Please also see the above response to Comment #1 about the importance of this study.

3. Another concern is related to the missing an application with a real comparison with experimental data. I see only ruminations between numerical solutions. A strong limit of the paper is that it is only one example and it is a synthetic test.

In literature, there are many examples of theoretical and numerical pioneering research papers, which do not provide any comparison with the experimental data, yet are important and valuable contributions to the hydrologic and hydraulic literature. Several of these papers were cited in Kavvas and Ercan (2016) in the context of fractional differential equations for the transport process.

We believe that experimental and/or field applications can be performed in the future studies. If suggested, we will be happy to increase the number of numerical examples. Please also see the response to Comment #1 for the importance of this study.

4. countless numerical solvers exist today for the numerical solution of the physical problem the authors solve. I think it is appropriate that the authors provide guidance on the applicability of their model, as well as the benefits to apply their model instead of other solvers.

The numerical algorithm provided here is the only available algorithm to solve the governing equations of unsteady open channel flow in fractional time-space, which was derived by Kavvas and Ercan (2016). Therefore, such a comparison with other solvers is impossible.

5. Authors should:

a) provide the treatment of the boundary conditions

Please see lines 66-70 as copied below:

*“Within this context, time-space fractional governing equations of unsteady/non-uniform open channel flow process were developed within Caputo fractional derivative framework by Kavvas and Ercan (2016). The advantage of the fractional derivatives in Caputo framework is that the traditional initial and boundary conditions, which are physically interpretable, can be utilized (Podlubny, 1999).”*

Please also see lines 156-159 as copied below:

*The first order derivative of the function  $f$  can be estimated as  $f'(0) = \frac{f(h) - f(0)}{h}$  at the upstream boundary and as  $f'(a) = \frac{f(a) - f(a-h)}{h}$  at the downstream boundary when the further upstream and further downstream function values are not available, which is usually the case in open channel flow problems.*

b) present some applications with experimental data. It would be appropriate to see how their solvers behaves in a real case with real river topography.

The purpose of this paper is not to propose another solver to solve governing equations of the traditional open channel flow problem. The equations under consideration are generalizations of the well-known Saint Venant equations. Please see the responses to Comments #1 and 3.

c) indicate, on the basis of these applications, the ranges of applicability of their proposed methodology, that is, if in some cases it would be better to solve equations with exponents equal to 1 (de Saint Venant equations), to achieve higher stability of the numerical method.

The exponents were not introduced to achieve higher numerical stability. Please also see the response to Comments #1.

d) provide an analysis of the computational costs, which could direct the reader to the choice of a numerical method rather than another

The numerical algorithm provided here is the only available algorithm to solve the governing equations of unsteady open channel flow in fractional time-space, which was derived by Kavvas and Ercan (2016). Therefore, such a comparison is impossible.

In conclusion, I believe that the authors should review the article on the basis of the indications reported above, and that the work in its present form is not ready to be published

After seeing Reviewer's comments on the numerical technique being obsolete (Comment #1), the availability of "countless numerical solvers" (Comment #4), misunderstanding of the usage of the fractional exponents (Comment 5c), we believe that the Reviewer did not fully understand the scope of this manuscript, and the value of the time-space fractional governing equations of one-dimensional unsteady open channel flow process under consideration. Maybe because of this, the Reviewer thinks that the paper is not suitable for HESS (Comment #2).

As discussed in the response to Comment #1, we believe that this manuscript provides important results for the hydrologic and hydraulic community. In the revised manuscript, we will clarify some of the issues mentioned by the reviewer.