Response to Interactive comment by Anonymous Referee #1, which was received and published on 1 September 2016, on "Numerical Solution and Application of Time-Space Fractional Governing Equations of One-Dimensional Unsteady Open Channel Flow Process" by Ali Ercan and M. Levent Kavvas

We thank Anonymous Referee #1 for his/her comments. For clarity in responses and to facilitate cross-referencing, we numbered the comments according to the original order provided by Anonymous Referee #1. Our responses are provided in red color below:

1. I do not see which are the scientific questions relevant for hydrology. The paper provides a numerical method that solves time-space fractional differential equations that reduce to St Venant equations if powers equal 1.The only conclusion is the solving of these latter equations, which does not seem to be an innovation. Thus, it is not clear what the present paper provides in addition to previous papers among which some are from the same authors. However, the paper is clearly built and the reader can follow easily.

Authors (Kavvas and Ercan, 2016) presented a detailed derivation of the complete continuity and momentum equations of unsteady open channel flow in fractional time-space, for the first time in hydrology and hydraulics, from the basic mass conservation law and the Newton's second law of motion and this manuscript provides an approach on the numerical solution of these new equations and new insights for the capabilities of these equations by the help of a numerical solution algorithm and its numerical application. Numerical simulations presented here showed that the flow discharge and water depth can exhibit heavier tails in downstream locations as space and time fractional derivative powers decrease from 1. The new governing equations in the fractional order differentiation framework have the capability of modeling nonlocal flow processes both in time and in space by taking the global relations into consideration (see Eqn. 4 and Eqn. 7 in this manuscript, which provides the algorithms to estimate time and space fractional derivatives). The derived fractional governing equations under consideration are generalizations of the well-known Saint Venant equations. When powers of the time and space fractional derivatives become one, the proposed governing equations of unsteady/non-uniform open channel flow process in fractional differentiation framework reduce to Saint Venant equations in integer order differentiation framework (Kavvas and Ercan, 2016), and the proposed numerical algorithm reduces to the explicit finite difference scheme similar to the one reported by Viessman, et al. (1977). As such, we believe the reported material is quite significant and relevant to hydrology and hydraulics. Furthermore, the numerical solution of the newly-developed complete equations of open channel flow in fractional time-space is new and original material. Please also see our responses below, which will clarify some of the misunderstandings.

Kavvas, M. and Ercan, A. (2016). "Time-Space Fractional Governing Equations of Unsteady Open Channel Flow." J. Hydrol. Eng. ,10.1061/(ASCE)HE.1943-5584.0001460 , 04016052.

2. Figures 2 and 3 show zooms of the differences between results for various powers but the authors did not explain why these differences are important. Particularly, they did not propose a more realistic power value (instead of the classical value of 1).

Figure 3 does not show any zoom of the differences, only Figure 2 does. Please see lines 212-216 for importance of Figure 2 as copied below:

"In order to visualize the tails better, non-dimensional flow discharges (Q/Q0) and non-dimensional water depths (y/y0), which were depicted in Figures 1a and 1b, are zoomed over the [0.9-1.1] range and presented in Figures 2a and 2b, respectively. As presented in Figures 2a and 2b, non-dimensional flow discharges and non-dimensional water depths exhibit heavier tails as space and time fractional derivative powers decrease from 1."

For Figure 3, please see lines 225-237 for the discussion and importance of the differences between results for various powers through longitudinal length x.

We believe that the reviewer thought that Viessmann et al. (1977), depicted by blue circles in Figures 1-2, were the target solution that we were trying to achieve. However, this understanding is not true. Please see lines 196-201 for the purpose of providing numerical solution of Viessmann et al. (1977). Please see also the response to Comment #7.

3. The text is not always clear although the authors detail some steps of their method (but the various steps are not fully explained then one can re-build the numerical method but it is not straightforward). For instance, they seem to write that the hydraulic radius is not calculated using the ratio of the wetter area over the wetted perimeter but they do not write the equation they are proposing.

In standard hydraulic engineering practice, the hydraulic radius is calculated using the ratio of the wetted area over the wetted perimeter. Since we thought it was obvious in standard practice, we did not explicitly write it. In the revised manuscript, we can write it. We will be happy to clarify if the reviewer has other concerns about the clarity of the methodology.

4. There are plenty of references including a lot by the authors but often it is not clear how these references are related to the topic of the paper. So, I think too much useless references are quoted.

We believe that a comprehensive literature review was necessary to give a perspective on the importance of the fractional calculus applications and its implications on open channel flow. If the reviewer thinks specific references are irrelevant, we can consider removing these specific references.

5. The title quotes " application " but I did not see to which problem the described method is applied. Particularly, the " heavy tailed distribution of particle displacements " are not treated in the paper.

A numerical application was provided in lines 184-237 for the solution of continuity and momentum equations of unsteady open channel flow in fractional time-space for various fractional parameters. Numerical simulations showed that the flow discharge and water depth can exhibit heavier tails for the fractional equations, when compared to the conventional integer equations, in downstream locations as space and time fractional derivative powers decrease from 1. We also showed numerically that the proposed numerical algorithm reduces to the explicit finite difference scheme similar to one reported by Viessman, et al. (1977) when the powers of time and space fractional derivatives become one (the conventional integer equation case).

The new governing equations in the fractional order differentiation framework have the capability of modeling nonlocal flow processes both in time and in space by taking the global relations into consideration (see Eqn. 4 and Eqn. 7 in this manuscript, which provides the algorithms to estimate time and space fractional derivatives, and Kavvas and Ercan, 2016). We believe that the generalized flow process that was dealt with in this manuscript may shed light into understanding the theory of the anomalous transport processes and observed heavy tailed distributions of particle displacements because the flow process is the main mechanism for the movement of particles in transport. However, this is a hypothesis that needs to be validated in the future. The reviewer is correct that heavy tailed distributions of particle displacements were not treated in this paper. Please see lines 16-29 in the “Abstract”, and lines 61-86 in the “Introduction” for the objectives of this paper.

6. The paper is well structured with the presentation of the equations and of the numerical method.

Thank you for the supportive comment.

7. One example is shown but it is not clear why this example was selected.

The reason of selection of this problem was explained in lines 196-201, as copied below:

“A classical solution of this problem by the Saint Venant equations in integer order differentiation framework was provided by Viessmann et al. (1977). In addition to the above proposed numerical solution for the time-space fractional governing equations, this problem was also simulated by the Viessmann et al. (1977)’s explicit numerical scheme for the Saint Venant equations,…”

Please also see lines 77-82:

“When orders (or powers) of the time and space fractional derivatives become one, the proposed governing equations of unsteady/non-uniform open channel flow process in

fractional differentiation framework reduce to Saint Venant equations in integer order differentiation framework (Kavvas and Ercan, 2016), and the proposed numerical algorithm reduces to the explicit finite difference scheme reported in Viessman, et al. (1977)."

8. Generally, language is clear but objectives are not clearly defined.

Thank you for your comment about the clarity of the language. Please see lines 16-29 in the "Abstract", and lines 61-86 in the "Introduction" for the objectives. We believe that the objectives are clear.

9. Symbol of velocity "V" is not defined.

Thank you, it will be added in the revised version of the manuscript.