



Improvement of hydrological model calibration by selecting multiple parameter ranges

Qiaofeng. Wu¹, Shuguang Liu^{1,2}, Yi Cai^{1,2}, Xinjian Li³ and Yangming Jiang⁴

¹Department of Hydraulic Engineering, College of Civil Engineering, Tongji University, Shanghai, 200092, China

5 ²The Yangtze river water environment key laboratory of the ministry of education, Tongji University, Shanghai, 200092, China

³Irrigation Experiment Center of Guangxi Zhuang Autonomous Region, Guilin, 541105, China

⁴Hydrology & Water Resources Bureau of Guangxi Zhuang Autonomous Region, Guilin, 541001, China

Correspondence to: Yi Cai (caiyi@tongji.edu.cn)

Abstract. The parameters of hydrological models are usually calibrated to achieve a good performance of the model, owing
10 to the highly non-linear problem of hydrology process modelling. However, parameter calibration efficiency has a direct
relation with parameter range. Furthermore, parameter range selection is affected by probability distribution of parameter
values, parameter sensitivity and correlation. A newly proposed method is introduced to select and coordinate parameter ranges
for improving the calibration of hydrological models with multiple parameters. At first, the probability distribution
15 characteristics of single parameter value was analysed based on 100 samples obtained from independent calibration with initial
parameter range and the distribution type (i.e. normal, exponential and uniform distributions) determined for single parameter.
Then, the way to select the optimal range for single parameter was demonstrated by comparing different reduced and extended
ranges corresponding to the distribution. Next, parameter correlation and sensibility were estimated to coordinate range
selection of single parameter and the optimal combination of ranges for all parameters obtained. The results show that the
20 probability of calibrated parameter values of Xinanjiang model takes on the normal or exponential distributions. For normal
distribution, selecting the range of high probability density from the initial range is much more efficient for calibration. For
exponential distribution, if the initial range can not be extended, selecting the range of high probability density contributes to
high objective function. If the initial range can be extended, it is better to make the exponential distribution convert into normal
distribution by doubling the range along X-axis direction and subsequently select the range according to normal distribution.
Moreover, the coordination of range selection of single parameters makes the calibration of models with multiple parameters
25 more efficient and effective.

Key words: hydrological model, calibration, parameter ranges, probability distribution

1. Introduction

Hydrological process modelling is an important method for research on water resources management, flood control and disaster
30 mitigation, water conservancy project planning and design, hydrological response to climate change and so on (Zanon et al.,



2010;Papathanasiou et al., 2015). The hydrological model is a type of black-box model in 1932 originally (Sherman, 1932), and conceptual models and distributed models are subsequently put forward in 1960s (Freeze and Harlan, 1969). The three kinds of hydrological models have been significantly improved in recent years and their structures become more and more mature. Theoretically, distributed models have definite physical mechanism of water cycle and all parameters can be measured in-situ (Abbott et al., 1986;Huang et al., 2014). Conceptual models express hydrological processes in form of some abstract models which come from some physical phenomenon and experience. For example, the interflow and base flow are simplified as the flow from linear reservoirs (Caviedes-Voullième et al., 2012;Lü et al., 2013). As a result, some parameters of conceptual models need calibrating. In general, conceptual models have better performance of modelling the streamflow at catchment outlet than distributed models do, especially for catchments lacking sufficient data (Bao et al., 2010;Cullmann et al., 2011). Thus, many conceptual models such as HBV model, TOPMODEL, Tank model and Xinanjiang model are of strong vitality (Abebe et al., 2010;Vincendon et al., 2010;Hao et al., 2015). Additionally, the performance of distributed models can be improved after calibration of some parameters. Therefore, all of the hydrological models should be calibrated before engineering applications.

There are two kinds of calibration methods for hydrological models, the trial-error method and auto-calibration method. The trial-error method depends on plenty of trials for reducing the error of the objective. However, it is difficult to obtain exact optimal solution due to limited enumeration (Boyle et al., 2000). The auto-calibration method is based on stochastic or mathematical methods, having wide application in the non-linear parameter optimization. Compared with the trial-error method, it is more efficient and effective, avoiding the interference of anthropogenic factors (Madsen, 2000;Getirana, 2010). The initial automatic optimization methods, such as the Rosenbrock Method (Rosenbrock, 1960) and the Simplex Method (Nelder and Mead, 1965), are classical and useful methods, but has its limitation of initial value ranges of parameters. Therefore, it can only be regarded as local optimization algorithms (Gupta and Sorooshian, 1985). Different from classical methods above, the Genetic Algorithm (GA) is of random search strategy that avoids problem of local search, being a global optimization algorithm in a real sense (Wang, 1991, 1997;Sedki et al., 2009;Chandwani et al., 2015). After that, many global optimization algorithms have been proposed inheriting the random search strategy. The Shuffled Complex Evolution (SCE-UA) method combines many advantages of Genetic Algorithm and Simplex Method, having powerful capability of calibrating the rainfall-runoff model (Duan et al., 1994;Zhang and Shi, 2011). The Particle Swarm Optimization (PSO) based on random solution can directly obtain the identification parameters through the iterative search for the optimal solution (Kennedy, 1997;Zambrano-Bigiarini and Rojas, 2013). Though the auto-calibration method has been intensively employed to calibrate parameters in the field of hydrology, the most advanced algorithm inevitably falls into local solution because of the strong non-linear problem of the hydrological model and parameter correlation (Chu et al., 2010;Jiang et al., 2010;Jiang et al., 2015).

In general, parameter variables obey some types of probability distribution in the given range after multiple independent repeat calibration by an auto-calibration method (Viola et al., 2009;Jin et al., 2010;Li et al., 2010). Graziani et al. (2008) stated that the shapes of the parameter value probability distributions can be significantly affected by their ranges. Ben et al. (2013) studied the effects of different probability distributions (e.g., Normal distribution and uniform distribution) of parameters



values on parameter sensitivity, and found that the probability distribution can be provide a clue to realize parameter sensitivity. Although Normal and uniform distributions are greatly studied in practice, other types of probability distributions seldom were investigated in previous researches (Kucherenko et al., 2012;Esmaeili et al., 2014).

Most hydrological models contain many parameters of different sensitive characteristics and correlation behaviour. Some researchers believe that the sensitive parameter should be calibrated, but the insensitive parameter can be set as a fixed value by experience (Beck, 1987;Cheng et al., 2006). Inappropriate parameter ranges or fixed values may result in the instability of calibrated results. Furthermore, the range setting of one parameter may influence the calibration of other parameters correlated with it (Song et al., 2015). The model parameter sensitivity analysis has been a growing concern in recent years. Parameters sensitivity varies with catchment characteristic, objective function and parameter ranges (van Griensven et al., 2006). Wang et al. (2013) noted the different parameter ranges lead to changes in parameter sensitivity. Shin et al. (2013) reported that reducing or extending the ranges would affect the parameters sensitivity, making insensitive parameters become sensitive ones or vice versa. Thus, parameter ranges and correlation should be taken into considered when the calibration of multi-parameters models is performed.

Parameter ranges are generally given roughly due to lack of knowledge concerning physical settings of a local catchment (Song et al., 2013;Hao et al., 2015). The more deviation between true ranges and given range, the more instability of calculated results. Appropriate parameter ranges selection is critical for calibrating the model efficiently. However, few literature reported how to select the appropriate parameter range for improving the calibration of hydrological models. Furthermore, the calibration of multiple parameters is more complex due to the parameter sensitivity and correlation. Hence, it is necessary to find a way to coordinate the range settings of all parameters.

Considering the effect of parameter ranges on calibration efficiency of hydrological models, an approach of parameter ranges selection (PRS) is put forward to improve the calibration of hydrological models with multiple parameters. At first, probability distribution characteristics of parameter values were analysed based on the parameter value samples that calibrated by using a GA method. Then the optimal range of single parameter is selected for calibration according to its probability distribution. Finally, parameter correlation and sensitivity were estimated to determine the optimal combination of multiple parameters ranges. The proposed method is expected to be helpful for an effective and efficient calibration of hydrological models with multiple parameters.

2. Study area and data collection

The Chaotianhe River catchment is located in the northeast of Guangxi Zhuang Autonomous Region in Southwest China (Fig. 1). The Chaotianhe River is the major tributary of the Lijiang River of well-known karst landscape. The total catchment area is 476.24 km². The annual precipitation is approximately 1704 mm and 78% precipitation concentrates in flood reason (March to August). The thickness of soil varies in most karst areas tremendously different with space: limestone exposed in some peak-cluster region, 2-10 m thickness clay covered in the depression and valley bottom. In clastic rock mountain areas, the thickness of the soil is usually less than 0.5 m. Thus the soil moisture storage capacity varies significantly with space. Moreover, the



underground rivers are very well developed in the karst area, which makes the flood gather rapidly and recess slowly due to higher underground flow rate.

The daily data concerning precipitation, evaporation and streamflow were collected from national gauging stations for the 5-year period of 1996–2000. Four precipitation stations, one streamflow gauging station and one evaporation station are selected for the investigation. Areal precipitation was calculated using data from the four precipitation stations by using Thiessen polygon method under GIS environment (Cai et al., 2014). The streamflow gauging station is at the catchment outlet. Some metro-hydrological statistical data of the studied catchment are summarized in Table 1. From 1996–2000, the maximum of streamflow is about 719 m³/d, the minimum 0.53 m³/d and the average is 13.31 m³/d at the outlet. The maximum areal precipitation of the studied catchment varies with year, the value is 235 mm/d of 1996 while 107 mm/d of 2000. The average streamflow decreases from 14.38 to 11.37 m³/d during the studied period.

3. Methodology

3.1 Hydrological model selection

The method of parameters ranges selections (PRS) is designed for most of hydrological models. At present, there have been many hydrological models for hydrological processes simulation. Considering the climate characteristics of the study area, the Xinanjiang model which is suitable for humid regions was chosen to serve as the hydrological model for the investigation. The Xinanjiang model mainly includes three evapotranspiration layers and three runoff components (i.e. surface-, subsurface runoff and groundwater) (Zhao et al., 1980; Zhao, 1992). The surface runoff is routed by the Unit Hydrograph (UH) which is derived from observed streamflow and other runoff components are simplified as linear reservoirs (Ju et al., 2009). With regard to the Xinanjiang model, there are 10 parameters that should be calibrated. The meaning and the common range of the parameters are given in Table 2 (Lin et al., 2014; Hao et al., 2015). The proposed PRS method is introduced as follows, taking a Xinanjiang model for example.

3.2 Probability distribution analysis of calibrated parameter value

3.2.1 Sample collection of calibrated parameter value

In theory, the results of calibration by using a stochastic-based auto-calibration method are not completely same but similar in a reasonable convergence condition, which obey some probability distributions (Jiang et al., 2015). In order to analyse the probability distribution of calibrated parameter values, a stochastic-based auto-calibration is used to calibrate the model, and samples of calibrated parameters values are obtained. As far as the sample size is concerned, 100 samples are enough to estimate the probability distribution of calibrated parameter values, which is deduced from plenty of tests by comparing the similitude of distributions and computing efficiency.

A Genetic Algorithm was selected as the auto-calibration method in the investigation, because GAs are common and widespread used global optimization algorithm based on stochastic and evolutionary optimization technique. Many studies



showed that the evolutionary algorithms could provide equal or better performance than other algorithms (Cooper et al., 1997; Jha et al., 2006; Zhang et al., 2009). The Nash–Sutcliffe efficiency (E_{NS}) was chosen as the objective function (Eq. (1)) for GA, which representing agreement between observed and simulated data.

$$E_{NS} = 1 - \frac{\sum_{i=1}^n (Q_{obs,i} - Q_{sim,i})^2}{\sum_{i=1}^n (Q_{obs,i} - Q_{mean})^2} \quad (1)$$

- 5 where E_{NS} is Nash–Sutcliffe efficiency, i serial number of the step; n total number of observed streamflow data, $Q_{obs,i}$ observed streamflow at step i , $Q_{sim,i}$ simulated streamflow at step i , and Q_{mean} is mean value of observed streamflow.

3.2.2 Determination of probability distribution types

The probability distributions of calibrated parameters values can be determined by using box-plot charts, cumulative frequency curves and frequency histograms. Figure 2 shows the three types of probability distribution based on 100 samples of parameter values of the Xinanjiang model. The symmetry of the box-plot chart (including box and whiskers) and the length ratio of whisker to the box, the shapes of the cumulative frequency curve and the frequency histogram are important indicators to determine the distribution types. Based on these indicators, three types of probability distribution are listed as follows: (1) Normal distributions, the box and whiskers are symmetrical along Y-axis direction, the length of whiskers longer than the height of the box in box-plot chart (Fig. 2a), the cumulative frequency curve S shaped and the histogram is bell shaped (Fig. 10 2b); (2) Exponential distributions, the whole chart is distinct asymmetrical in the Y-axis direction which means the average value (small hollow square) deviates from the median value (a centre line in box), the box close to one side where the whisker is extreme shorter than that on the opposite side (Fig. 2a), the cumulative frequency curve is parabola shaped, and the histogram tends to increase or decline gradually (Fig. 2c); (3) Uniform distribution, the box and whiskers are symmetrical along Y-axis direction, the length of whiskers approximates to that of the box (Fig. 2a), the cumulative frequency curve tends a line and the 15 histogram varies little along x-axis (Fig. 2d).

3.3 Parameters ranges selections

3.3.1 Single parameter range selection (S-PRS)

In order to improve E_{NS} , the initial range of parameter is required adjusting properly. In consideration of the three probability distribution types mentioned above, the different ways to adjust the ranges for parameters are presented in the investigation. 25 For uniform distribution, it is better to keep initial range due to little influence of the range on calibration results. For normal distribution, the cumulative frequency curve is employed to seek several of reduced ranges with a given cumulative frequency, and the minimum and maximum ranges (namely MINR and MAXR) are obtained as depicted in Fig. 3. The MINR and MAXR represents the ranges of maximum and minimum probability density of parameter values with a given cumulative frequency, respectively. As for exponential distribution, the initial range can be doubled from the boundary of high probability density to 30 the outside, if the parameter has reasonable meaning in the new range. Thus, the exponential distribution can be converted into



normal distribution and then the optimal range can be selected by using the method for normal distribution. If the initial range can not be extended, the MINR and MAXR are sought out according to the cumulative frequency curve. Through plenty of tests, a cumulative frequency value of 50% was adopted to search the MINR and MAXR, which reduces sampling errors in case of smaller percentage, and increases difference between MINR and MAXR in case of larger percentage. Through extending or reducing the ranges, the probability distribution of calibrated parameter values can transform and finally convert into approximate uniform distribution.

3.3.2 Multiple parameters ranges selections (M-PRS)

In general, there is more or less correlation between parameters for most hydrological models. As far as the Xinanjiang model is concerned, both parameter WM and B refer to the water storage volume – area curve that representing the spatial variability of soil moisture storage. If the curve is fixed, the larger WM results in the smaller B (Zhao et al., 1980). The change of a parameter range may more or less effect the calibration of other parameters. The correlations among parameters, therefore, should be taken into account, if several parameters ranges require adjusting. If the change of one parameter range has positive influence on calibration of other parameters, the selected ranges for the parameter will contributes to better calibration results. On the contrary, the negative influence may make the contribution of the selected ranges against model calibration. Thus, some coordination measures should be taken to deal with such contradiction. The index R_C (Eq. (2)) were quantified to analyse the influencing degree of one parameter range change to the calibration of other parameters. The more close value of $R_{C,Y,X}$ to 1, the greater positive influence of range change of parameter X on calibration of parameter Y. If $R_{C,Y,X}$ less than 0, it means the negative influence.

$$R_{C,Y,X} = 1 - \frac{L_{Y,X} - L_{Y,Y}}{L_{Y,Initial} - L_{Y,Y}} \quad (2)$$

Where $R_{C,Y,X}$ is the influencing degree of range change of parameter X on calibration of parameter Y; $L_{Y,X}$ the range of parameter Y calibrated with selected range of parameter X and initial ranges of other parameters, $L_{Y,Y}$ the range of parameter Y calibrated with selected range of parameter Y and initial ranges of other parameters, and $L_{Y,Initial}$ is the range of parameter Y calibrated with initial ranges of all parameters. The calibrated range of the parameter is calculated except extreme outliers.

If there is negative influence between two parameters, the parameter of high sensitivity is ranked as primary one and its selected ranges can be kept in the range combination for all parameters, while the initial range is used in place of the selected range to minimize the negative effect for the other parameter of low sensitivity. It is due to the fact that sensitive parameters play more important roles than insensitive parameters do during multi-parameter calibration. In order to assess the sensitivity of parameter range change to E_{NS} , index S_E as expressed in Eq. (3) is computed by performing S-PRS method on each parameter. The larger R_E , the more concentrated E_{NS} distribution, which means parameter calibration is stable and efficient. Thus, the parameter of high S_E is given priority to use the selected range when R_C of two parameters is minus.

$$S_E = 1 - \frac{E'_{NS Max} - E'_{NS Min}}{E_{NS Max} - E_{NS Min}} \quad (3)$$



where S_E is sensitivity of parameter range change to E_{NS} , $E_{NS \text{ Max}}$ and $E_{NS \text{ Min}}$ maximum and minimum E_{NS} calibrated with initial range, and $E'_{NS \text{ Max}}$ and $E'_{NS \text{ Min}}$ are maximum and minimum E_{NS} calibrated with selected range. The statistic analysis of E_{NS} excludes extreme outliers.

Considering there are more than two parameters in most hydrological models, the accumulative influence and the coordination of range selection are investigated in the study. Parameters of positive influence on other parameters can be taken into account, while the selected ranges is substituted for the initial ranges for the parameters of negative influence. The mean value of R_C ($R_{C \text{ mean}}$) is the index to judge the accumulative influence of one parameter range change on the calibration of the other parameters. Thus, for parameters of negative $R_{C \text{ mean}}$, the initial ranges instead of the selected one is adopted for calibration of multiple parameters.

The flow chart of the parameter range selection method is shown in Fig. 4. In stage 1, a set of initial parameter ranges is given for a hydrological model and the probability distribution analysed based on the 100 independent parameters values calibrated by an auto-calibration method. In stage 2, there are three range adjustment methods with response to parameter value probability distribution: for normal distribution, the optimal range for single parameter is obtained by reducing the initial range; for exponential distribution, the initial range of single parameter is extended to convert to the normal distribution and the optimal range determined according to normal distribution, or the initial range is reduced to seek the optimal range for calibration in the case of the limitation on range extension; for uniform distribution, the initial range is kept. In stage 3 the method of single parameter range selection (S-PRS) is performed on each parameter. Based on the indexes S_E and R_C estimated, the optimal combination of ranges is determined by coordinating the ranges selection for all parameters.

4. Results and discussion

4.1 Probability distribution characteristics of calibrated parameter values of Xinanjiang model

A series of calibrated parameters values were obtained through 100 times independent calibration by using GA method. The initial and calibrated ranges of parameter are presented in Table 3. The ratio of calibrated parameter range to initial one in Table 3 is less than 60% for most parameters (i.e. parameter CI, Kc, KI, SM, B, and WM), which implies that reducing the ranges can help calibrate the parameter efficiently. The 100 calibrated values for single parameters were normalized by dividing them by the corresponding initial range, and the box-plot chart of the results is shown in Fig. 5. It is obvious that the box and whiskers are symmetrical and the length of whiskers is longer than that of the box along the direction of Y axis for parameter CI, SM and Kc. But for other parameters, it is shown from the box-plot chart that the mean value deviates from the median one, which means a considerably asymmetric chart. According to these characteristics of the box-plot chart, it is indicated that the probability distribution of calibrated values is normal distribution for parameter CI, SM, and Kc, while that is exponential distribution for other parameters. The ratio of calibrated parameter range to initial one is less than 30% for parameters CI, SM, and Kc, while the ratio varies from 23% to 100% for parameters such as KI, B, CG, and Im. It suggest that



reducing the ranges is suitable to improve calibration for parameters whose values obey normal distributions, whereas that is not enough for parameters whose values obey exponential distributions.

4.2 Effect of range adjustment pattern on calibration results

5 Since the probability distribution of parameter value has a direct relation with parameter range selection, the range adjustment of parameters for calibration is discussed on the basis of probability distribution type of parameters in the investigation.

To normal distribution, reducing the range is generally used to select the appropriate range. Figure 6 shows the calibration results of parameter CI when the different parameter range are selected. The MINR (0.679–0.713) and MAXR (0.623–0.694) were picked out based on the cumulative frequency curve derived from calibration with initial range (0–0.900). From the cumulative curves and the histograms in Fig. 6a, 6b and 6c, it is found that the probability distribution of parameter CI values is converted from normal distribution to uniform distribution when the initial range is reduced to MINR, whereas the normal distribution is changed to the exponential one when the range is cut to MAXR. Figure 6d reveals that the contribution of parameter ranges selection to E_{NS} . It is found that the minimum E_{NS} except extreme outliers rises convincingly and E_{NS} concentrates at larger value zone when MINR is used instead of the initial range. It is indicated that the reduced range of high probability density is helpful to make calibration more steady and efficient.

15 To exponential distribution, both reduced range and the extended range of reasonable meaning can be used to select the appropriate range for calibration. Figure 7 shows the calibration results of parameter KI. Since the initial range of parameter KI can not be extended, the reduced range was searched by using the cumulative frequency curve, the MINR (0.660–0.700) and MAXR (0.522–0.660) were picked out. From the cumulative curves and the histograms in Fig. 7a, 7b and 7c, it is found that the probability distribution of parameter KI values is converted from exponential distribution to uniform distribution when the initial range is reduced to MINR, whereas the exponential distribution is still kept when the range is cut to MAXR. The contribution of parameter ranges to E_{NS} is shown in Fig. 7d. Similar to the results of parameter CI, MINR is best for calibration when compared with MAXR or initial range. It is demonstrated that MINR is better than MAXR to improve calibration when reducing the range for parameters whose value obeys normal or exponential distribution. Because the parameter values in MINR indicate high probability to be pick out to achieve high E_{NS} , vice versa.

25 Figure 8 shows the calibration results of parameter B whose range can be extended. The initial range (B=0.1–0.4) of parameter B is common for most areas, but it is quite different for karst areas where the soil moisture storage varies remarkably with space, and as a result, the value of parameter B could be larger than 0.4. From Fig. 8a and 8b, it is shown that the distribution of parameter B is converted from exponential distribution to normal distribution when the initial range is extended to new one (B=0.1–0.6). After MINR selection is performed on the initial range and the extended range respectively, the two ranges, i.e. MINR (B=0.36–0.40) and extension-MINR (B=0.379–0.488) are obtained and then used to calibrate parameter B. From Fig. 8c and 8d, it is found that the probability distribution of parameter B values is converted into approximate uniform distribution when the range is reduced from initial range to MINR or from the extended range to extension-MINR. The box-plot chart of E_{NS} for different ranges are shown in Fig. 8e. It is indicated that there is a considerable improvement of both



maximum and minimum E_{NS} when extension-MINR is used for calibration. It suggests that an appropriate range extension followed by MINR selection is helpful to improve calibration for parameters whose probability distribution is exponential and ranges can be extended.

4.3 Effect of multiple parameters ranges combination on calibration results

5 The S-PRS method was employed to select the one-parameter optimal range for each parameter, and the optimal ranges, indexed R_C and S_E values are listed in Table 4. It is obvious that R_C value in columns of parameter CI and WM are positive, but most R_C values in column of parameter Im are negative. The negative R_C value between two parameters indicates that using the optimal range of one parameter is adverse to calibration of the other parameter. Specially, both $R_{C_{EX,Im}}$ and $R_{C_{Im,EX}}$ are negative in spite of small values. It means that using the optimal ranges of parameter EX and Im simultaneously is not
 10 conductive to multi parameter combined calibration. The mean of R_C ($R_{C_{mean}}$) varies with parameters. Parameter CI has the maximum $R_{C_{mean}}$ of 0.465, while parameter Im the minimum $R_{C_{mean}}$ of -0.026 . Furthermore, $R_{C_{mean}}$ values for all parameters are positive except for that for parameter Im. It is due to the accumulative negative influence of parameter Im on others.

To coordinate the contradiction between parameters, the index S_E is used to pick parameters of high sensitivity to E_{NS} . From Table 4, it is found that parameter CI has the maximum S_E of 54.7%, and parameter Im the minimum S_E of 0.3%. Most S_E
 15 values are more than 20% except those of parameters C, EX and Im. It suggests that parameters CI, B, SM, KI, K_c , WM and CG are of high sensitive to E_{NS} , and parameters C, EX and Im of low sensitivity for E_{NS} . CI is the most sensitive parameter while Im the most insensitive parameter, which agrees with the work of Lü et al. (2013) and Song et al. (2013). For the well-developed karst areas, the thin layer of soil and strong permeability of limestone make rainfall easy to penetrate into the ground. Moreover, the existence of karst caves and subsurface streams contribute to great interflow storage which accounts for a large
 20 proportion of streamflow. As a result, parameter KI representing penetrate ability of free water to interflow, and parameter CI representing recession capacity of interflow storage have significant influence on rainfall-runoff simulation results. Hence, parameters KI and CI are very sensitive in the investigation. It can be deduced that the optimal range of insensitive parameter Im can not be taken into account when there is contradiction owing to it, in order to improve calibration.

In order to determine the optimal range combination of multi parameter, seven cases are investigated with different range
 25 combinations of parameters (Table 5). The results of seven cases are compared in Fig. 9. There is a little decrease in E_{NS} when Case 4 compared with Case 1, Case 2 and Case 3. It can be explained that both $R_{C_{EX,Im}}$ and $R_{C_{Im,EX}}$ are negative and the combination of the optimal ranges corresponding to the two parameters leads to a worse result. As S_E of parameter Im is less than that of parameter EX, parameter EX is given priority to select the optimal range, that is why the calibration result of Case 3 is better than that of Case 2. As for the cases of multi-parameter range selection (i.e. Case 5, Case 6 and Case 7), the results
 30 are much better than that of Case 1–4. There are some differences in E_{NS} between Case 5, Case 6 and Case 7 when their box-plot charts are magnified. The box and whisker of E_{NS} for Case 6 rise, which means Case 6 has a better performance of calibration than Case 5 does, when the optimal range of parameter CG is included. But the box and whisker of E_{NS} for Case 7



decline when the optimal range of parameter I_m is included. Because the mean R_C value of parameter I_m is negative and its S_E much less than that of others, using the optimal range of I_m is adverse to multi-parameter combined calibration.

5. Conclusions

Considering that there is the relation between the parameter ranges and probability distributions of parameter value, an approach to determine the optimal range combination for multi parameters of hydrological models is put forward by analysing the parameter value probability distribution, parameter sensitivity and parameter correlation. A case of improving the calibration of the GA-based Xinanjiang model for karst areas is studied, and some findings are presented as follows:

The proposed parameter range selection (PRS) method improves the minimum E_{NS} and the maximum E_{NS} , which makes the results concentrate at high E_{NS} . The PRS-based calibration is, therefore, more efficient and effective. In the Xinanjiang model for karst areas, the parameters CI , K_c , SM and B obey normal probability distribution, and parameters WM , C , EX , KI , CG and I_m obey exponential probability distributions. For normal distribution, the minimum ranges (MINR) with a given cumulative frequency of the parameter is preferred to be selected as the optimal range for calibration. For exponential distribution, if the parameter range can be extended outside the boundary of high probability, the extension range followed by MINR is recommended to be selected, otherwise MINR of the initial range is selected as the optimal range for calibration.

The R_C and S_E are two important indexes to help coordinate ranges of different parameter to form the optimal combination of ranges. As far as Xinanjiang model is concerned, the initial ranges of parameter of low S_E and negative $R_{C\text{ mean}}$ are preferred to adopted, and the optimal ranges of parameters of positive $R_{C\text{ mean}}$ are used for multi parameter combined calibration.

Acknowledgements. The investigation is financially supported by special funds for scientific research on public causes of Chinese Ministry of Water Resources (No. 201401057) and the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry (No. 2013–1792).

Reference

- Abbott, M. B., Bathurst, J. C., Cunge, J. A., O'Connell, P. E., and Rasmussen, J.: An introduction to the European Hydrological System — Systeme Hydrologique Europeen, "SHE", 1: History and philosophy of a physically-based, distributed modelling system, *J. Hydrol.*, 87, 45-59, doi:10.1016/0022-1694(86)90114-9, 1986.
- Abebe, N. A., Ogden, F. L., and Pradhan, N. R.: Sensitivity and uncertainty analysis of the conceptual HBV rainfall–runoff model: Implications for parameter estimation, *J. Hydrol.*, 389, 301-310, doi:10.1016/j.jhydrol.2010.06.007, 2010.
- Bao, H., Wang, L., Li, Z., Zhao, L., and Zhang, G.: Hydrological daily rainfall-runoff simulation with BTOPMC model and comparison with Xin'anjiang model, *Water Sci. Eng.*, 3, 121-131, doi:10.3882/j.issn.1674-2370.2010.02.001, 2010.
- Beck, M. B.: Water quality modeling: A review of the analysis of uncertainty, *Water Resour. Res.*, 23, 1393-1442, doi:10.1029/WR023i008p01393, 1987.
- Ben Touhami, H., Lardy, R., Barra, V., and Bellocchi, G.: Screening parameters in the Pasture Simulation model using the Morris method, *Ecol. Modell.*, 266, 42-57, doi:10.1016/j.ecolmodel.2013.07.005, 2013.



- Boyle, D. P., Gupta, H. V., and Sorooshian, S.: Toward improved calibration of hydrologic models: Combining the strengths of manual and automatic methods, *Water Resour. Res.*, 36, 3663-3674, doi:10.1029/2000wr900207, 2000.
- Cai, Y., Esaki, T., Liu, S., and Mitani, Y.: Effect of Substitute Water Projects on Tempo-Spatial Distribution of Groundwater Withdrawals in Chikugo-Saga Plain, Japan, *Water Resour. Manage.*, 28, 4645-4663, doi:10.1007/s11269-014-0764-2, 2014.
- 5 Caviedes-Voullième, D., García-Navarro, P., and Murillo, J.: Influence of mesh structure on 2D full shallow water equations and SCS Curve Number simulation of rainfall/runoff events, *J. Hydrol.*, 448-449, 39-59, doi:10.1016/j.jhydrol.2012.04.006, 2012.
- Chandwani, V., Vyas, S. K., Agrawal, V., and Sharma, G.: Soft Computing Approach for Rainfall-runoff Modelling: A Review, *Aquat. P.*, 4, 1054-1061, doi:10.1016/j.aqpro.2015.02.133, 2015.
- 10 Cheng, C.-T., Zhao, M.-Y., Chau, K. W., and Wu, X.-Y.: Using genetic algorithm and TOPSIS for Xinanjiang model calibration with a single procedure, *J. Hydrol.*, 316, 129-140, doi:10.1016/j.jhydrol.2005.04.022, 2006.
- Chu, W., Gao, X., and Sorooshian, S.: Improving the shuffled complex evolution scheme for optimization of complex nonlinear hydrological systems: Application to the calibration of the Sacramento soil-moisture accounting model, *Water Resour. Res.*, 46, W09530, doi:10.1029/2010wr009224, 2010.
- 15 Cooper, V. A., Nguyen, V. T. V., and Nicell, J. A.: Evaluation of global optimization methods for conceptual rainfall-runoff model calibration, *Water Sci. Technol.*, 36, 53-60, doi:10.1016/S0273-1223(97)00461-7, 1997.
- Cullmann, J., Krausse, T., and Saile, P.: Parameterising hydrological models – Comparing optimisation and robust parameter estimation, *J. Hydrol.*, 404, 323-331, doi:10.1016/j.jhydrol.2011.05.003, 2011.
- Duan, Q., Sorooshian, S., and Gupta, V. K.: Optimal use of the SCE-UA global optimization method for calibrating watershed 20 models, *J. Hydrol.*, 158, 265-284, doi:10.1016/0022-1694(94)90057-4, 1994.
- Esmaili, S., Thomson, N. R., Tolson, B. A., Zebarth, B. J., Kuchta, S. H., and Neilsen, D.: Quantitative global sensitivity analysis of the RZWQM to warrant a robust and effective calibration, *J. Hydrol.*, 511, 567-579, doi:10.1016/j.jhydrol.2014.01.051, 2014.
- Freeze, R. A., and Harlan, R. L.: Blueprint for a physically-based, digitally-simulated hydrologic response model, *J. Hydrol.*, 25 9, 237-258, doi:10.1016/0022-1694(69)90020-1, 1969.
- Getirana, A. C. V.: Integrating spatial altimetry data into the automatic calibration of hydrological models, *J. Hydrol.*, 387, 244-255, doi:10.1016/j.jhydrol.2010.04.013, 2010.
- Graziani, F., Tong, C., and Graziani, F.: A Practical Global Sensitivity Analysis Methodology for Multi-Physics Applications, in, edited by: Graziani, F., *Lect. Notes Comput. Sci. Eng.*, 277-299, doi:10.1029/WR021i004p00473, 2008.
- 30 Gupta, V. K., and Sorooshian, S.: The Automatic Calibration of Conceptual Catchment Models Using Derivative-Based Optimization Algorithms, *Water Resour. Res.*, 21, 473-485, doi:10.1029/WR021i004p00473, 1985.
- Hao, F., Sun, M., Geng, X., Huang, W., and Ouyang, W.: Coupling the Xinanjiang model with geomorphologic instantaneous unit hydrograph for flood forecasting in northeast China, *Int. Soil Water Conserv. Res.*, 3, 66-76, doi:10.1016/j.iswcr.2015.03.004, 2015.



- Huang, X., Liao, W., Lei, X., Jia, Y., Wang, Y., Wang, X., Jiang, Y., and Wang, H.: Parameter optimization of distributed hydrological model with a modified dynamically dimensioned search algorithm, *Environ. Modell. Softw.*, 52, 98-110, doi:10.1016/j.envsoft.2013.09.028, 2014.
- Jha, M. K., Kumar, A., Nanda, G., and Bhatt, G.: Evaluation of traditional and nontraditional optimization techniques for determining well parameters from step-drawdown test data, *J. Hydrol. Eng.*, 11, 617-630, doi:10.1061/(asce)1084-0699(2006)11:6(617), 2006.
- Jiang, Y., Liu, C., Huang, C., and Wu, X.: Improved particle swarm algorithm for hydrological parameter optimization, *Appl. Math. Comput.*, 217, 3207-3215, doi:10.1016/j.amc.2010.08.053, 2010.
- Jiang, Y., Liu, C., Li, X., Liu, L., and Wang, H.: Rainfall-runoff modeling, parameter estimation and sensitivity analysis in a semiarid catchment, *Environ. Modell. Softw.*, 67, 72-88, doi:10.1016/j.envsoft.2015.01.008, 2015.
- Jin, X., Xu, C.-Y., Zhang, Q., and Singh, V. P.: Parameter and modeling uncertainty simulated by GLUE and a formal Bayesian method for a conceptual hydrological model, *J. Hydrol.*, 383, 147-155, doi:10.1016/j.jhydrol.2009.12.028, 2010.
- Ju, Q., Yu, Z., Hao, Z., Ou, G., Zhao, J., and Liu, D.: Division-based rainfall-runoff simulations with BP neural networks and Xinanjiang model, *Neurocomputing*, 72, 2873-2883, doi:10.1016/j.neucom.2008.12.032, 2009.
- Kennedy, J.: The particle swarm: social adaptation of knowledge, *IEEE Int. Conf.*, 1997.
- Kucherenko, S., Tarantola, S., and Annoni, P.: Estimation of global sensitivity indices for models with dependent variables, *Comput. Phys. Commun.*, 183, 937-946, doi:10.1016/j.cpc.2011.12.020, 2012.
- Lü, H., Hou, T., Horton, R., Zhu, Y., Chen, X., Jia, Y., Wang, W., and Fu, X.: The streamflow estimation using the Xinanjiang rainfall runoff model and dual state-parameter estimation method, *J. Hydrol.*, 480, 102-114, doi:10.1016/j.jhydrol.2012.12.011, 2013.
- Li, L., Xia, J., Xu, C., and Singh, V. P.: Evaluation of the subjective factors of the GLUE method and comparison with the formal Bayesian method in uncertainty assessment of hydrological models, *J. Hydrol.*, 390, 210-221, doi:10.1016/j.jhydrol.2010.06.044, 2010.
- Lin, K., Lv, F., Chen, L., Singh, V. P., Zhang, Q., and Chen, X.: Xinanjiang model combined with Curve Number to simulate the effect of land use change on environmental flow, *J. Hydrol.*, 519, 3142-3152, doi:10.1016/j.jhydrol.2014.10.049, 2014.
- Madsen, H.: Automatic calibration of a conceptual rainfall-runoff model using multiple objectives, *J. Hydrol.*, 235, doi:276-288, 10.1016/s0022-1694(00)00279-1, 2000.
- Nelder, J. A., and Mead, R.: A simplex method for function minimization, *Comput. J.*, 4, 308-313, 1965.
- Papathanasiou, C., Makropoulos, C., and Mimikou, M.: Hydrological modelling for flood forecasting: Calibrating the post-fire initial conditions, *J. Hydrol.*, 529, 1838-1850, doi:10.1016/j.jhydrol.2015.07.038, 2015.
- Rosenbrock, H.: An automatic method for finding the greatest or least value of a function, *Comput. J.*, 3, 175-184, 1960.
- Sedki, A., Ouazar, D., and El Mazoudi, E.: Evolving neural network using real coded genetic algorithm for daily rainfall-runoff forecasting, *Expert Syst. Appl.*, 36, 4523-4527, doi:10.1016/j.eswa.2008.05.024, 2009.
- Sherman, L. K.: Stream Flow from Rainfall by the Unit-Graph Method, *Eng. News Rec.*, 108, 501-505, 1932.



- Shin, M., Guillaume, J. H. A., Croke, B. F. W., and Jakeman, A. J.: Addressing ten questions about conceptual rainfall–runoff models with global sensitivity analyses in R, *J. Hydrol.*, 503, 135-152, doi:10.1016/j.jhydrol.2013.08.047, 2013.
- Song, X., Kong, F., Zhan, C., Han, J., and Zhang, X.: Parameter identification and global sensitivity analysis of Xin'anjiang model using meta-modeling approach, *Water Sci. Eng.*, 6, 1-17, doi:10.3882/j.issn.1674-2370.2013.01.001, 2013.
- 5 Song, X., Zhang, J., Zhan, C., Xuan, Y., Ye, M., and Xu, C.: Global sensitivity analysis in hydrological modeling: Review of concepts, methods, theoretical framework, and applications, *J. Hydrol.*, 523, 739-757, doi:10.1016/j.jhydrol.2015.02.013, 2015.
- van Griensven, A., Meixner, T., Grunwald, S., Bishop, T., Diluzio, M., and Srinivasan, R.: A global sensitivity analysis tool for the parameters of multi-variable catchment models, *J. Hydrol.*, 324, 10-23, doi:10.1016/j.jhydrol.2005.09.008, 2006.
- 10 Vincendon, B., Ducrocq, V., Saulnier, G.-M., Bouilloud, L., Chancibault, K., Habets, F., and Noilhan, J.: Benefit of coupling the ISBA land surface model with a TOPMODEL hydrological model version dedicated to Mediterranean flash-floods, *J. Hydrol.*, 394, 256-266, doi:10.1016/j.jhydrol.2010.04.012, 2010.
- Viola, F., Noto, L. V., Cannarozzo, M., and La Loggia, G.: Daily streamflow prediction with uncertainty in ephemeral catchments using the GLUE methodology, *Phys. Chem. Earth*, 34, 701-706, doi:10.1016/j.pce.2009.06.006, 2009.
- 15 Wang, J., Li, X., Lu, L., and Fang, F.: Parameter sensitivity analysis of crop growth models based on the extended Fourier Amplitude Sensitivity Test method, *Environ. Modell. Softw.*, 48, 171-182, doi:10.1016/j.envsoft.2013.06.007, 2013.
- Wang, Q. J.: The Genetic Algorithm and Its Application to Calibrating Conceptual Rainfall-Runoff Models, *Water Resour. Res.*, 27, 2467-2471, doi:10.1029/91wr01305, 1991.
- Wang, Q. J.: Using genetic algorithms to optimise model parameters, *Environ. Modell. Softw.*, 12, 27-34, doi:10.1016/s1364-
20 8152(96)00030-8, 1997.
- Zambrano-Bigiarini, M., and Rojas, R.: A model-independent Particle Swarm Optimisation software for model calibration, *Environ. Modell. Softw.*, 43, 5-25, doi:10.1016/j.envsoft.2013.01.004, 2013.
- Zanon, F., Borga, M., Zoccatelli, D., Marchi, L., Gaume, E., Bonnifait, L., and Delrieu, G.: Hydrological analysis of a flash flood across a climatic and geologic gradient: The September 18, 2007 event in Western Slovenia, *J. Hydrol.*, 394, 182-197,
25 doi:10.1016/j.jhydrol.2010.08.020, 2010.
- Zhang, S., and Shi, J.: A Microwave Wetland Surface Emissivity Calibration Scheme Using SCE-UA Algorithm and AMSR-E Brightness Temperature Data, *P. Environ. Sci.*, 10, 2731-2739, doi:10.1016/j.proenv.2011.09.424, 2011.
- Zhang, X., Srinivasan, R., Zhao, K., and Liew, M. V.: Evaluation of global optimization algorithms for parameter calibration of a computationally intensive hydrologic model, *Hydrol. Process.*, 23, 430-441, doi:10.1002/hyp.7152 %/ John Wiley &
30 Sons, Ltd., 2009.
- Zhao, R. J., Zhang, Y. L., Fang, L. R., Liu, X. R., and Zhang, Q. S.: The Xinanjiang model, in *Hydrological forecasting*, P. Oxford Sym. IAHS, 129, 351–356, doi:10.1007/s11269-012-0244-5, 1980.

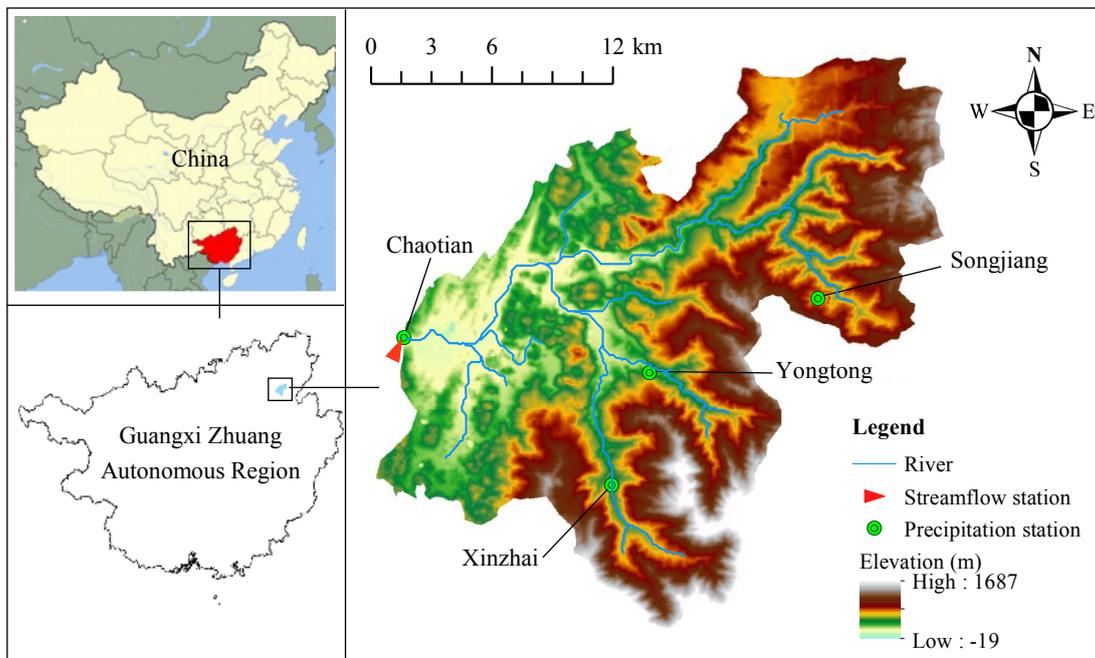


Fig. 1. Location of the study area

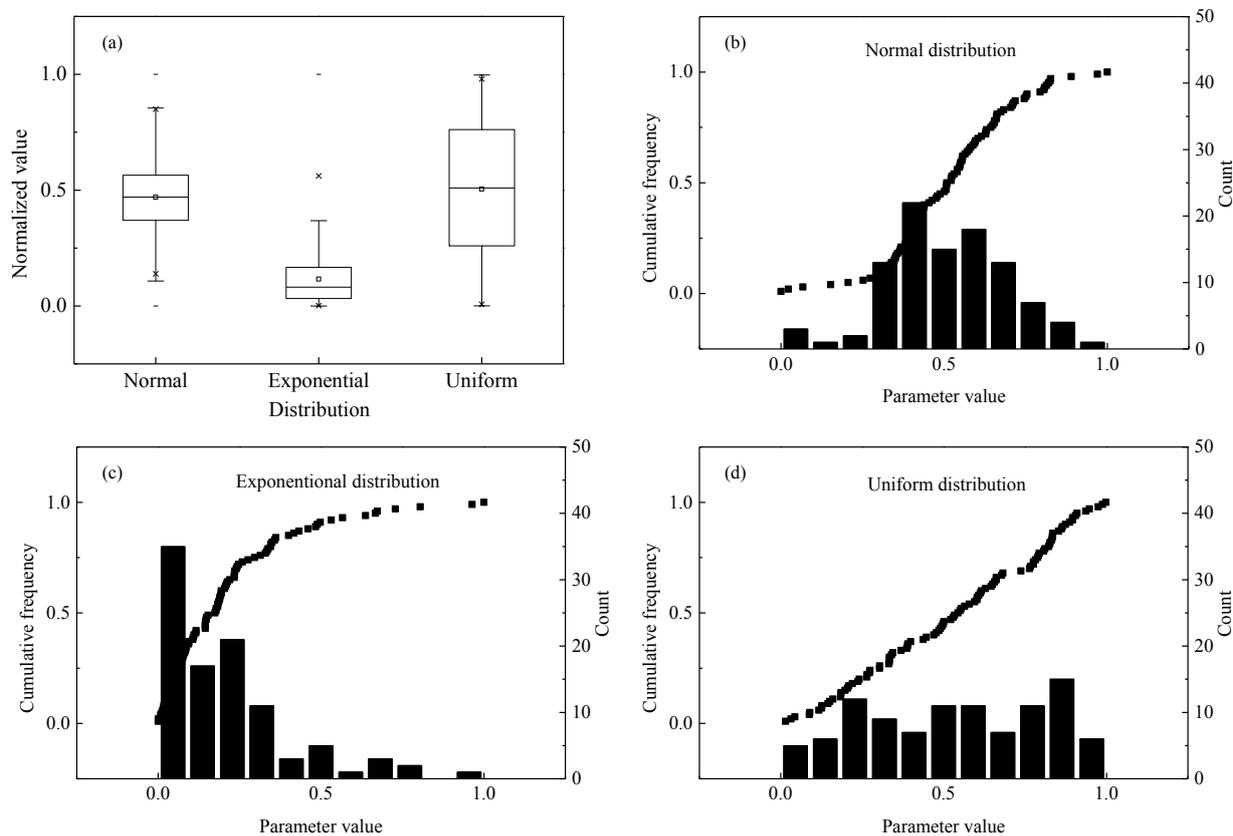


Fig. 2. Different probability distribution types of calibrated parameter values

- (a) Box-plot charts of normal, exponential and uniform distribution (b) Cumulative frequency curve and histogram of normal distribution
 (c) Cumulative frequency curve and histogram of exponential distribution (d) Cumulative frequency curve and histogram of uniform distribution

5

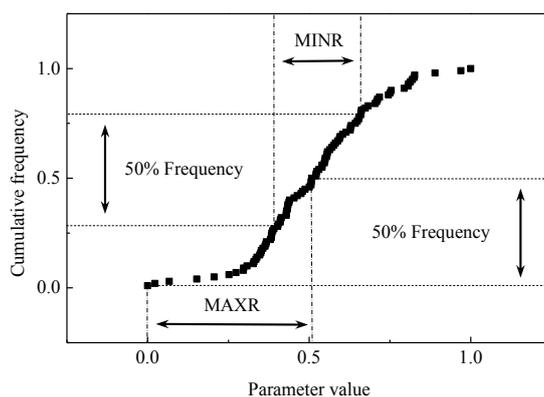


Fig. 3. Selection of minimum and maximum range (MINR and MAXR) with a cumulative frequency of 50%

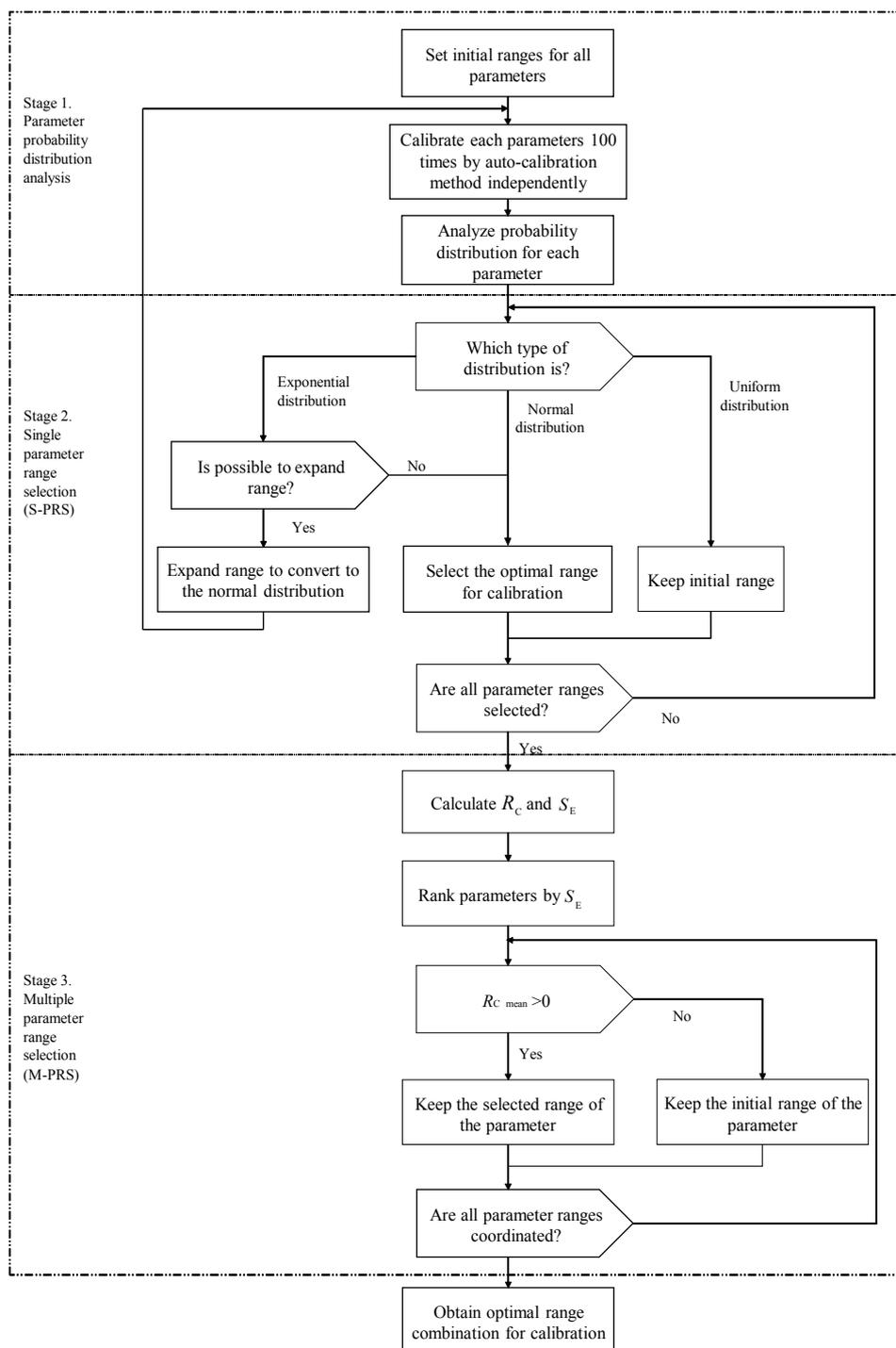


Fig. 4. Flow chart of multiple parameters ranges selections

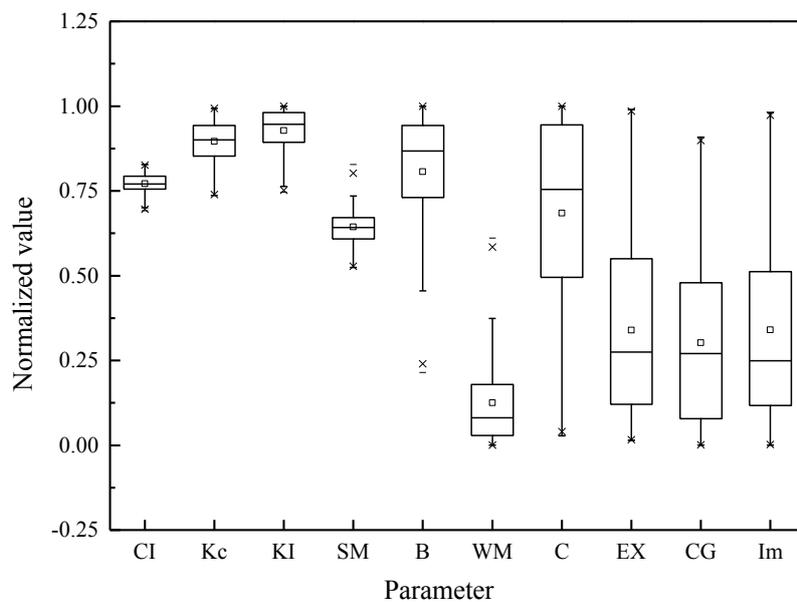


Fig. 5. The box-plot chart of normalized calibrated values for parameters of Xinanjiang model

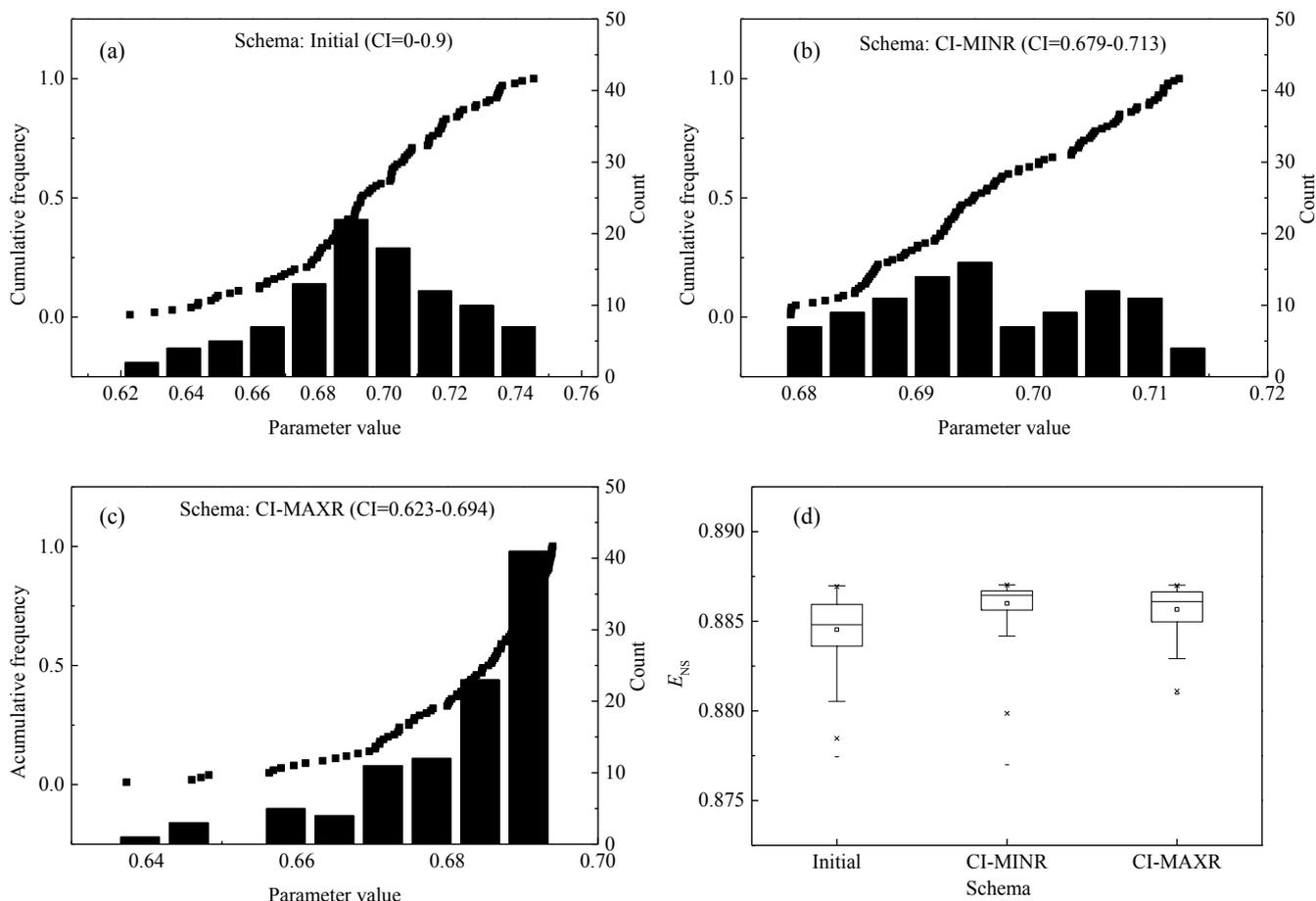


Fig. 6. Results of range selection of parameter CI

- (a) probability distribution of parameter values for schema initial range (b) probability distribution of parameter value for schema CI-MINR
 (c) probability distribution of parameter values for schema CI-MAXR (d) box-plot chart of E_{NS} for three schemas

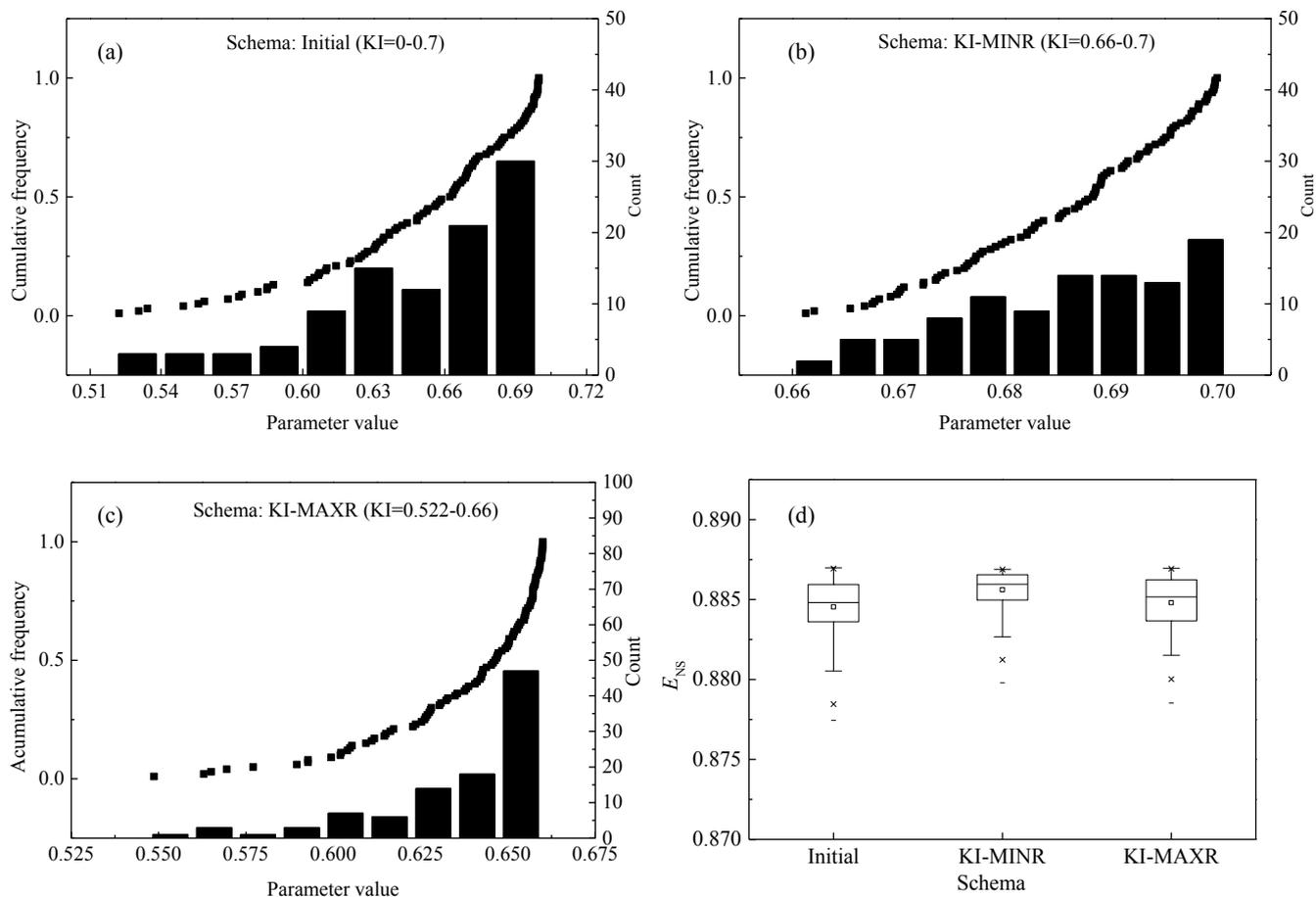


Fig. 7. Results of range selection of parameter KI

- (a) probability distribution of parameter values for schema initial range (b) probability distribution of parameter values for schema KI-MINR
 (c) probability distribution of parameter values for schema KI-MAXR (d) box-plot chart of E_{NS} for three schemas

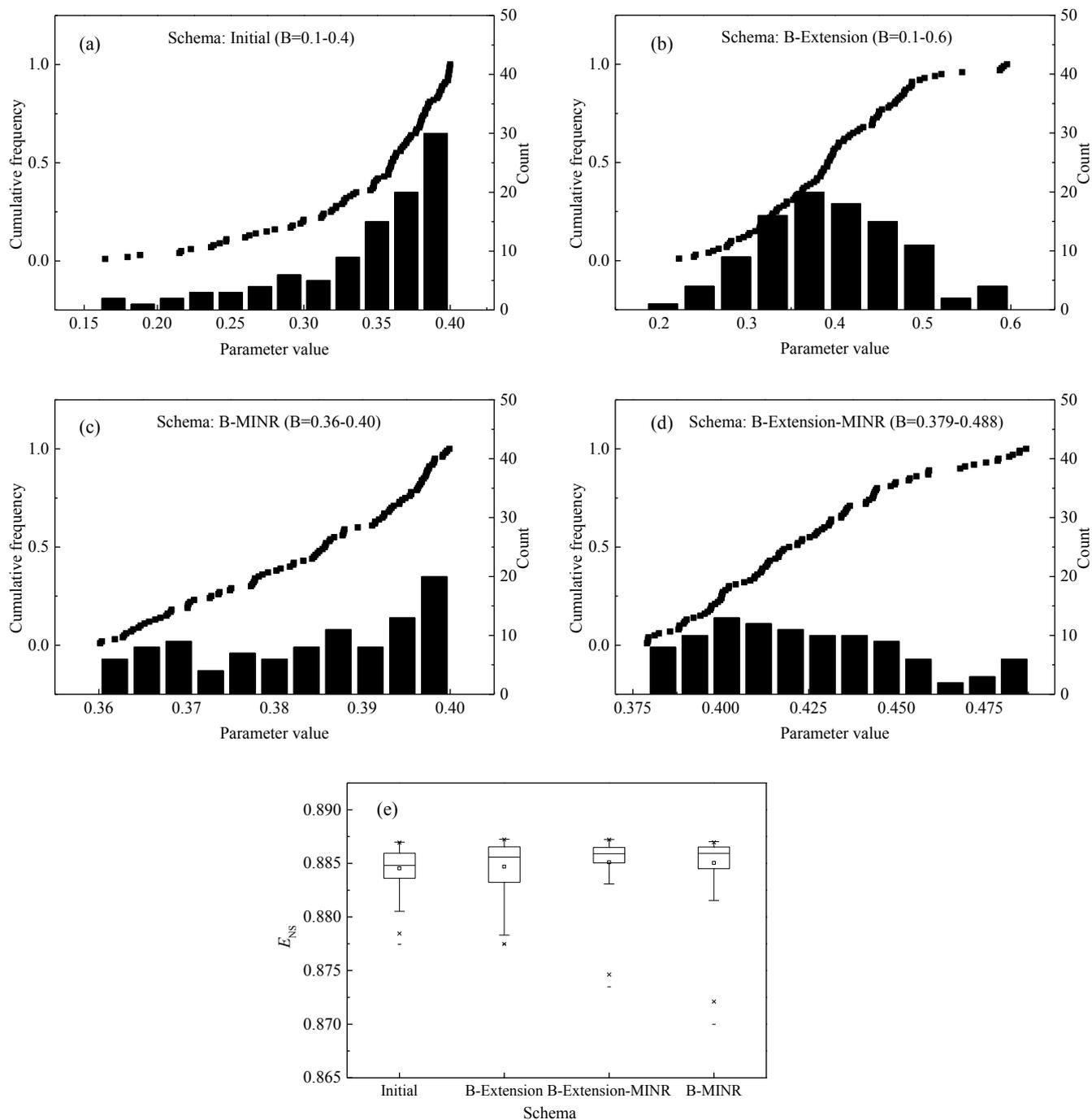


Fig. 8. Results of range selection of parameter B

(a) probability distribution for schema initial range (b) probability distribution for schema B-Extension (c) probability distribution for schema B-MINR (d) probability distribution for schema B-Extension-MINR (e) box-plot chart of E_{NS} for four schemas

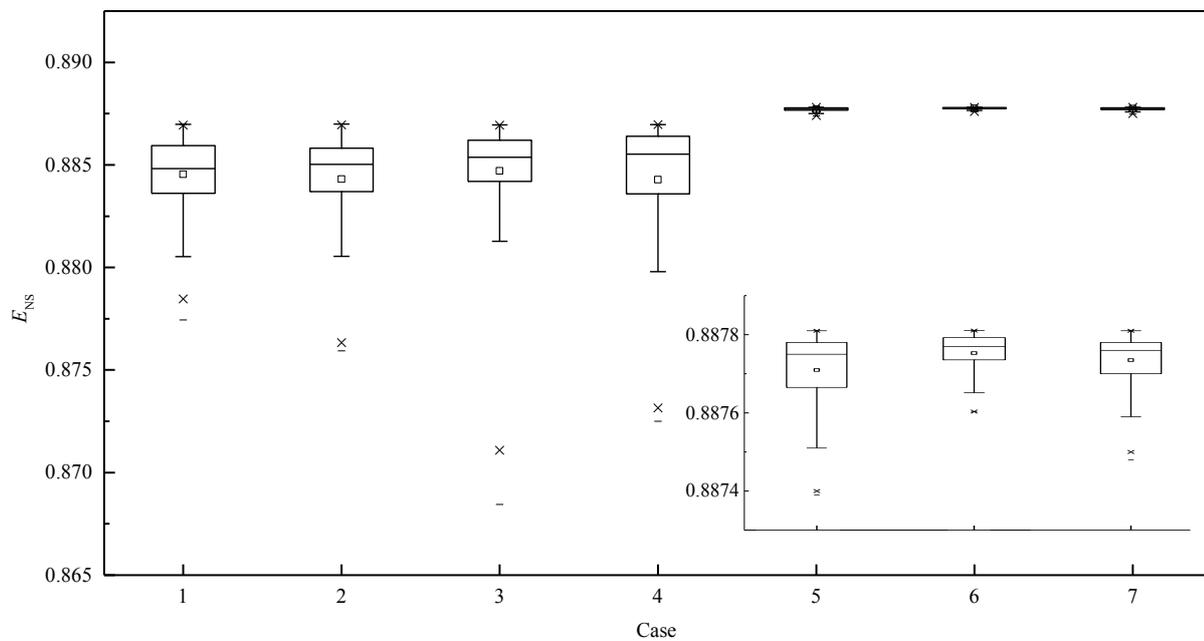


Fig. 9. Box-plot chart of E_{NS} for different cases



Table 1. Metro-hydrological statistical data

Year	Q_{Max} (m^3/d)	Q_{Min} (m^3/d)	Q_{Avg} (m^3/d)	P_{Max} (mm/d)
1996	719	0.76	14.38	235
1997	308	0.76	14.32	155
1998	369	0.66	13.67	157
1999	282	0.53	12.81	144
2000	339	1.14	11.37	107

Q means streamflow and P means average precipitation.

Table 2. Parameters of Xinanjiang model

Parameter	Definition	Range
Kc	Ratio of potential evapotranspiration to pan evaporation	0–1.1
C	Coefficient of the deep layer, that depends on the proportion of the basin area covered by vegetation with deep roots	0.1–0.2
WM	Averaged soil moisture storage capacity of the whole layer	120–200 (mm)
B	Exponential parameter with a single parabolic curve, which represents the non-uniformity of the spatial	0.1–0.4
Im	Percentage of impervious and saturated areas in the catchment	0.01–0.04
SM	Areal mean free water capacity of the surface soil layer, which represents the maximum possible deficit of free water storage	10–50 (mm)
EX	Exponent of the free water capacity curve influencing the development of the saturated area	1.0–1.5
KI	Outflow coefficients of the free water storage to interflow	0–0.7
KG	relationships Outflow coefficients of the free water storage to groundwater relationships	KG+KI=0.7
CG	Recession constants of the groundwater storage	0.950–0.998
CI	Recession constants of the lower interflow storage	0–0.9



Table 3. Range changes of parameters in schema Initial

Parameter	Initial parameter range	Calibrated parameter range*	Ratio** (%)
CI	0–0.9	0.630–0.745	12.78
Kc	0–1.1	0.81–1.09	25.45
KI	0–0.7	0.534–0.7	23.71
SM	10–50	31–39.4	21.00
B	0.1–0.4	0.238–0.4	54.00
WM	120–200	120–150	37.50
C	0.1–0.2	0.1–0.2	100.00
EX	1.0–1.5	1.0–1.5	100.00
CG	0.950–0.998	0.950–0.994	91.67
Im	0.01–0.04	0.01–0.04	100.00

* the calibrated parameter range except the extreme outlier

** the ratio is the ratio of calibrated parameter range to initial parameter range

5 Table 4. The indexed R_C and S_E of parameters when optimal range for single parameter is performed for calibration

Parameter*	CI	Kc	KI	SM	B	WM	C	EX	CG	Im	
Optimal range of single parameter	0.679– 0.713	0.95– 1.05	0.66– 0.7	35–39	0.379– 0.488	105– 110	0.175 –0.2	1–1.118	0.95– 0.966	0.01– 0.0245	
R_C	CI	1.000	0.334	0.371	0.462	0.322	0.113	0.105	0.115	–0.128	0.272
	Kc	0.689	1.000	0.467	0.429	0.504	0.503	0.389	0.102	0.284	0.150
	KI	0.778	0.315	1.000	0.445	0.574	0.268	0.456	0.328	0.060	0.258
	SM	0.508	–0.199	0.422	1.000	–0.089	0.009	–0.063	0.383	0.218	–0.032
	B	0.914	0.560	0.698	–0.017	1.000	0.972	–0.175	0.007	–0.319	–0.722
	WM	0.575	0.311	0.439	0.553	0.325	1.000	0.229	0.360	–0.069	–0.235
	C	0.208	0.273	0.083	0.151	0.277	0.335	1.000	0.077	0.200	0.210
	EX	0.054	0.047	–0.011	0.018	0.371	0.045	0.009	1.000	–0.021	–0.025
	CG	0.221	0.246	–0.135	0.022	0.010	0.198	–0.034	–0.009	1.000	–0.112
	Im	0.238	0.073	–0.025	0.045	0.031	0.030	–0.026	–0.020	0.001	1.000
Mean of R_C	0.465	0.218	0.257	0.234	0.258	0.275	0.099	0.149	0.025	–0.026	
S_E (%)	54.7	47.9	36.6	41.7	48.1	39.9	10.8	14.7	21.9	0.3	

* The parameter represent the parameter X in Eq. 2.



Table 5. Parameter ranges setting for different cases

Case	Range setting of parameter									
	WM	C	B	SM	EX	KI	CI	CG	Kc	Im
1	I	I	I	I	I	I	I	I	I	I
2	I	I	I	I	I	I	I	I	I	O
3	I	I	I	I	O	I	I	I	I	I
4	I	I	I	I	O	I	I	I	I	O
5	O	O	O	O	O	O	O	I	O	I
6	O	O	O	O	O	O	O	O	O	I
7	O	O	O	O	O	O	O	O	O	O

The symbol ‘I’ represents the initial range of the parameter in Table 3, and ‘O’ the optimal range of the parameter in Table 4.