

We thank the reviewer for his time and comments, but we would like to rectify a number of arguments before the discussion gets out of hand.

1. The term with v in the governing equation does not imply that the soil moves in the z -direction, it implies that heat is advected through the movement of the fluid in the soil, in a direction opposite the z -direction. This is logical because the z -axis is positive upwards (stated immediately after equation A1), so if v is positive flow will be downwards.

Stating that in this equation the soil is moving, is similar to stating that the gravity term in the Richards equation is making the soil move (which is not the case, it makes the fluid move), or that in the continuity equation the v (or Q) term makes the control volume move (again, the fluid is moving).

But then again, this discussion is not very important, because immediately before equation A1 we clearly state that we use this term with v because it makes the analytical solution easier. We then calculate the limit case for v equal to zero and use this solution. This, even if the argument that in this equation the soil column is moving were true (which it is not), it is irrelevant, because we use the limit case for v equal to zero.

2. The mathematical solution is correct. One simply needs to look at any book on Laplace transforms to see that this is a correct way to perform an inverse transform. Furthermore, this strategy has been used in numerous studies in hydrology, for example in Verhoest and Troch (2000) or Brutsaert (1994), to name just a few (there are many more papers that use this method).

We clearly show in section 6.2 that the analytical solution is identical to the numerical simulation if the spatial and temporal resolutions are fine enough. There are deviations in the case of coarse resolutions, but that can be expected with any numerical solution. One can simply plug equation A5 back into equation A3 to see that the solution is correct. We did not do this in the paper (as it would not add value to the paper), but the first derivative of equation A5 is:

$$\frac{dT}{dz} = \frac{vC}{\kappa} (T_{b,0} - T_{t,0}) \frac{\frac{vC}{e \kappa} z}{\frac{vC}{e \kappa} z_b - \frac{vC}{e \kappa} z_t} \quad (1)$$

The second derivative is:

$$\frac{d^2T}{dz^2} = \frac{v^2C^2}{\kappa^2} (T_{b,0} - T_{t,0}) \frac{\frac{vC}{e\kappa}z}{\frac{vC}{e\kappa}z_b - \frac{vC}{e\kappa}z_t} \quad (2)$$

Plugging this back into equation A1 we obtain:

$$\kappa \frac{d^2T}{dz^2} - vC \frac{dT}{dz} = \left(\frac{v^2C^2}{\kappa} - \frac{v^2C^2}{\kappa} \right) (T_{b,0} - T_{t,0}) \frac{\frac{vC}{e\kappa}z}{\frac{vC}{e\kappa}z_b - \frac{vC}{e\kappa}z_t} = 0 \quad (3)$$

So the governing equation is fulfilled. Furthermore, if z is equal to z_b then:

$$T = T_{b,0} \quad (4)$$

If z is equal to z_t then:

$$T = T_{b,0} + (T_{b,0} - T_{t,0}) \frac{\frac{vC}{e\kappa}z_t - \frac{vC}{e\kappa}z_b}{\frac{vC}{e\kappa}z_b - \frac{vC}{e\kappa}z_t} = T_{b,0} - T_{b,0} + T_{t,0} = T_{t,0} \quad (5)$$

So here we prove analytically that the solution is correct, and in the paper we prove it numerically. To show that the methodology is completely correct, we will do the same for equation A24. We will first plug the fifth and sixth terms into the governing equation. The first derivative becomes:

$$\begin{aligned} \frac{\partial T(z, t)}{\partial z} &= -(T_{b,M} - T_{b,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_b)} \\ &\quad \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[b \sin \left(\frac{z - z_t}{z_b - z_t} x_n \right) + \frac{x_n}{z_b - z_t} \cos \left(\frac{z - z_t}{z_b - z_t} x_n \right) \right] e^{y_n t} \\ &\quad - (T_{t,M} - T_{t,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_t)} \\ &\quad \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[b \sin \left(\frac{z - z_b}{z_t - z_b} x_n \right) + \frac{x_n}{z_t - z_b} \cos \left(\frac{z - z_b}{z_t - z_b} x_n \right) \right] e^{y_n t} \end{aligned} \quad (6)$$

The second derivative is:

$$\begin{aligned}
\frac{\partial T(z, t)}{\partial z} = & \\
& -(T_{b,M} - T_{b,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_b)}. \\
& \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[\left(b^2 - \frac{x_n^2}{(z_b - z_t)^2} \right) \sin \left(\frac{z - z_t}{z_b - z_t} x_n \right) + \frac{2bx_n}{z_b - z_t} \cos \left(\frac{z - z_t}{z_b - z_t} x_n \right) \right] e^{y_n t} \quad (7) \\
& -(T_{t,M} - T_{t,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_t)}. \\
& \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[\left(b^2 - \frac{x_n^2}{(z_t - z_b)^2} \right) \sin \left(\frac{z - z_b}{z_t - z_b} x_n \right) + \frac{2bx_n}{z_t - z_b} \cos \left(\frac{z - z_b}{z_t - z_b} x_n \right) \right] e^{y_n t}
\end{aligned}$$

The transient part of the PDE is:

$$\begin{aligned}
\kappa \frac{\partial^2 T}{\partial z^2} - vC \frac{\partial T}{\partial z} = & \\
& -(T_{b,M} - T_{b,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_b)} \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[\left(\kappa b^2 - \kappa \frac{x_n^2}{(z_b - z_t)^2} - vCb \right) \sin \left(\frac{z - z_t}{z_b - z_t} x_n \right) \right] e^{y_n t} \\
& -(T_{b,M} - T_{b,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_b)} \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[\left(\frac{2\kappa b x_n}{z_b - z_t} - \frac{vC x_n}{z_b - z_t} \right) \cos \left(\frac{z - z_t}{z_b - z_t} x_n \right) \right] e^{y_n t} \\
& -(T_{t,M} - T_{t,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_t)} \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[\left(\kappa b^2 - \kappa \frac{x_n^2}{(z_t - z_b)^2} - vCb \right) \sin \left(\frac{z - z_b}{z_t - z_b} x_n \right) \right] e^{y_n t} \\
& -(T_{t,M} - T_{t,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_t)} \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} \left[\left(\frac{2\kappa b x_n}{z_t - z_b} - \frac{vC x_n}{z_t - z_b} \right) \cos \left(\frac{z - z_b}{z_t - z_b} x_n \right) \right] e^{y_n t} \quad (8)
\end{aligned}$$

Following the definition of b the cosine terms become zero. We can further simplify:

$$\begin{aligned}
\kappa \frac{\partial^2 T}{\partial z^2} - vC \frac{\partial T}{\partial z} = & \\
& -(T_{b,M} - T_{b,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_b)} \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} C y_n \sin \left(\frac{z - z_t}{z_b - z_t} x_n \right) e^{y_n t} \quad (9) \\
& -(T_{t,M} - T_{t,M-1}) \frac{2\kappa}{C(z_t - z_b)^2} e^{b(z-z_t)} \sum_{n=1}^{\infty} (-1)^n \frac{x_n}{y_n} C y_n \sin \left(\frac{z - z_b}{z_t - z_b} x_n \right) e^{y_n t}
\end{aligned}$$

This is equal to the temporal derivative. A similar reasoning can be made for the last two terms in A24. If we set z equal to z_b in the fifth term, we obtain the sine of x_n , which is by definition zero. z equal to z_t leads to the sine of zero, which is again zero. A similar reasoning can be made for the sixth term. So, if the steady-state terms satisfy

the boundary conditions, the entire solution satisfies them.

We will now analyse the first four terms (the steady-state terms). For z_b this becomes:

$$T(z_b, t) = T_{b,M} - T_{b,0} + T_{b,0} = T_{b,M} \quad (10)$$

For z_t this becomes:

$$T(z_t, t) = T_{t,M} - T_{t,0} + (T_{t,0} - T_{b,0}) + T_{b,0} = T_{t,M} \quad (11)$$

So the boundary conditions are fulfilled. We calculate the first derivative:

$$\begin{aligned} \frac{dT(z, t)}{dz} &= (T_{b,M} - T_{b,0})e^{b(z-z_b)} \frac{b \sinh(b(z-z_t)) + b \cosh(b(z-z_t))}{\sinh(b(z_b-z_t))} \\ &+ (T_{t,M} - T_{t,0})e^{b(z-z_t)} \frac{b \sinh(b(z-z_b)) + b \cosh(b(z-z_b))}{\sinh(b(z_t-z_b))} \\ &+ (T_{t,0} - T_{b,0}) \frac{\frac{vC}{\kappa} z}{\frac{vC}{\kappa} z_t - \frac{vC}{\kappa} z_b} \end{aligned} \quad (12)$$

The second derivative is:

$$\begin{aligned} \frac{d^2T(z, t)}{dz^2} &= (T_{b,M} - T_{b,0})e^{b(z-z_b)} \frac{(b^2 + b^2) \sinh(b(z-z_t))}{\sinh(b(z_b-z_t))} \\ &+ (T_{b,M} - T_{b,0})e^{b(z-z_b)} \frac{2b^2 \cosh(b(z-z_t))}{\sinh(b(z_b-z_t))} \\ &+ (T_{t,M} - T_{t,0})e^{b(z-z_t)} \frac{(b^2 + b^2) \sinh(b(z-z_b))}{\sinh(b(z_t-z_b))} \\ &+ (T_{t,M} - T_{t,0})e^{b(z-z_t)} \frac{2b^2 \cosh(b(z-z_b))}{\sinh(b(z_t-z_b))} \\ &+ (T_{t,0} - T_{b,0}) \frac{\frac{v^2 C^2}{\kappa^2} z}{\frac{vC}{\kappa} z_t - \frac{vC}{\kappa} z_b} \end{aligned} \quad (13)$$

The steady-state part of the PDE is:

$$\begin{aligned}
& \kappa \frac{\partial T^2}{\partial z^2} - vC \frac{\partial T}{\partial z} = \\
& (T_{b,M} - T_{b,0}) e^{b(z-z_b)} \frac{(2\kappa b^2 - bvC) \sinh(b(z-z_t))}{\sinh(b(z_b-z_t))} \\
& + (T_{b,M} - T_{b,0}) e^{b(z-z_b)} \frac{(2\kappa b^2 - vCb) \cosh(b(z-z_t))}{\sinh(b(z_b-z_t))} \\
& + (T_{t,M} - T_{t,0}) e^{b(z-z_t)} \frac{(2\kappa b^2 - bCv) \sinh(b(z-z_b))}{\sinh(b(z_t-z_b))} \\
& + (T_{t,M} - T_{t,0}) e^{b(z-z_t)} \frac{(2\kappa b^2 - vCb) \cosh(b(z-z_b))}{\sinh(b(z_t-z_b))} \\
& + (T_{t,0} - T_{b,0}) \left(\frac{v^2 C^2}{\kappa} - \frac{v^2 C^2}{\kappa} \right) \frac{e^{\frac{vC}{\kappa} z}}{e^{\frac{vC}{\kappa} z_t} - e^{\frac{vC}{\kappa} z} - e^{\frac{vC}{\kappa} z_b}}
\end{aligned} \tag{14}$$

Application of the definition of b shows that this is equal to zero, so the equation is solved correctly.

So we have analytically shown that the solution satisfies the boundary conditions and the governing equation. Furthermore, in the paper we also show this numerically. The solution methodology is very well known in literature. We really do not see what else we can do to show that our solution is correct.

References:

- Brutsaert, W. (1994), The unit response of groundwater outflow from a hillslope, *Water Resour. Res.*, 30(10), 2759-2763.
 - Verhoest, N. E. C., and P. A. Troch (2000), Some analytical solutions of the linearized Boussinesq equation with recharge for a sloping aquifer, *Water Resour. Res.*, 36(3), 793-800.
3. Wang et al (2011) use different boundary conditions, more specifically a flux at the bottom and top of the soil profile. We use a temperature boundary condition, not a flux. We do not see how we can compare the two solutions.
 4. We devote five lines to the calibration algorithm, which cannot be considered excessive. We then explain the objective function that is minimized, which is very important for the remainder and the objectives of the paper.

We appreciate the effort that the reviewer undertook to review the paper, but there are clearly some misunderstandings that need to be rectified in order to continue the review of the paper in a correct manner.