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Matching the Turc-Budyko functions with the complementary evaporation relationship: consequences for the drying power of the air and the Priestley-Taylor coefficient

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Abstract. The Turc-Budyko functions $B_1(\Phi_p)$ are dimensionless relationships relating the ratio E/P (actual evaporation over precipitation) to the aridity index $\Phi_p = E_p/P$ (potential evaporation over precipitation). They are valid on long timescales at catchment scale with E_p generally defined by Penman's equation. The complementary evaporation (CE) relationship stipulates that a decreasing actual evaporation enhances potential evaporation through the drying power of the air which becomes higher. The Turc-Mezentsev function with its shape parameter λ is chosen as example among various Turc-Budyko curves and the CE relationship is implemented in the form of the Advection-Aridity model. First, we show that there is a functional dependence between the Turc-Budyko curve and the drying power of the air. Then, we examine the case where potential evaporation E_0 is calculated by means of the Priestley-Taylor equation with a varying coefficient α_0. Introducing the CE relationship into the Turc-Budyko function leads to a new transcendental form of the Turc-Budyko function $B_1'(\Phi_0)$ linking E/P to $\Phi_0 = E_0/P$. The two functions $B_1(\Phi_p)$ and $B_1'(\Phi_0)$ are equivalent only if α_0 has a specified value which is determined. The functional relationship between the Priestley-Taylor coefficient, the Turc-Mezentsev shape parameter and the aridity index is specified and analysed.

1 Introduction

The Turc-Budyko curves are analytical formulations of the functional dependence of actual evaporation E on moisture availability represented by precipitation P and atmospheric water demand represented by potential evaporation E_p. They are valid on long timescales at catchment scale. More precisely, the Turc-Budyko relationships relate the evaporation fraction E/P to an aridity index defined as $\Phi_p = E_p/P$. Empirical formulations have been obtained by simple fitting to observed values (Turc, 1954; Budyko, 1974). Analytical derivations have also been developed (Mezentsev, 1955; Fu, 1981; Zhang et al., 2004; Yang et al., 2008). The Turc-Budyko relationships have been extensively used in the scientific literature up to now and interpreted with physical models (Gerrits et al., 2009) or thermodynamic approaches (Wang et al., 20015). For some of the formulations the shape of the curve is determined by a parameter linked to catchment characteristics in terms of vegetation

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and soil water storage (Li et al., 2013; Yang et al., 2007). The most representative functions $E/P = B(\Phi_p)$ are shown in Table 1 (see Lebecherel et al. (2013) for an historical overview) and one of them (Turc-Mezentsev) is represented in Fig. 1 for different values of the shape parameter. Steady-state conditions are assumed, considering that all the water consumed by evaporation E comes from the precipitation P and that the change in catchment water storage is nil: $P\text{-}E = Q$ with Q the total runoff. All the Turc-Budyko functions should necessarily verify the following conditions: (i) $E = 0$ *if* $P = 0$, (ii) $E \leqslant P$ (water limit), (iii) $E \leqslant E_p$ (energy limit), (iv) $E \rightarrow E_p$ *if* $P \rightarrow +\infty$. These conditions define a physical domain where the Turc-Budyko curves are constrained (Fig. 1). It is interesting to note also that any Turc-Budyko function B_1 relating E/P to Φ_p can be transformed into a corresponding function B_2 relating E/E_p to $\Phi_p^{-1} = P/E_p$ (Zhang et al., 2004; Yang et al., 2008). Indeed we have

$$\frac{E}{E_p} = B_2(\Phi_p^{-1}) = \frac{E}{P}\frac{P}{E_p} = B_1(\Phi_p)\Phi_p^{-1} = \Phi_p^{-1}B_1\left(\frac{1}{\Phi_p^{-1}}\right). \quad (1)$$

Potential evaporation, which establishes an upper limit to the evaporation process in a given environment, is generally given by a Penman-type equation (Lhomme, 1997a). It is the sum of two terms: a first term depending on the radiation load R_n and a second term involving the drying power of the ambient atmosphere E_a

$$E_P = \frac{\Delta}{\Delta+\gamma}R_n + \frac{\gamma}{\Delta+\gamma}E_a . \quad (2)$$

In Eq. (2) γ is the psychrometric constant and Δ the slope of the saturated vapour pressure curve at air temperature. E_a represents the capacity of the ambient air to extract water from the surface. It is an increasing function of the vapour pressure deficit of the air D_a and of wind speed u through a wind function $f(u)$: $E_a = f(u)\, D_a$. Contrary to precipitation, potential evaporation E_p is not a forcing variable independent of the surface. E_p is in fact coupled to E by means of a functional relationship known as the complementary evaporation relationship (Bouchet, 1963), which stipulates that potential evaporation increases when actual evaporation decreases. This complementary behaviour is made through the drying power of the air E_a: a decreasing actual evaporation makes the ambient air drier, which enhances E_a and thus potential evaporation. Eq. (2) takes into account this complementary behaviour through the drying power E_a, which adjusts itself to the conditions generated by the rate of actual evaporation. It is also the case, for instance, when E_p is calculated as a function of pan evaporation.

However, in most of Turc-Budyko functions encountered in the literature, E_p is not accurately defined. Choudhury (1999, p. 100) noted that "varied methods were used to calculate E_p, and these methods can give substantially different results". Many formulae, in fact, can be used to calculate the potential rate of evaporation, each one involving different weather variables and yielding different values. Some formulae are based upon temperature alone, others on temperature and radiation (Carmona et al., 2016). In the present study we examine the case where E_p is estimated via a Priestley-Taylor type equation (Priestley and Taylor, 1978) with a variable coefficient α_0:

$$E_0 = \alpha_0 \frac{\Delta}{\Delta+\gamma}R_n . \quad (3)$$

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Soil heat flux is neglected on large timescale. The coefficient α_0, generally named Priestley-Taylor coefficient, is supposed to increase with climate aridity, from around *1.25* up to *1.75* (Shuttleworth, 2012), which can be seen as a direct consequence of the complementary evaporation relationship. Lhomme (1997b) made a thorough examination of the coefficient α_0 by means of a convective boundary layer model.

In the present paper, the behaviour of the drying power of the air E_a will be examined, together with its physical boundaries, in relation to the actual rate of evaporation predicted by the Turc-Budyko functions. It will be also shown that the coefficient α_0 has a functional relationship with the shape parameter of the Turc-Budyko curve and the aridity index. The standpoint used in this study differs from various previous attempts undertaken in the literature to examine from different perspectives the links between Bouchet and Turc-Budyko relationships, investigating their apparent contradictory behaviour. For example, Zhang et al. (2004) established a parallel between the assumptions underlying Fu's equation and the complementary relationship. In a study by Yang et al. (2006) concerning numerous catchments in China, the consistency between Bouchet, Penman and Turc-Budyko hypotheses was theoretically and empirically explained. Lintner et al. (2015) examined the Budyko and complementary relationships using an idealized prototype representing the physics of large-scale land-atmosphere coupling in order to evaluate the anthropogenic influences. Zhou et al. (2015) developed a complementary relationship for partial elasticities to generate Turc-Budyko functions, their relationship fundamentally differing from Bouchet's one. Carmona et al. (2016) proposed a power law to overcome a physical inconsistency of the Budyko curve in humid environments, this new scaling approach implicitly incorporating the complementary evaporation relationship.

The paper is organized as follows. First, the basic equations used in the development are detailed: the choice of a particular Turc-Budyko function is discussed and the complementary evaporation relationship, implemented through the Advection-Aridity model (Brutsaert and Stricker, 1979) is presented. Second, the feasible domain of the drying power of the air E_a is examined, together with the correspondence between E_a and actual evaporation in dimensionless form. Third, the functional relationship linking the Priestley-Taylor coefficient α_0 to the shape parameter of the Turc-Budyko function and the aridity index is inferred. In the following development, "Turc-Budyko" will be abbreviated in TB and "complementary evaporation" in CE.

2 Basic equations

Among the TB functions given in Table 1, one particular form is retained in our study: the one initially obtained by Turc (1954) and Mezentsev (1955) through empirical considerations and then analytically derived by Yang et al. (2008) through the resolution of a Pfaffian differential equation with particular boundary conditions. Three reasons guided this choice: (i) the function is one of the most commonly used; (ii) it involves a model parameter λ which allows it to evolve within the Turc-Budyko framework; (iii) it has a notable simple mathematical property expressed as: $F(1/x) = F(x)/x$. This last property means that the same mathematical expression is valid for B_1 and B_2 (Eq. 1). The so-called Turc-Mezentsev function is expressed as:

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$$\frac{E}{P} = B_1(\Phi_p) = \Phi_p\left[1 + (\Phi_p)^{\lambda}\right]^{-\frac{1}{\lambda}} = \left[1 + (\Phi_p)^{-\lambda}\right]^{-1/\lambda} . \quad (4)$$

It is written here with an exponent noted λ instead of the n generally used (Yang et al., 2009). The slope of the curve for $\Phi_p = 0$ is 1. When the model parameter λ increases from 0 to $+\infty$, the curves grow from the x-axis (zero evaporation) to an upper limit (water and energy limits), as shown in Fig. 1. In other words, when λ increases, actual evaporation gets closer to its maximum rate and when Φ_p tends to infinite E/P tends to 1. The intrinsic property of Eq. (4) allows it to be transformed into a similar equation with E/E_p replacing E/P and Φ_p^{-1} replacing Φ_p (see Figs. 2a, b):

$$\frac{E}{E_p} = B_2(\Phi_p^{-1}) = \Phi_p^{-1}\left[1 + (\Phi_p^{-1})^{\lambda}\right]^{-\frac{1}{\lambda}} = \left[1 + (\Phi_p^{-1})^{-\lambda}\right]^{-1/\lambda} . \quad (5)$$

Fu (1981) and Zhang et al. (2004) derived a very similar equation with a shape parameter ω (see Table 1) and Yang et al. (2008) established a simple linear relationship between the two parameters ($\omega = \lambda + 0.72$). In the rest of the paper, the development and calculations are made with the Turc-Mezentsev formulation. However, similar (but less straightforward) results can be obtained with the Fu-Zhang formulation (see the supplementary material S4).

The complementary evaporation (CE) relationship expresses that actual evaporation E and potential evaporation E_p are related in a complementary way following

$$E + bE_p = (1 + b)E_w . \quad (6)$$

E_w is the wet environment evaporation, which occurs when $E = E_p$ and b is a proportionality coefficient (Han et al., 2012). Various forms of the CE relationship exist in the literature (Xu et al., 2005). In our analysis, it is interpreted in the widely accepted framework of the Advection-Aridity model (Brutsaert and Stricker, 1979), where $b = 1$, potential evaporation E_p is calculated using Penman's equation (Eq. 2) and E_w is expressed by the Priestley-Taylor equation

$$E_w = \alpha_w \frac{\Delta}{\Delta+\gamma} R_n \ , \quad (7)$$

where the coefficient α_w has an estimated and fixed value of 1.26. E_w only depends on net radiation and air temperature through Δ. As already said in the introduction, the complementarity between E and E_p is essentially made through the drying power of the air E_a: a decrease in regional actual evaporation, consecutive to a decrease in water availability, generates a drier air, which enhances E_a and thus E_p. The fact that E_0 (Eq. 3), as a substitute for E_p, should also verify the CE relationship implies that: $\alpha_w \leqslant \alpha_0 \leqslant 2\alpha_w$.

3 Feasible domain of the drying power of the air and correspondence with the evaporation rate

As a consequence of the CE relationship, the drying power of the air E_a is linked to the evaporation rate. Its feasible domain is examined hereafter by determining its bounding frontiers and its behaviour is assessed as a function of the evaporation rate. Inverting Eq. (2) and replacing its radiative term by E_w (Eq. 7) yields to

$$E_a = \left(1 + \frac{\Delta}{\gamma}\right)\left(E_p - \frac{E_w}{\alpha_w}\right) . \quad (8)$$

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Taking into account the CE relationship (Eq. 6 with b=1) and scaling by E_p leads to

$$\frac{E_a}{E_p} = \left(1+\frac{\Delta}{\gamma}\right)\left[1-\frac{1}{2\alpha_w}\left(1+\frac{E}{E_p}\right)\right]. \tag{9}$$

Inserting Eq. (5) into Eq. (9) gives

$$\frac{E_a}{E_p} = D\left(\Phi_p^{-1}\right) = \left(1+\frac{\Delta}{\gamma}\right)\left(1-\frac{1}{2\alpha_w}\left\{1+\Phi_p^{-1}\left[1+\left(\Phi_p^{-1}\right)^{\lambda}\right]^{-\frac{1}{\lambda}}\right\}\right). \tag{10}$$

This means that the ratio E_a/E_p can be also expressed and drawn as a function of Φ_p^{-1} like the TB functions. Given that there is a water limit expressed by $0 < E < P$ and an energy limit expressed by $0 < E < E_p$, the function $E_a/E_p = D(\Phi_p^{-1})$ should meet the following three conditions:

(i) $E > 0$ implies that $E_a < E_{a,x}$ given by:

$$\frac{E_{a,x}}{E_P} = \left(1+\frac{\Delta}{\gamma}\right)\left(1-\frac{1}{2\alpha_w}\right). \tag{11}$$

(ii) $E < P$ implies that $E_a > E_{a,n1}$ given by:

$$\frac{E_{a,n1}}{E_p} = \left(1+\frac{\Delta}{\gamma}\right)\left[1-\frac{1}{2\alpha_w}\left(1+\frac{P}{E_P}\right)\right]. \tag{12}$$

(iii) $E < E_p$ implies that $E_a > E_{a,n2}$ given by:

$$\frac{E_{a,n2}}{E_p} = \left(1+\frac{\Delta}{\gamma}\right)\left(1-\frac{1}{\alpha_w}\right). \tag{13}$$

With E_p as scaling parameter, the feasible domain of E_a/E_p in the dimensionless space ($\Phi_p^{-1} = P/E_p$, E_a/E_p) is shown in Fig. 2c: when evaporation is nil, $E_a = E_{a,x}$ is maximum (upper boundary in Fig. 2c); when evaporation is maximal, E_a is minimal (lower boundary in Fig. 2c). The maximum dimensionless difference D^* between the upper boundary ($E_{a,x}/E_p$) and the lower boundary is obtained by subtracting Eq. (13) from Eq. (11):

$$D^* = \frac{1}{2\alpha_w}\left(1+\frac{\Delta}{\gamma}\right). \tag{14}$$

There is a correspondence between the TB curves $E/P = B_1(\Phi_p)$ and $E/E_P = B_2(\Phi_p^{-1})$ drawn into Figs. 2a, b and the one of $E_a/E_p = D(\Phi_p^{-1})$ drawn in Fig. 2c. Figs. 2a, b, c show this correspondence for a particular case defined by $\lambda = 1$ and $T = 15°C$ ($\Delta = 110\ Pa\ °C^{-1}$). When the TB curves reach their upper limit, i.e. in very evaporative environments, the corresponding curve E_a/E_p reaches its lower limit. Conversely, when the TB curves reach their lower limit, i.e. the x-axis (no-evaporative environment), the corresponding E_a/E_p curve reaches its upper limit.

It is interesting to note that the parameter λ of the Turc-Mezentsev function has a clear graphical expression. Denoting by d^* the maximum difference between the Turc-Mezentsev curve and its upper limit (Fig. 2a), this difference ($0 < d^* < 1$) obviously occurring for $\Phi_p = P/E_p = 1$, we have from Eq. (4)

$$d^* = 1 - 2^{-\frac{1}{\lambda}}, \tag{15}$$

which leads to

$$\lambda = \frac{-ln2}{\ln(1-d^*)} \;. \quad (16)$$

When d^* varies from 1 to 0, the parameter λ varies from 0 to $+\infty$. The value corresponding to d^* in the graphical representation of $E_a/E_p= D(\Phi_p^{-1})$ (Fig. 2c) is the difference δ^* between the E_a/E_p curve (Eq. 10) and its lower boundary (Eq. 13) for $P/E_p = 1$. It is given by

$$\delta^* = \left(1+\frac{\Delta}{\gamma}\right)\frac{1}{2\alpha_w}\left(1-2^{-\frac{1}{\lambda}}\right) = D^* d^* \;. \quad (17)$$

This simple relationship shows that the dimensionless differences d^* and δ^* vary simultaneously in the same direction with a proportionality coefficient equal to D^*, whose value is close to 1. It is a direct consequence of the CE relationship. When d^* decreases, i.e. the dimensionless evaporation rate (E/P or E/E_p) increases, δ^* decreases, i.e. the drying power of the air E_a decreases: for a constant wind speed, the air becomes wetter.

In the next section, another consequence of the CE relationship will be examined in relation to the value of the Priestley-Taylor coefficient and its dependence on the rate of actual evaporation.

4 Linking the Priestley-Taylor coefficient to the TB functions

Using the CE relationship as a basis, this section examines the link existing between the Priestley-Taylor coefficient α_0 defined by Eq. (3) and the Turc-Mezentsev shape parameter λ (Eq. 4). Combining Eqs. (3), (6) and (7) potential evaporation can be written as

$$E_p = 2\frac{\alpha_w}{\alpha_0}E_0 - E \;. \quad (18)$$

Substituting E_p in Eq. (4) by its value given by Eq. (18) and putting $\Phi_0 = E_0/P$ gives

$$\frac{E}{P} = \left(\frac{2\alpha_w}{\alpha_0}\Phi_0 - \frac{E}{P}\right)\left[1+\left(\frac{2\alpha_w}{\alpha_0}\Phi_0 - \frac{E}{P}\right)^{\lambda}\right]^{-\frac{1}{\lambda}} . \quad (19)$$

Eq. (19) can be rewritten as

$$\Phi_0 = B_1'^{-1}\left(\frac{E}{P}\right) = \frac{\alpha_0}{2\alpha_w}\left\{\left[\left(\frac{E}{P}\right)^{-\lambda} - 1\right]^{-1/\lambda} + \frac{E}{P}\right\} . \quad (20)$$

Eq. (20) represents a transcendental form of the Turc-Mezentsev function (Eq. 4) issued from the complementary relationship and written with $\Phi_0 = E_0/P$ instead of $\Phi_p = E_p/P$. Calling B_1' this new function $E/P = B_1'(\Phi_0)$, Eq. (20) represents in fact its inverse function $\Phi_0 = B_1'^{-1}(E/P)$. The function $E/P = B_1'(\Phi_0)$ has properties similar to the Turc-Mezentsev function (Eq. 4) (see the demonstrations in the supplementary materials S1): i) when Φ_0 tends to zero, $B_1'(\Phi_0)$ tends to zero with a slope equal to α_w/α_0 ($\leqslant 1$); ii) when Φ_0 tends to infinite, E/P tends to 1. A transcendental form of Eq. (5), called B_2', can be obtained by expressing E/E_0 as a function of $\Phi_0^{-1} = P/E_0$

$$\Phi_0^{-1} = B_2'^{-1}\left(\frac{E}{E_0}\right) = \left[\left(\frac{E}{E_0}\right)^{-\lambda} - \left(\frac{2\alpha_w}{\alpha_0} - \frac{E}{E_0}\right)^{-\lambda}\right]^{-1/\lambda} . \quad (21)$$

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Function B_2' has the following properties at its limits (see the supplementary materials S2): i) when Φ_0^{-1} tends to zero, $B_2'(\Phi_0^{-1})$ tends to zero with a slope equal to 1; ii) when Φ_0^{-1} tends to infinite, E/E_0 tends to α_w/α_0 ($\leqslant 1$).

For a given value of the exponent λ, a fixed value of α_0 and with $\alpha_w = 1.26$, the relationship between E/P and Φ_0 (or between E/E_0 and Φ_0^{-1}) can be obtained by using numerical methods to resolve Eqs. (20) and (21). Similar calculations, more or less complicated, could be made with any Turc-Budyko function. These results show that a Turc-Mezentsev curve (or any TB curve) generates a different curve when potential evaporation is given by E_0 instead of E_p. This new curve is represented in Fig. 3 by comparison with the original one for two values of the shape parameter λ (0.5 and 2) assuming $\alpha_0 = \alpha_w = 1.26$. The new curve has a form similar to the original one, with the same limits at 0 and $+\infty$, but it is higher or lower depending on the value of α_0. It is worthwhile noting also that B_2' is different from B_1', contrary to B_2 (Eq. 5) which is identical to B_1 (Eq. 4). Nevertheless the two curves are very close, as shown in Fig. 4, and it is easy to verify they have the same value for $\Phi_0 = \Phi_0^{-1} = 1$.

We have now two sets of TB functions: B'_1 and B'_2 (Eqs. 20 and 21) involving $\Phi_0 = E_0/P$ and their corresponding original formulations B_1 and B_2 (Eqs. 4 and 5) as a function of $\Phi_p = E_p/P$. The question now is to find out the value of α_0 which allows B'_1 to be equivalent (or the closest) to the original Turc-Mezentsev function B_1. Both equations expressing E/P as a function of an aridity index Φ (Φ_p or Φ_0), the expression of α_0 can be inferred by matching Eq. (20) and Eq. (4): for a given value of the aridity index Φ, B_1 and B_1' should give the same value of E/P. This leads to

$$\alpha_0 = \frac{2\alpha_w}{1+\left(1+\Phi^{\lambda}\right)^{-1/\lambda}} \ . \tag{22}$$

The same relationship (Eq. 22) is obtained by matching B'_2 with B_2. It is worthwhile noting that when α_0 is expressed by Eq. (22) and Φ_0 tends to zero (or Φ_0^{-1} tends to infinite), α_w/α_0 in Eqs. (20) and (21) tends to 1. This means that these equations have the same limits as their original equations (Eqs. 4 and 5). Putting the value of α_0 defined by Eq. (22) into B_1' and B_2' (Eqs. 20 and 21) leads to new transcendental equations linking E/P and Φ_0 (or E/E_0 and Φ_0^{-1}) which are exactly equivalent to the original Turc-Mezentsev functions (Eqs. 4 and 5). Function B_1' transforms into

$$\frac{E}{P} + \left[\left(\frac{E}{P}\right)^{-\lambda} - 1\right]^{-1/\lambda} = \Phi_0 + \left(1+\Phi_0^{-\lambda}\right)^{-1/\lambda} , \tag{23}$$

and B_2' into

$$\left\{1 + \left[1 + (\Phi_0^{-1})^{-\lambda}\right]^{-1/\lambda} - \frac{E}{E_0}\right\}^{-\lambda} = \left(\frac{E}{E_0}\right)^{-\lambda} - (\Phi_0^{-1})^{-\lambda} . \tag{24}$$

In the supplementary material (S3) we show that the original Turc-Mezentsev functions are the solutions of these transcendental equations.

For every value of λ and Φ, a unique value of α_0 can be calculated by means of Eq. (22), α_w being fixed. In this equation $\alpha_0 = f(\lambda, \Phi)$, Φ represents climate aridity and λ catchments characteristics in relation to its ability to evaporate (the greater λ, the higher its evaporation capability). The Priestley-Taylor coefficient α_0 appears to be an increasing function of Φ and a decreasing function of λ. Fig. 5a shows the relationship between α_0 and λ for different values of Φ. α_0 tends to $2\alpha_w$ when λ

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tends to zero (non-evaporative catchment) whatever the value of Φ. When λ tends to infinity (i.e. very evaporating catchment), the limit of α_0 depends on the value of Φ. For $\Phi \leqslant 1$ the limit is α_w and for $\Phi > 1$ the limit is the branch of the hyperbole $2\alpha_w\Phi/(1+\Phi)$. Fig. 5b shows the relationship between α_0 and Φ for different values of λ. When Φ tends to $+\infty$ (very arid catchment), the coefficient α_0 tends to $2\alpha_w$. When Φ tends to 0 (very humid catchment), α_0 tends to α_w. These results illustrate the simple functional relationship existing between the Priestley-Taylor coefficient, the TB shape parameter and the aridity index. Similar results are obtained when the Fu-Zhang formulation is used, as detailed in the supplementary material S4.

5 Summary and conclusion

The TB curves have two different and equivalent dimensionless expressions: B_1 where E/P is a function of the aridity index $\Phi_p = E_p/P$, and B_2 where E/E_p is a function of $\Phi_p^{-1} = P/E_p$; any B_1 curve can be transformed into an equivalent B_2 curve and conversely. Among various TB type curves, the Turc-Mezentsev one (Eq. 4) with the shape parameter λ was chosen because it is commonly used and has the remarkable property of having the same mathematical expression in both representations B_1 or B_2. Using Penman's equation (Eq. 2) to express potential evaporation and introducing the complementary evaporation relationship in the form of the Advection-Aridity model with its parameter α_w (Eqs. 6 and 7), it was shown that the dimensionless drying power of the air $D = E_a/E_p$ expressed as a function of Φ_p^{-1} has upper and lower boundaries and that there is a functional correspondence between the TB and D curves. Next, we examined the case where potential evaporation is expressed by the Priestley-Taylor equation (E_0 given by Eq. 3) with a varying coefficient α_0 instead of the sounder Penman's equation. Introducing the CE relationship in the form of the Advection-Aridity model shows that the Turc-Mezentsev function linking E/P to $\Phi_p = E_p/P$ (Eq. 4) transforms into a new transcendental form of the Turc-Budyko function B_1' linking E/P to $\Phi_0 = E_0/P$ (Eq. 20), only numerically resolvable. The Priestley-Taylor coefficient α_0 should have a specified value as a function of α_w, λ and $\Phi_0 = \Phi_p$ so that the two curves B_1 and B_1' be equivalent. This means that the coefficient α_0 ($\alpha_w \leqslant \alpha_0 \leqslant 2\alpha_w$) is intrinsically linked to the shape parameter λ of the Turc-Mezentsev function and to the aridity index.

6 List of symbols

B_1	function linking E/P to $\Phi_p = E_p/P$.
B_1'	function linking E/P to $\Phi_0 = E_0/P$ given by Eq. (20).
B_2	function linking E/E_P to $\Phi_p^{-1} = P/E_p$.
B_2'	function linking E/E_0 to $\Phi_0^{-1} = P/E_0$ given by Eq. (21).
D	function linking E_a/E_p to P/E_p.

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D^*	difference between the upper and lower boundaries of D [-].
d^*	maximum difference between the Turc-Budyko curve and its upper limit [-].
E	actual evaporation [LT^{-1}].
E_p	potential evaporation expressed by Penman's equation [LT^{-1}].
E_0	potential evaporation expressed by Priestley-Taylor equation [LT^{-1}].
E_w	wet environment evaporation in the CE relationship [LT^{-1}].
P	precipitation [LT^{-1}].
E_a	drying power of the air [LT^{-1}].
$E_{a,n1}$	lower limit of E_a given by Eq. (12) [LT^{-1}].
$E_{a,n2}$	lower limit of E_a given by Eq. (13) [LT^{-1}].
$E_{a,x}$	upper limit of E_a given by Eq. (11) [LT^{-1}].
R_n	net radiation [LT^{-1}].
α_0	coefficient of the Priestley-Taylor equation [-].
α_w	$=1.26$ [-].
γ	psychrometric constant [$M\ L^{-1}T^{-2}\ °C^{-1}$].
Δ	slope of the saturated vapour pressure curve at air temperature [$M\ L^{-1}T^{-2}\ °C^{-1}$].
δ^*	maximum difference between the E_a/E_p curve and its lower boundary [-].
λ	shape parameter of the Turc-Mezentsev equation ($\lambda > 0$) [-].
Φ_0	aridity index calculated with E_0 ($\Phi_0 = E_0/P$) [-].
Φ_p	aridity index calculated with E_p ($\Phi_p = E_p/P$) [-].

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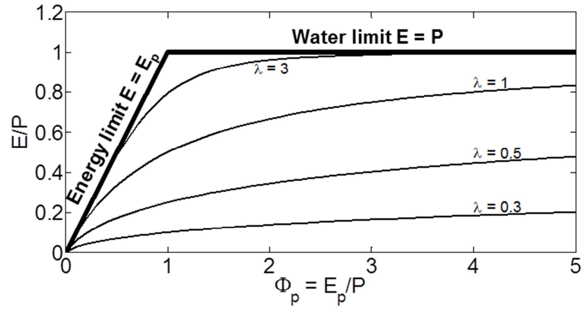

Figure 1: The Turc-Mezentsev relationship Eq. (4) between the ratio *E/P* and the aridity index $\Phi_p = E_p/P$ for four values of the parameter λ (*0.3, 0.5, 1* and *3*). The bold line indicates the upper limit of the feasible domain.

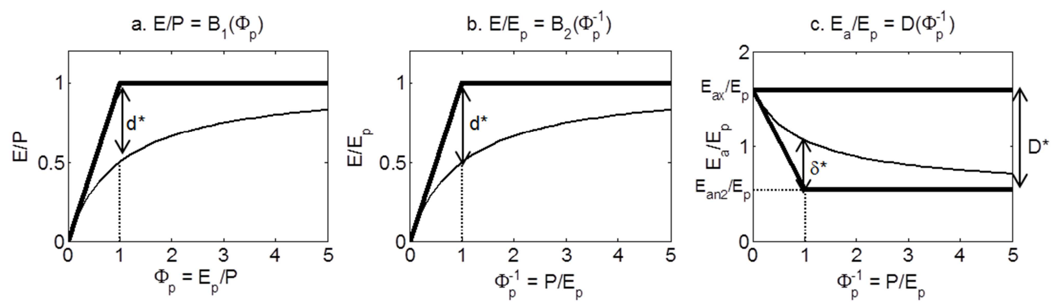

Figure 2: Correspondence between the two forms of the Turc-Mezentsev functions ($E/P = B_1(\Phi_p)$, given by Eq. (4) and $E/E_p = B_2(\Phi_p^{-1})$ given by Eq. (5)) and the function defining the drying power of the air $E_a/E_p = D(\Phi_p^{-1})$ given by Eq. (10). The calculations are made with a shape parameter $\lambda = 1$ and a temperature of *15°C*: $E_{a,x}/E_p = 1.59$, $E_{a,n2}/E_p = 0.54$, $d^* = 0.50$, $D^* = 1.05$ and $\delta^* = 0.52$. The bold lines indicate the upper limit of the feasible domain.

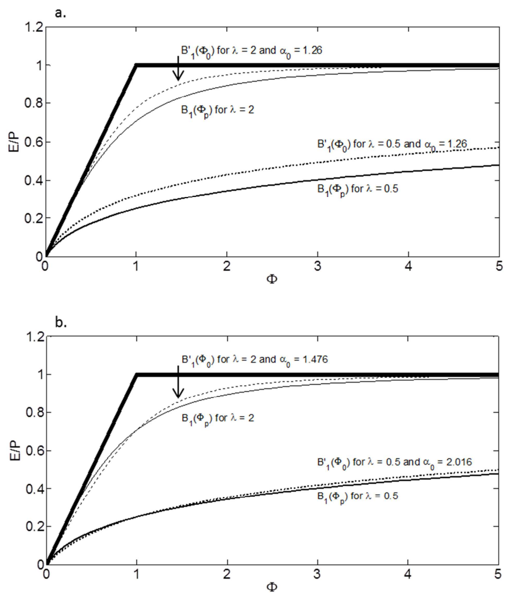

Figure 3: Comparison between the Turc-Mezentsev function $B_1(\Phi_p)$ (Eq. 4) in solid line and its corresponding function $B_1'(\Phi_0)$ (Eq. 20) in dotted line for two values of λ (0.5 and 2): (a) with $\alpha_0 = \alpha_w = 1.26$; (b) with α_0 adjusted according to Eq. (22) for $\Phi = 1$. The x-axis legend Φ represents either Φ_p for $B_1(\Phi_p)$ or Φ_0 for $B_1'(\Phi_0)$.

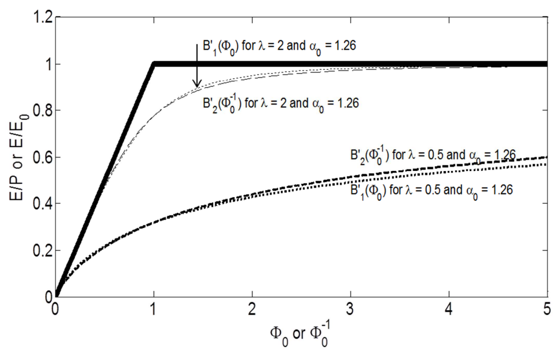

Figure 4: Comparison of functions $E/P = B_1'(\Phi_0)$ (Eq. 20) and $E/E_0 = B_2'(\Phi_0^{-1})$ (Eq. 21) for two different values of the shape parameter λ (0.5 and 2) and the same value of $\alpha_0 = 1.26$.

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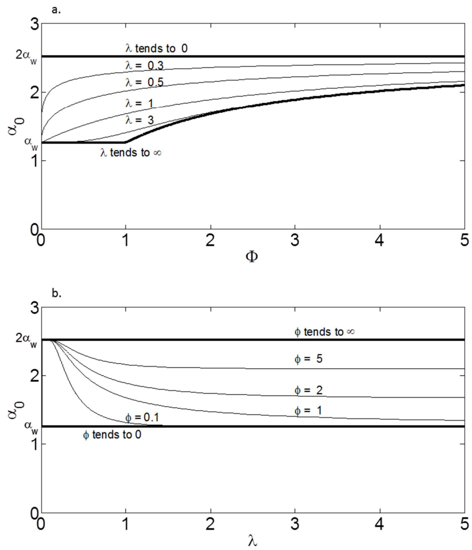

Figure 5: Variation of the Priestley-Taylor coefficient α_0: (a) as a function of the aridity index Φ for different values of the shape parameter λ of the Turc-Mezentsev function; (b) as a function of λ for different values of the aridity index Φ (Eq. (22) with α_w = *1.26*). The bold lines indicate the upper and lower limits of the feasible domain.

Table 1: Different expressions for the Turc-Budyko curves as a function of the aridity index Φ_p.

Equation	Reference
$E/P = \left\{\Phi_0 \tanh(\frac{1}{\Phi_p})[1 - \exp(-\Phi_p)]\right\}^{1/2}$	Budyko (1974)
$E/P = \Phi_p \left[1 + (\Phi_p)^{\lambda}\right]^{-\frac{1}{\lambda}}$	Turc (1954) with λ = *2*, Mezentsev (1955), Yang et al. (2008)
$E/P = 1 + \Phi_p - \left[1 + (\Phi_p)^{\omega}\right]^{\frac{1}{\omega}}$	Fu (1981), Zhang et al. (2004)
$E/P = \frac{1 + w\Phi_p}{1 + w\Phi_p + {\Phi_p}^{-1}}$	Zhang et al. (2001)
$E/P = \Phi_p \left(\frac{k}{1 + k{\Phi_p}^{n}}\right)^{1/n}$	Zhou et al. (2015)