GENERAL COMMENTS

The paper offers an interesting perspective on the equations that govern water age evolution in catchments. Building upon previous works (in particular Botter et al., 2011, van der Velde et al., 2012, Harman 2015 and Benettin et al., 2015), the manuscript explores the hydrologic balance equation in the travel time dimension, and all the age distributions associated with it. Although the paper is written in formal mathematical language, some parts may be difficult to follow and should be expanded, possibly including some physical interpretation. I encourage the authors to improve the readability of the paper and I list below some suggestions.

DETAILED COMMENTS

Notation

Everyone is of course free to use the notation which is most suitable to explain their research. However, last year many researchers made a big effort to converge upon a unifying and useful notation for the travel time literature, based on their experience. The notation was presented in Harman, (2015) and Rinaldo et al., (2015), and has been used in many other papers. If the authors want to clarify some concepts in the travel time formulation, I think the use of a different notation doesn't help the reader. I can understand the use of a probabilistic approach and the conditional probability notation, but I don't see the need for changing symbol for the most elementary variables. In particular I refer to:

- τ instead of t_{in} and ι instead of t_{ex} , for the injection and exit time of a water parcel in the domain
- SSF instead of SAS, to denote the StorAge Selection functions
- The lack of a subscript S to denote the age distributions that refer to the water storage

Use of probability distributions

The travel time literature was often related to stochastic hydrology and therefore it is natural to formulate its theory using probability formalism. However, in such a case, a probabilistic framework should be given i.e. one should define what the relevant random variables and the related sources of randomness are. Note, however, that the connection with stochasticity is often weak in applications. When one estimates the age distribution of streamwater at a certain time, he doesn't mean that there is a certain probability that water has a certain age. He wants to say that there is a distribution of water particles, each with a different estimated age, and those ages together can explain the measured solute concentration. Another example pertains the marginal distributions: the “probability” of observing an input (or an output) is not taken from stochastic hydrology concepts, it is instead a normalized timeseries of actual precipitation (or discharge) measurements.

Section 2

Eq 1: I think it should be explained that the time variables refer to a parcel of water that moves inside a hydrologic volume, entering at a time τ and leaving at a time ι.

Line 72: I have often seen this integral expressed between $-\infty$ and t. Maybe it would avoid the need to specify that time $t = 0$ comes before any input to the system?

Line 85: it may be worth saying that eq. (85) is a spatially-integrated equation that can be easily related to previous works in the literature, in particular Ginn (1999), Delhez et al., (1999) and Dagan (1984). See Benettin et al., (2013) for a review on this.

Section 3 (backward and forward approaches)

I think this section is very important but not sufficiently developed. The authors present 4 different kind of probabilities and it may be worth to specify which probabilities pertain to the different elements of the water budget. In particular, when the time t is to be interpreted as a general time at which the system is observed, the probabilities refer to the water particles in storage. Instead, when the time t has the special meaning of time at which particles enter or leave the system, the probabilities refer to the water particles in the fluxes.

Line 90: Please if possible refer to the paper Benettin et al., 2015 published on Hydrological Processes, instead of the Ph.D. thesis Benettin 2015, as the latter has no DOI.

Line 91: this a bit imprecise: in the mentioned paper the concept of backward refers to the residence time (or age). Instead, the concept of travel time is both a forward or backward concept, depending on the point of view (i.e. if ones focuses on the entrance or on the exit).

Line 95-96: given your definition in eq. (1), $t - \tau$ is a residence time and not a travel time, so the distributions should be residence time probabilities and not travel time probabilities.

Section 4

This section, in my opinion, does need to be further explored. I recommend the authors to include a physical interpretation of the processes, besides the mathematical description. Also, I think it may help the reader if the probability distributions associated with the water storage were denoted with a subscript S (e.g. p_S), just like the authors did for the probability associated with discharge (p_Q) or evapotranspiration (p_{E_T})

Line 105: Benettin (2015) used the notation $\overleftarrow{p}_S(T_R,t)$ and not $\overleftarrow{p}(T_R,t)$

Line 108-109: I did not understand this sentence (and maybe you meant eq (5) instead of eq (1))

Line 112: as this is a definition, the symbol $:=$ should be used?

Line 120: please enclose the first term at left hand side of eq. (14) within brackets, otherwise the derivative refers to $S(t)$ only. Please also explain what the symbol $\delta(t-\tau)$ refers to and why.

Line 129: please enclose the first term at left hand side of eq. (17) within brackets

Line 131: please specify that this is only valid in case eq. (17) is linear, i.e. $\omega(t,\tau)$ is not a function of $p(T_R|t)$

Line 135-145: Figures 2,3 and 4 need to be given more context and be better explained, because I had a hard time to interpret them and I am not sure the indicated t and τ are all in place . Please also specify what $P(T_R|t)$ is. I think the main point is letting the reader understand what happens when one keeps the chronologic time t fixed and let the injection time τ change, or vice versa.

Section 5

Just like Section 4, I think more interpretation should be provided. E.g. what does $p(t-\tau,\tau)$ represent? How is it related to $p_Q(t-\tau,\tau)$ and $p_{E_T}(t-\tau,\tau)$?

I think there may be some misunderstanding on the fact that the forward distributions cannot be known after time t. The problem is more general and equally applies to backward distributions, which are unknown for any time t lower than the first available measurement. Moreover, unless one needs to do real-time predictions, travel time computations can be done on datasets which are much longer than the period of interest.

Line 1: Please explain what you mean by “integral form”. It’s important to specify that you are integrating over dt, hence you follow a single injection and track its evolution while crossing the catchment.

Line 68 (eq. 29): I don't see why the asymptotic value is that important. I believe the knowledge of $q(t,\tau)$ is much more important, and $\Theta(\tau)$ is just its integral over dt.

Section 6

Line 181: as the authors are focusing on a parcel of water that enters at time $t = \tau$ and exits at $t = \iota$, I think the equations should be rewritten using ι instead of t.

Line 187: the symbol S for eq (34) and following is a bit ambiguous as the symbol $S(t)$ already appears at eq. (5) with a different meaning. Moreover, as the integral is defined from 0 to infinity, it appears to me that $S \to \infty$.

Please give an interpretation of the marginal pdf's.

I think the notation could be made more "symmetrical". Why is $p(\tau)$ the marginal pdf of the input and $p_Q(t)$ is the marginal probability of the output? Shouldn't they be either $p(\tau)$ and $p(\iota)$ or $p_Q(t)$ and $p_J(t)$?

Line 197: given the notation in eq. (37), it should be $p_Q(t)$ instead of $p(t)$

Section 7

In section 7 and 8 I found several typos and little errors in the formulas. I am not sure I detected them all, so please carefully revise these sections.

As in section 5, I think the authors should not assume by default that our knowledge limits to time t, and everything between t and ι is unknown. That is just a very special case, limited to real time forward modeling.

Line 226: $\iota - t$ instead of $\iota - s$

Line 229: isn't the canonic convolution symbol different?

Line 235: typo: time time

Line 237: typo: τs

Section 8

Please specify that the balance equation is in terms of substance mass.

Line 251: please add the dependence on entrance time at left hand side of the equation

Line 261: I would suggest, for consistency with eq. (53) and (54) to name it $C_Q^i(t)$ (note in the table of symbols at the end of the manuscript it is listed $C_Q^i(t)$)

Line 263: I don't understand the expression "it is usually assumed the validity of an integral expression like…". Given the definitions of the terms involved in the equation, the integral is just an expression of mass conservation.

Line 265: Θshould be a function of τ. Please note that here you use the product $\Theta(\tau)p(t-\tau|\tau)$ but in the subsequent equations $\Theta(\tau)$ disappears.

Line 271: shouldn't it be $p_Q(t-\tau|t)$ instead of $p(t-\tau|t)$? In your notation, the latter denotes the age probability of the water storage (eq. 10).

Line 287 and 294-296: why you did not use a dirac delta function (like in eq. 17) for the age distribution of precipitation?

Line 291-293: this sounds interesting but I am not sure it is actually feasible. I can't say what the issue may be from a mathematical point of view, but I have two examples in mind which are in conflict with the possibility of deriving $p(t-\tau|t)$ and $\omega(t,\tau)$ by coupling different budget equations. The first is that I am not sure the added mass budget equation is actually bringing different information with respect to the water budget equation. If the solute is conservative, it is transported just like water, so its concentration will simply be proportional to that of water, where $C_J^i(t)=1$. The second is that in case of multi-solute information one could get a system with 3 or more equations and just two unknown functions, so following your reasoning it would be impossible to solve, and this sounds strange to me.

Section 9

I think there is a very important assumption here, which is not explicitly stated. The fact that discharge is a linear function of storage does not automatically mean that each parcel of discharge (from an age-rank point of view) is a linear function of the corresponding parcel in storage. This only happen when a well-mixed (o random sampling) scheme is assumed.

I think it is also important that the authors acknowledge here the difference between the storage which can be estimated from a purely hydrologic balance (i.e. the "active" storage which gets displaced during the hydrologic response) and the total storage of a system. This is very important considering the recent debate about water displacement and water travel times (e.g. McDonnell and Beven, 2014, Rinaldo et al., 2015, Kirchner, 2016). If the equations derived in this section do not take into account the so called “passive” storage (Kirchner, 2009), they may be of limited practical use.

Conclusions

Line 394: typo “to the obtain understanding”

Appendix A

Please define $\Theta(t, \tau)$, as only $\Theta(\tau)$ has been defined so far, and use consistently the upper-case or lower-case symbols. Also, I guess some SSF for both discharge and evapotranspiration was imposed to produce the figures and I think it should mentioned.

REFERENCES

Benettin, P., Rinaldo, A., and Botter, G. (2015). Tracking residence times in hydrological systems: forward and backward formulations, Hydrological Processes.

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