Hydrol. Earth Syst. Sci. Discuss.,
doi:10.5194/hess-2016-210-AC3, 2016

Hydrology and Earth System Sciences Discussions

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HESSD

Interactive comment

Interactive comment on "Age-ranked hydrological budgets and a travel time description of catchment hydrology" *by* R. Rigon et al.

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Received and published: 20 July 2016

Answer to reviewer #2, Dr. P. Benettin

We thank Dr. Paolo Benettin for his detailed comments. This will help us to improve the manuscript.

GENERAL COMMENTS

The paper offers an interesting perspective on the equations that govern water age evolution in catchments. Building upon previous works (in particular Botter

et al., 2011, van der Velde et al., 2012, Harman 2015 and Benettin et al., 2015), the manuscript explores the hydrologic balance equation in the travel time dimension, and all the age distributions associated with it. Although the paper is written in formal mathematical language, some parts may be difficult to follow and should be expanded, possibly including some physical interpretation. I encourage the authors to improve the readability of the paper and I list below some suggestions.

We will try to add further explanations, trying be clearer and to improve the readability of the paper in the revised version.

DETAILED COMMENTS

Notation

Everyone is of course free to use the notation which is most suitable to explain their research. However, last year many researchers made a big effort to converge upon a unifying and useful notation for the travel time literature, based on their experience. The notation was presented in Harman, (2015) and Rinaldo et al., (2015), and has been used in many other papers. If the authors want to clarify some concepts in the travel time formulation, I think the use of a different notation doesn't help the reader. I can understand the use of a probabilistic approach and the conditional probability notation, but I don't see the need for changing symbol for the most elementary variables. In particular I refer to:

- **instead of tin and instead of tex for the injection and exit time of a water parcel in the domain**
- **SSF instead of SAS, to denote the StorAge Selection functions**

- **The lack of a subscript S to denote the age distributions that refer to the water storage**

We corrected the notation according to reviewer's request.

Use of probability distributions

The travel time literature was often related to stochastic hydrology and therefore it is natural to formulate its theory using probability formalism. However, in such a case, a probabilistic framework should be given i.e. one should define what the relevant random variables and the related sources of randomness are. Note, however, that the connection with stochasticity is often weak in applications. When one estimates the age distribution of streamwater at a certain time, he doesn't mean that there is a certain probability that water has a certain age. He wants to say that there is a distribution of water particles, each with a different estimated age, and those ages together can explain the measured solute concentration. Another example pertains the marginal distributions: the "probability" of observing an input (or an output) is not taken from stochastic hydrology concepts, it is instead a normalized timeseries of actual precipitation (or discharge) measurements.

Here, Dr. Benettin supports the idea that some variables (travel time, residence time, and life expectancy) can have a distribution and, in reality, the physics are not random, but deterministic, and the distributions are simply due to some causal dynamics. There are a few difficulties to accept his point of view. Reality is reality, and mathematics (the model) is mathematics (the model). All the probability distribution functions we use obey all the axioms required to be called probability. What the theory we presented does is to try to constrain the limits where the choice is random. If we assumed well-mixed waters (SSF $\omega(t,\tau)$ = 1), there is no doubt in the literature that we pick particles of water at random, without thinking of their age. If (SSF $\omega(t,\tau) \neq 1$) we reduce our (uniform) random sampling to the set of water of the same age, while we select the proportion of each age over the total storage. We agree with Dr. Benettin that hydrology could be ultimately different from the model. But there is no doubt that the model (we use and he, too, in his papers) obeys the axioms of probability and is based on random sampling.

Section 2

Eq 1: I think it should be explained that the time variables refer to a parcel of water that moves inside a hydrologic volume, entering at a time and leaving at a time .

Changed.

Line 72: I have often seen this integral expressed between $-\infty$ and t. Maybe it would avoid the need to specify that time t=0 comes before any input to the system?

We put the 0 limit to be more close to applications where the modeler has to chose an instant of time at which to start the modelling. After the comments of reviewer #3 we were actually push to change the notation to well defined time intervals.

Line 85: it may be worth saying that eq. (9) is a spatially-integrated equation that can be easily related to previous works in the literature, in particular Ginn (1999),

Delhez et al., (1999) and Dagan (1984). See Benettin et al., (2013) for a review on this.

We will add information about in the revised manuscript. However, the above statement is true for Ginn and Dehlez, but not for Dagan (an extraordinarily beautiful paper). As we wrote in the answer to reviewer $\#1$, Ginn equation (10) and Dehlez equation (4) convey the same concepts with a very different formalism, and it is not so easy to grasp that actually they are talking about the same equation. Dagan's paper, instead, talks about stationary fields. So our paper, dealing with non-stationary situations, is the integrated form of a generalisation of Dagan's result.

Section 3 (backward and forward approaches)

I think this section is very important but not sufficiently developed. The authors present 4 different kind of probabilities and it may be worth to specify which probabilities pertain to the different elements of the water budget. In particular, when the time t is to be interpreted as a general time at which the system is observed, the probabilities refer to the water particles in storage. Instead, when the time t has the special meaning of time at which particles enter or leave the system, the probabilities refer to the water particles in the fluxes.

We added further explanations after the definition of the travel time and evapotranspiration time pdfs.

Line 90: Please if possible refer to the paper Benettin et al., 2015 published on Hydrological Processes, instead of the Ph.D. thesis Benettin 2015, as the latter has no DOI.

Done

Line 91: this a bit imprecise: in the mentioned paper the concept of backward refers to the residence time (or age). Instead, the concept of travel time is both a forward or backward concept, depending on the point of view (i.e. if ones focuses on the entrance or on the exit).

We corrected the sentence and added further explanations.

Line 95-96: given your definition in eq. (1), $t - \tau$ is a residence time and not a travel time, so the distributions should be residence time probabilities and not travel time probabilities.

Right. We corrected the mistake.

Section 4

This section, in my opinion, does need to be further explored. I recommend the authors to include a physical interpretation of the processes, besides the mathematical description. Also, I think it may help the reader if the probability distributions associated with the water storage were denoted with a subscript S (e.g.p_S), just like the authors did for the probability associated with discharge (p_Q) or evapotranspiration (p_{ET}).

We changed the notation accordingly.

Line 105: Benettin (2015) used the notation $\overleftarrow{p_S}(T_r, t)$ and not $\overleftarrow{p}(T_r, t)$

Corrected accordingly.

Line 108-109: I did not understand this sentence (and maybe you meant eq (5) instead of eq (1))

Corrected accordingly.

Line 112: as this is a definition, the symbol := should be used?

Yes, corrected accordingly.

Line 120: please enclose the first term at left hand side of eq. (14) within brackets, otherwise the derivative refers to S(t) only. Please also explain what the symbol $\delta(t-\tau)$ refers to and why.

Corrected accordingly.

Line 129: please enclose the first term at left hand side of eq. (17) within brackets

Corrected accordingly.

Line 131: please specify that this is only valid in case eq. (17) is linear, i.e. $\omega(t,\tau)$ is not a function of $p(T_r|t)$

Corrected accordingly.

Line 135-145: Figures 2,3 and 4 need to be given more context and be better explained, because I had a hard time to interpret them and I am not sure the indicated t and are all in place . Please also specify what $P(T_r|t)$ is. I think the main point is letting the reader understand what happens when one keeps the chronological time t fixed and let the injection time change, or vice versa

The figures were changed accordingly to both the #1 and #2 reviewer's observation, further explanation added and all the variables updated according to the new notation.

Section 5

Just like Section 4, I think more interpretation should be provided. E.g. what does $p(t-\tau,\tau)$ represent? How is it related to $p_Q(t-\tau,\tau)$ and $pET(t-\tau,\tau)$?

Maybe we are pedantic about this, but the notation is $p(t-\tau|\tau)$ which different from $p(t-\tau,\tau)$. The first expression in standard probability notation means a conditional probability, the second a joint probability. They convey different concepts and return different numbers, so they should not be confused. In this case the first is a forward probability, since it is conditioned to the injection time. In particular, as it should be clear from the notation, the first refers to the forward distribution of the residence time, while the latter expression once rewritten properly as $p_Q(t-\tau|\tau)$ and $p_{ET}(t-\tau|\tau)$, refers to the forward distribution of travel and evapotranspiration time. The three are related through the partitioning coefficient, which is another one of the main aspects in the forward approach.

I think there may be some misunderstanding on the fact that the forward distributions cannot be known after time t. The problem is more general and equally

applies to backward distributions, which are unknown for any time t lower than the first available measurement. Moreover, unless one needs to do real-time predictions, travel time computations can be done on datasets which are much longer than the period of interest.

We don't see the misunderstanding here. Instead the reference to real-time simulations and ex-post simulations is a little misleading. The main problem, that maybe we did not convey properly, is about causation. If the age-ranked approach is deployed on the basis of the solution of the water budget equation, from the definitions we gave it follows that the backward probability is completely known up to actual time t (and, obviously, if no measurement or estimation is made, nothing can be known). Whatever happens after the actual time, is not known (and should not, unless all the future story of discharges and evapotranspiration would). The same happens for the forward probability. The latter, however, has a further element unknown, which is the partition coefficient, which is known only at infinity. We will try to make this point more clear in the revised manuscript.

Line 1: Please explain what you mean by "integral form". It's important to specify that you are integrating over dt, hence you follow a single injection and track its evolution while crossing the catchment.

We added an explanation about the integral over dt.

Line 168 (eq. 29): I don't see why the asymptotic value is that important. I believe the knowledge of $q(t,\tau)$ is much more important, and $\Theta(\tau)$ is just its integral over dt.

The importance of the partitioning coefficient is explained in Appendix A. Its asymptotic values, as shown in figure 8, summarize relevant characteristics of the investigated basin in the partitioning of the hydrological fluxes. The final value of Θ is achieved after an initial oscillating period, which is a characteristic of the basin too, due to hourly and daily oscillation, especially in evapotranspiration. Without the knowledge of the asymptotic value of Θ, the proper forward probabilities are not known, not even for time past the actual time t, because their definitive form depends on future times. This is overlooked in the past literature to which we owe everything, but, in our opinion, is a conceptual passage which is very useful to fully understand the formalism. As we say in the paper, this does not prevent us from knowing the state variables and the backward probability up to time t, as it is required by the fact that the past is known. On practical bases, as we shown in Appendix A, Θ value stabilises after a finite time. So for practical engineering approximations, we just need to wait for a reasonable elapsed time to have the information we require. This time is, in our example, less than the concentration time of the basin of interest.

Section 6

Line 181: as the authors are focusing on a parcel of water that enters at time $t = \tau$ and exits at $t = \iota$, I think the equations should be rewritten using instead of t.

We left t but we added further explanations in the text, specifying that t is the exit time.

Line 187: the symbol S for eq (34) and following is a bit ambiguous as the symbol S(t) already appears at eq. (5) with a different meaning. Moreover, as the integral is defined from 0 to infinity, it appears to me that $S \to \infty$.

We changed the notation from S to V_S but we think that the integral is correct since it is considering the entire history of the water particles.

Please give an interpretation of the marginal pdf's. This will be added in the revised versionof the paper. An explanation is given in the response to reviewer #3

I think the notation could be made more "symmetrical". Why is $p(\tau)$ the marginal pdf of the input and pQ(t) is the marginal probability of the output? Shouldn't they be either $p(\tau)$ and $p(\iota)$ or $p_Q(t)$ and $p_J(t)$?

We changed it in $p_Q(t)$ and $p_J(t)$.

Line 197: given the notation in eq. (37), it should be $p_Q(t)$ instead of p(t)

Changed

Section 7

In section 7 and 8 I found several typos and little errors in the formulas. I am not sure I detected them all, so please carefully revise these sections. As in section 5, I think the authors should not assume by default that our knowledge limits to time t, and everything between t and ι is unknown. That is just a very special case, limited to real time forward modeling.

We tried to correct all the typos we found. However, we insist that, if we derive probabilities "empirically", i.e. after having measured discharge and evapotranspiration, everything after the actual time (clock time) is unknown. Obviously, in practice, in many situations we do not perform real-time forecasting, but use given time series of past events, so the way we operate is going forward in time as it is needed (for instance for decently estimating $\Theta(\tau)$. But this works because those events are opportunely remote, and er remain well before our clock time.

The limitation to clock time is actually present also in backward probabilities, which are, as well, not known after the clock time. The backward formalism, however, does not require future information to work, and the probability is completely defined up clock time, which is not the case for forward probabilities.

Differently, if we use approaches where probabilities are assigned, we obviously know any future event. But if we assign only the probabilities for discharge (or one of the relevant processes), still the uncertainty of the partition coefficient remains. This is a well known problem in rainfall-runoff model and is related to the determination of the runoff coefficient.

Line 226: – t instead of – s

Corrected

Line 229: isn't the canonic convolution symbol different?

Changed

Line 235: typo: time time

Changed

Line 237: typo: τs

Changed with "all injection times"

Section 8

Please specify that the balance equation is in terms of substance mass.

Done

Line 251: please add the dependence on entrance time at left hand side of the equation

Added.

Line 261: I would suggest, for consistency with eq. (53) and (54) to name it $C_Q^i(t)$ note in the table of symbols at the end of the manuscript it is listed $C_Q^i(t)$).

Changed.

Line 263: I don't understand the expression "it is usually assumed the validity of an integral expression like...". Given the definitions of the terms involved in the equation, the integral is just an expression of mass conservation.

We changed in: "mass conservation implies"

Line 265: should be a function of $\Theta(\tau)$. Please note that here you use the product $\Theta(\tau)p(t-\tau|t)$ but in the subsequent equations $\Theta(\tau)$ disappears.

Right, there was a mistake. We correct it in the revised version.

Line 271: shouldn't it be $p_Q(t-\tau|t)$ instead of $p(t-\tau|t)$? In your notation, the latter denotes the age probability of the water storage (eq. 10).

Corrected accordingly the reviewer suggestions.

Line 287 and 294-296: why you did not use a Dirac delta function (like in eq. 17) for the age distribution of precipitation?

Corrected accordingly the reviewer suggestions.

Line 291-293: this sounds interesting but I am not sure it is actually feasible. I can't say what the issue may be from a mathematical point of view, but I have two examples in mind which are in conflict with the possibility of deriving $p(t-\tau|t)$ and $\omega(t,\tau)$ by coupling different budget equations. The first is that I am not sure the added mass budget equation is actually bringing different information with respect to the water budget equation. If the solute is conservative, it is transported just like water, so its concentration will simply be proportional to that of water, where $C^i_J(t)=1$. The second is that in case of multi-solute information one could get a system with 3 or more equations and just two unknown functions, so following your reasoning it would be impossible to solve, and this sounds strange to me.

The main point here is that the SSF cannot be imposed arbitrarily but is computed coupling the the water and solute budgets. What the reviewer asks is if equation (14) and (59), after omitting evapotranspiration in the first, are actually the same equation. It is easy to see that they are not, except for the trivial case in which solute concentration not only is equal in input, output and storage, but also constant.

The second question does not imply problems. In case of multiple solutes, we are measuring the same quantity with two different tools, like we measure a distance of a moving object from a reference point with a laser gun and with a GPS. The two measures have to coincide up to measurements errors. In practice, if the measure is very error prone (as can be the case), Statistics can help to get from the two measures one improved estimate.

Section 9

I think there is a very important assumption here, which is not explicitly stated. The fact that discharge is a linear function of storage does not automatically mean that each parcel of discharge (from an age-rank point of view) is a linear function of the corresponding parcel in storage. This only happen when a well-mixed (o random sampling) scheme is assumed.

This is certainly true. Linear storage implies just a time invariant and exponential forward distribution. In the paragraph it is actually shown, e.g. equation (69), that backward probabilities are not time invariant.

I think it is also important that the authors acknowledge here the difference between the storage which can be estimated from a purely hydrologic balance (i.e. the "active" storage which gets displaced during the hydrologic response) and the total storage of a system.

We are interested here in the formal part of the problem. This is completely, we believed, pretty well explained, especially after the clarification came from reviewers' comments. As we present it (but this is what we understood from the Benettin's himself dissertation) probabilities are derived from the water budget, and the SSF. It can be that

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Discussion paper

representing the hydrologic response unit (HRU) as a single storage would be insufficient for representing reality with fidelity (e.g. Kirchner, 2003). In this case the single storage equation (9) must be enhanced by adding further storages. This would imply a generalization of our approach that would be straightforward, and, in substance, was already pursued in various papers (e.g., McMillan, 2012; Hrachowitz, 2013). We are pretty aware of it, and we will do some additions in this sense, in the revised version of the paper.

This is very important considering the recent debate about water displacement and water travel times (e.g. McDonnell and Beven, 2014, Rinaldo et al., 2015, Kirchner, 2016). If the equations derived in this section do not take into account the so called "passive" storage (Kirchner, 2009), they may be of limited practical use

We already this topic in the answer to first reviewer, and the first author wrote this blog post: http://abouthydrology.blogspot.it/2016/06/celerity-vs-velocity.html to further clarify the issue. The passive storage, in case, has to be present in the water budget equations. If this is accomplished, what it causes is translated in probabilities by the algebra we described.

Conclusions

Line 394: typo "to the obtain understanding"

Corrected accordingly the reviewer suggestions.

Appendix A

Please define $\omega(t,\tau)$, as only $\omega(\tau)$ has been defined so far, and use consistently the upper-case or lower-case symbols. Also, I guess some SSF for both discharge and evapotranspiration was imposed to produce the figures and I think it should mentioned.

Corrected accordingly the reviewer suggestions.

References

Benettin, P., Rinaldo, A., and Botter, G. (2015). Tracking residence times in hydrological systems: forward and backward formulations, Hydrological Processes.

Benettin, P., Rinaldo, A., and Botter, G. (2013). Kinematics of age mixing in advection-dispersion models, Water Resources Research.

McDonnell, J. J., & Beven, K. (2014). Debates - The future of hydrological sciences: A (common) path forward? A call to action aimed at understanding velocities, celerities and residence time distributions of the headwater hydrograph, Water Resources Research.

Kirchner, J. W. (2003). A double paradox in catchment hydrology and geochemistry. Hydrological Processes, 17(4), 871–874. http://doi.org/10.1002/hyp.5108

Kirchner, J. W. (2009). Catchments as simple dynamical systems: Catchment characterization, rainfall-runoff modeling, and doing hydrology backward, Water Resources Research.

Kirchner, J. W. (2016). Aggregation in environmental systems - Part 2: Catchment mean transit times and young water fractions under hydrologic nonstationarity, Hydrology and Earth System Sciences

Dagan, G. (1984). Solute transport in heterogeneous porous formations, Journal of Fluid Mechanics

Ginn, T. (1999). Distribution of multicomponent mixtures over generalized exposure time in subsurface flow and reactive transport: Foundations, and formulations for groundwater age, Water Resources Research

Delhez, E. J. M., Campin, J.-M., Hirst, A. C., & Deleersnijder, E. (1999). Toward a general theory of the age in ocean modelling, Ocean Modelling

Rinaldo, A., Benettin, P., Harman, C. J., Hrachowitz, M., McGuire, K. J., van der Velde, Y., et al. (2015). Storage selection functions: A coherent framework for quantifying how catchments store and release water and solutes. Water Resources Research, 51(6), 4840–4847. http://doi.org/10.1002/2015WR017273