

Interactive comment on “Age-ranked hydrological budgets and a travel time description of catchment hydrology” by R. Rigon et al.

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Received and published: 20 July 2016

Answer to reviewer #3, Mr. D. Wilusz

Thank you for writing and sharing this manuscript. As a graduate student using storage selection functions and other transit time models, I benefited from seeing the theory synthesized and presented in a new way. Although I lack the depth of understanding of the reviewers and other experts in the field, I wanted to share some notes I made to myself as I read the paper, in case any of them can be helpful.

We thank Mr. D. Wilusz for his reading. We appreciate his observations and comments, and we believe that they will result in effective improvements of the final manuscript.

C1

Line 7, line 33-34: I was intrigued by the idea of deriving SSF functions (1) for the Nash cascade (line 7) and (2) for relatively complex cases with real-world data (line 33-34), and would have enjoyed more development in both these areas of the manuscript.

We will add more information to allow everybody to understand complex storage combinations. By the way, our source code, which uses more than one storage for producing the probability figures of the paper, is open source. It is available at <https://github.com/geoframecomponents> and commented on <http://geoframe.blogspot.com>

Figure 2 - Should this be labeled as a 'residence time backward' cdf? What data was used to produce these and the other plots?

We changed the notation according to the review of Dr. Benettin (reviewer #2), introducing the subscript S to denote the storage and thus the residence time distributions. Now the figure makes it clearer that we are talking about the backward residence time cdf. The figures were obtained using Posina River data. In particular, the data available were rainfall, temperature and discharge time series. Thanks to the model JGrass-NewAge we were able to delineate the HRUs, handle spatial data, simulate the radiation balance (shortwave and longwave), estimate the ETp and simulate the discharge. The solution of the water budget is also implemented.

Figure 3 - What is the residence time backward pdf over a continuum of injection times (the x axis) for a single injection time (e.g., tau1)? Shouldn't tau1 just be a slice on the x-axis?

The residence time is a backward pdf, which properly integrates to a value of one (1) when it is integrated in the injection time dimension. Both figures 2 and 4 show the

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evolution of the previous pdfs. In the first case (figure 2) we kept fixed the chronological time and let the injection time vary; in this case, the integral of the area under the curves (figure 3) is 1. Vice-versa, in figure 4 we kept fixed the injection time and let the chronological time to vary. We are going to explain the figures more thoroughly in the new version of the manuscript, according to the reviews.

Line 155-160: Is there any problems with using equations 26-28 when rainfall is some- times zero?

As we show in equation (21) and in figure 4, the probability remains constant when rainfall is not present and t varies.

Line 187: Will this integrate to infinity? Or is the second "=" supposed to be "-"? Line 193: The link to Bayes theorem is interesting. If Bayes theorem is $(P(A|B) * P(B) = P(A) * P(B|A))$, what are the equivalent events A and B in Niemi's relation?

We answer the two questions together. The equation is properly written. It integrates to infinity, and the second symbol is an "=". We will try to clarify this point better. S represents the net precipitation (input), which is equal to the total discharge (output). So one probability is the probability density of a single precipitation event, and the other is the probability density of a single discharge instant. However, we realised that this definition has a drawback that needs to be solved and brings to generalise Niemi's relation. The problem with the above definition is that any density is zero when the time domain tends to ∞ . However, the forward probability can be re-defined in a more general way to restrict our consideration to precipitations fallen in a finite interval $[0, t']$. For any $t \leq t'$ the algebra and the definition of the quantities in the Niemi's relation follows what already presented. Instead for $t > t'$, i.e. for actual time interval when rain

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is not falling or is being neglected, normalisation changes and:

$$p(t - \tau | t, \tau \in [0, t']) := \frac{q(t, \tau)}{Q_{t'}(t)} \quad \tau \in [0, t'] \quad (1)$$

and:

$$Q_{t'}(t) := \int_0^{t'} q(t, \tau) d\tau \quad (2)$$

Therefore $Q_{t'}(t)$ represents the fraction of discharge at time t generated by precipitation fallen for all the $\tau \in [0, t']$. At this point a generalised Niemi's relation holds as:

$$Q_{t'}(t) p(t - \tau | t, \tau \in [0, t']) = \Theta(\tau) p_q(t - \tau | \tau) J(\tau) \quad \tau \in [0, t'] \quad (3)$$

After restricting to a limited set of precipitations, the total amount of precipitation is:

$$S_{t'} = \int_0^{t'} J(\tau) d\tau \quad (4)$$

from which we have:

$$\frac{Q_{t'}(t)}{\Theta(\tau) S_{t'}} p(t - \tau | t, \tau \in [0, t']) = p_q(t - \tau | \tau) \frac{J(\tau)}{S_{t'}} \quad (5)$$

Defining:

$$p(\tau | \tau \in [0, t']) := \frac{J(\tau)}{S_{t'}} \quad (6)$$

and

$$p_Q(t | \tau \in [0, t']) := \frac{Q_{t'}(\tau)}{\Theta(\tau) S_{t'}} \quad (7)$$

We obtain again the Bayes theorem where $p(\tau | \tau \in [0, t'])$ is the fraction of the precipitation fallen at time τ with respect to the whole precipitation fallen $\tau \in [0, t']$. Similarly,

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$p_Q(t|\tau \in [0, t'])$ is the fraction of discharge at time t generated by precipitation fallen at τ . No precipitations fallen before or after the interval $[0, t']$ count, and the relative discharge has to be emended in the calculations. Remarkably, even if we would not be interested in the Bayes theorem analogy, the extended formalism properly identifies which are the normalisation factors to be used in defining the backward probabilities.

In the revised version of the paper we will modify the section about the Niemi's relation to account for the above considerations.

Line 338-340: Aren't other (including time-varying) SSF functions possible even if discharge is proportional to storage?

No, it follows from the definition of the backward probabilities that:

$$p(t - \tau|t) := \frac{s(t, \tau)}{S(t)} = \frac{\lambda q(t, \tau)}{\lambda Q(t)} = \frac{q(t, \tau)}{Q(t)} =: p_Q(t - \tau|t) \quad (8)$$

This happens because λ is not dependent on τ . Viceversa, as happens in equation (71) of the paper, the equality above does not hold because each injection time has its own (τ dependent) λ (which is, in fact, labeled λ_τ).