Reply to "Interactive comment on "Technical Note: Multiple wavelet coherence for untangling scale-specific and localized multivariate relationships in geosciences" by W. Hu and B. C. Si " by Referee #1

The manuscript of Multiple wavelet coherence by Hu and Si presented an important topic. In characterizing scale specific variations, wavelet coherence has been used in many field but was restricted to only two variables. Presentation of wavelet coherence produces a step forward on the methodological development aspect. The method will support a lot of different fields including soil science and hydrology. The scientific content is suitable for the journal and the readers of this journal will be interested in this topic. Therefore, my suggestion is for acceptance of the manuscript with some minor corrections such as English, which could be improved. Another thing, authors used the artificial series to compare with other multi-variate analysis. Just wondering, how will you confirm about you claimed superior information of the new method compare to other methods. I mean to say, how will you say that this variations, what is shown by other methods are also showing the right information. The variations showing here could be spurious as identified by different methods.

Response:

Thank you for the positive comments.

In terms of language, we have tried our best to correct it. We will ask an English editing company check the language again if we will be given a chance for revision.

We are not very sure we understand your second comment, but we will try to explain a bit here. The two existing methods (i.e., multiple spectral coherence and multivariate empirical mode decomposition) are widely used for spatial or temporal series analysis in different disciplines. Actually we have known that these two methods cannot deal with localized relationships between variables. Therefore, the advantages of the new method over these two methods is demonstrated mainly in terms of relationships between response and predictor variables at various scales of the response variable. The reason for using the artificial data is that the major features (e.g., scale) are known. Then, the superiority of the new method over these two methods can be assessed by whether the known major features of the artificial data are demonstrated by these methods. Our results clearly show that localized multivariate relationships are not available by the two existing methods and both methods are likely to underestimate the degree of multivariate relationships for non-stationary processes. Because the cosine-like artificial datasets mimic many time series and spatial series in geosciences. Therefore, we conclude that the new method is superior.

All above mentioned information can be found in the revised copy. Please refer to them at Lines 84-86, 156-162, 188-191, and 384-387.

Reply to "Interactive comment on "Technical Note: Multiple wavelet coherence for untangling scale-specific and localized multivariate relationships in geosciences" by W. Hu and B. C. Si " by Referee #2

General Comments

The multiple wavelet coherence methodology presented in the manuscript by Hu and Si represents an important contribution to wavelet analysis. In particular, Hu and Si build upon the previous work of Ng and Chan (2012) to extend multiple wavelet coherence to case of more than two predictor variables. The authors further demonstrate that the new multiple wavelet coherence methodology is better suited for situations where the predictor variables are cross-correlated. The problems with the traditional formulation are clearly stated and consistent with the objective of the paper proposed in the introduction section. Theoretical examples were also presented to highlight the advantages of the new methodology relative to existing ones. I their recommend that the manuscript be accepted after the substantial correction of grammatical errors and the consideration of more specific comments presented below.

Response:

Thank you for the positive comments.

Specific comments

The conclusion section simply summarizes the results of the paper. The authors could consider expanding the conclusion section into a discussion section to comment on limitations of the method. After all, wavelet analysis, while useful, is not a scientific panacea. More specifically, the inclusion of more predictor variables may result in the statistical significance threshold at a particular wavelet scale and time to approach unity, which would impose a limit on how much statistical information can be gained. This phenomenon occurs with the traditional multiple wavelet coherence formulation, where the threshold for 5% significance, for example, is higher than that for bivariate wavelet coherence at a given wavelet scale.

Response:

We agree with you that one of the limitation is that the critical values increase with the number of predictor variables. This is also why the percentage area of significant coherence (PASC) for three predictor variables (z2, z4, and noised z4) are even lower

than for only two predictor variables (z2 and z4) when the third predictor variable (noised z4) is not statistically significant to explain the variation of the response variable. Please see Lines 265-266 in the attached revision.

We put this limitation in the conclusion part as " Theoretically, any number of predictor variables can be included in the multiple wavelet analysis. However, the statistical significance threshold usually increases with the number of the predictor variables (Grinsted et al., 2004; Ng and Chan, 2012a), and inclusion of too many predictor variables may result in the statistical significance threshold at particular wavelet scales (e.g., the lowest and largest scales) to approach unity. This would restrict the availability of statistical information." (Lines391-397 in the attached revision).

The author may also consider discussing at least briefly the problem of simultaneously testing multiple statistical hypothesis, as discussed in Maraun and Kurths (2004), Maraun et al. (2007), Schulte et al. (2015), and Schulte (2016). Multiple-testing problem is a major problem in wavelet analysis and therefore merits consideration in a discussion section. Presenting clearly the methodological limitations will better guide the likely interdisciplinary readership in making decisions regarding what analysis tools to implement.

Response:

The multiple-testing problem has been briefly discussed in the conclusion part. "In addition, similar to bivariate wavelet analysis, the new method also suffers from the multiple-testing problem (Maraun and Kurths, 2004; Maraun et al., 2007; Schulte et al., 2015; Schulte, 2016). Therefore, a more robust statistical significance testing method may be beneficial to the new method." (Lines397-400 in the attached revision).

Throughout the manuscript, the authors mention how geoscience data are often nonstationary. Perhaps the term is used too loosely in some instances and is sometimes inconsistent with the strict time series analysis definition. Even white and red-noise processes contain time and scale-localized features in wavelet space, even though theirrespectivestatisticsarestationaryatallorders. Time-andscale-localizedfeatures are evident in the wavelet power spectrum of say, the North Atlantic Oscillation (NAO), even though the statistics of the NAO are consistent with a first-order Markov process (Feldstein,2000). Therefore,insomeinstances,Irecommendchangingtheword“nonstationary” to “transient” or “transitory”.

Response:

We agree. In the introduction, we made this more clear as " More often than not, geoscience data are transient, consisting of a variety of frequency regimes that may be localized in space or time (Torrence and Compo, 1998; Si and Zeleke, 2005; Graf et al.,

2014). The transient characteristics exists widely in non-stationary but also sometimes in stationary processes (Feldstein, 2000)." (Lines35-39 in the attached revision).

At many instances, we changed the "non-stationary" to "transient" when suitable, such as Line 41, 59, 67 in the attached revision.

Some Technical Corrections

Page 2 Line 3536. Change “geoscience data is” to “geoscience data are”.

Response:

Yes, done at L36.

Page 2 Line 39. Is it better to say bivariate wavelet coherency rather than “simple wavelet coherency”

Response:

Yes, we changed all throughout the paper.

Page 5, Line 97. Add comma before “respectively”.

Response:

Yes, we did throughout the paper.

Page9,Line169-171. The sentence can be slightly simplified by changing“ white noise with a mean of 0” to “zero-mean white noise”. Perhaps it is redundant to write that the white noise processes were generated. Authors could consider just saying that white noise was added to the predictor variables.

Response:

We agree. Now, it changed to" zero-mean white noises with a mean of 0 and standard deviations of 0.3, 1, and 4 are added to the predictor variables of y2 (or z2) and y4 (or z4).".

Page 9, Lines 171-173. The sentence “The resulting noised series are termed weakly, moderately, 172 and highly noised series respectively, and have a correlation coefficient of 0.9, 0.5, 173 and 0.1 respectively, with their original predictor variable” needs to be rewritten and simplified. Consider breaking the sentence into two separate sentences.

Response:

We changed it to two sentences. Now, it looks like "The resulting noised series have correlation coefficients of 0.9, 0.5, and 0.1, respectively, with their original predictor

variable. Therefore, we will refer them to weakly, moderately, and highly noised series, respectively." (Lines 177-180 in the attached revision copy).

The authors should carefully check for grammatical errors and make similar changes throughout the manuscript.

Response:

Yes, done.

Further English check will be made if a chance for revision will be given.

References

Feldstein SB (2000) The timescale, power spectra, and climate noise properties of teleconnection patterns. J Clim 13:4430–4440. doi:10.1175/15200442(2000)0132.0.CO;2 Maraun, D. and Kurths, J.: Cross wavelet analysis: significance testingand pitfalls, Nonlin. Processes Geophys., 11, 505–514, 2004. Maraun, D., Kurths, J., and Holschneider, M.: Nonstationary Gaussian processes in wavelet domain: synthesis,estimation, and significance testing, Phys. Rev. E, 75,doi:10.1103/PhysRevE.75.016707, 2007. Ng, E. K. W. and Chan, J. C. L.: Geophysical applications of partial wavelet coherence and multiple wavelet coherence, J. Atmos. Ocean. Tech., 29, 1845–1853, doi: 10.1175/JTECH-D-12-00056.1, 2012. Schulte, J. A., Duffy, C., and Najjar, R. G.: Geometric and topological approaches to significance testing in wavelet analysis, Nonlin.Processes Geophys., 22, 139–156, doi:10.5194/npg-22-139-2015, 2015. Schulte, J. A.: Cumulative areawise testing in wavelet analysis and its application to geophysical time series, Nonlin. Processes Geophys., 23, 45-57, doi:10.5194/npg-2345-2016, 2016.

Response:

Appreciate for the good references. We cited them when we made relevant discussion.