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Interactive comment on "A particle based model for soil water dynamics: how to match and step beyond Richards' equation?" *by* E. Zehe and C. Jackisch

Anonymous Referee #1

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The paper proposes a stochastic random walk particle tracking based approach for the modeling of soil-water dynamics in the unsaturated zone. First, I find this an interesting approach that is conceptually relatively simply despite the non-linearity (saturation-dependence) of the drift and diffusion coefficients, which impacts on the numerical efficiency of the scheme. The proposed approach is validated against numerical (finite difference?) solutions of the Richards equation and to a real-world benchmark. The random walk model does not fit perfectly with the finite-difference(?) solution of the Richards equation, the possible conceptual reasons of which will be discussed in the following.

The stochastic approach describes the motion of fluid particles by a stochastic differen-

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Discussion paper

tial equation, whose equivalent Fokker-Planck equation resembles (but is not equal to) the Richards equation. Equation (2) of the paper is not equivalent to Eq. (1). Rather, it corresponds to the Fokker-Planck equation (or convection-diffusion equation due to the diffusion correction), see, for example the textbook by Risken (The Fokker-Planck Equation),

(*) del \theta/del t = del/del z [k(theta) theta] + del/del z [D(theta) del/del z theta.

The difference between this Fokker-Planck equation and the Richards equation (1) in the manuscript is the advection term. In equation (1) it is k(theta) while in the above Fokker-Planck equation it is - del/del z [k(theta) theta] (Note that by convention the advection term in the Fokker-Planck equation is as in del theta /del t = - del / del z u theta + ...).

If we use the following more standard form (e.g., Eq. [9.4.100] in Bear, Dynamics of Fluids in Porous Media) of the Richards equation

del theta /del t = del/del z [K(theta) + D(theta) del/del z theta],

we can identify an advection term if we write it as

(**) del theta /del t = K'(theta) del/del z theta + del/del z [D(theta) del/del z theta]

where K' is the derivative of K with respect to theta.

The Richards equation in this form, however, is not a Fokker-Planck equation because it is not in a divergence-form (compare the advection terms in (*) and (**)). In order to achieve this one could

(i) consider the new variable

theta' = del theta/del z, which satisfies the Fokker-Planck equation

del theta' /del z = del /del z [K'(theta) theta'] + delˆ2/del zˆ2 D(theta) theta'.

The corresponding Langevin equation is given by d z = - K'(theta) dt + Z sqrt{6 D(theta) dt},

where z now is adjoint to the saturation gradient.

or

(ii) define the velocity u(theta) = K(theta)/theta (see also the paper by Zoia et al., PRE 81,031104, 2010)

This may be the more suitable definition for the purpose of the manuscript. Using this formulation, the Richards equation becomes the Fokker-Planck equation

del theta/del t = del/del z [u(theta) theta + D(theta) del/del z theta]

or equivalently

del theta/del t = del/del z [u(theta) theta - D'(theta) theta + del/del z D(theta) theta]

with D'(theta) = d D(theta)/d theta. This Fokker-Planck equation is equivalent to the Langevin equation

dz = - u(theta)dt - D'(theta)dt + Z sqrt{6 D(theta)dt}

In summary, my main concern arises from Eq. (1) and (2), which are not equivalent. If the implementation of the random walk method is indeed based on Eq. (2) with the drift terms specified as -k(theta), the random walk method is not equivalent to solving the Richards equation, but another problem, whose physical meaning is not clear. Thus, I encourage the authors to revise the manuscript along the lines detailed above.

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