

Reviewer 3: The paper deals with water flow in unsaturated soils simulated by a particle based model. The authors used the water content based formulation of the Richards equation to define an equivalent Fokker Planck equation. If the link between the PDE (eq. 1) and the stochastic equation (SE- eq.2) has been demonstrated for linear problems like the advection-dispersion equation, the analogy for highly nonlinear problem is fully intuitive, as well as the nice and smart implementations of the particle based method. Moreover, the easy way to extend the SE to preferential flow and mixing makes the particle based method very attractive as shown by the simulation of field data. As stated by the authors, this work is a first step in the development of this approach.

Erwin Zehe (EZ): We sincerely thank the anonymous reviewer for his encouraging comments and helpful comments. In fact the core objective of the study is to propose an alternative to the Richards equation for the case of rainfall driven conditions, which may cope with preferential flow and in an alternative manner and which avoids challenges of continuum approaches as for instance the dealing with partly wetted macropores or with macropore flow that hits the closed end of a macropore. The particle approach allows a straightforward accounting of a fast flow component by treating the event water as second particle fraction. We agree that we just presented a first tentative feasibility study in this respect, which is promising.

The scope of the particle model is thus clearly on infiltration and soil water dynamics during rainfall driven conditions. We think the ultimate model is a hybrid, which uses the particle approach during rainfall driven conditions, when time stepping needs to be in the order of minutes, due to the characteristic time scale of changes in rainfall intensity. The primary variable is thus soil moisture/particle density. During radiation driven conditions when water flow is slow and in local equilibrium, it is favorable to switch to a Richards solver, because it works well and it is much more computationally efficient and treatment of for instance root water uptake is much more straightforward. We will stress this point in the revised manuscript.

Reviewer 3: The paper needs some revisions according to the following comments:

EZ: Please note that we address the reviewers point 3 first.

Reviewer 3: The comparison with the three theoretical benchmarks is quite convincing for short times. However, the difference between the PDE formulation and SE increases with time (fig. 3, b&d, fig 5). Is it due to drainage? Is there a bias in the method? An additional simulation with drainage of an initial saturated soil may provide some information. This point is critical. It is probably not possible to demonstrate mathematically that PDE and SE are equivalent.

EZ. Our model was indeed biased. Equation 1) in the original manuscript is not the Richards equation! When starting with the correct version

$$\frac{\partial \theta}{\partial t} = \frac{\partial k(\theta)}{\partial z} + \frac{\partial}{\partial z} \left(D(\theta) \frac{\partial \theta}{\partial z} \right)$$

$$D(\theta) = k(\theta) \frac{\partial \psi}{\partial \theta}$$

This may be rewritten as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[\frac{k(\theta)}{\theta} \theta \right] + \frac{\partial}{\partial z} \left(D(\theta) \frac{\partial \theta}{\partial z} \right)$$

Or equivalently

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[\frac{k(\theta)}{\theta} \theta - \frac{\partial D(\theta)}{\partial z} \theta \right] + \frac{\partial^2}{\partial z^2} (D(\theta) \theta).$$

The latter is equivalent to the Fokker Planck Equation. The corresponding Langevin Equation is

$$z(t + \Delta t) = \left(-\frac{k(\theta(t))}{\theta(t)} - \frac{\partial D(\theta(t))}{\partial z} \right) \cdot \Delta t + Z \sqrt{6 \cdot D(\theta(t)) \cdot \Delta t}$$

, which differs from the equation (2) in the current manuscript. We already implemented the corrected version of the Langevin Equation. It yields simulations in much better accordance with the Richards solver as the former version, particularly when it is operated using normally distributed random numbers in case of uniformly distributed ones.

$$z(t + \Delta t) = \left(-\frac{k(\theta(t))}{\theta(t)} - \frac{\partial D(\theta(t))}{\partial z} \right) \cdot \Delta t + \xi \sqrt{2 \cdot D(\theta(t)) \cdot \Delta t}$$

$$\xi \in N(0,1)$$

Figure 1 corroborates for the sandy soil that the improved particle model matches the Richards solver much better than the old version, particularly for a reduced grid size of 2.5 cm the matching is almost perfect.

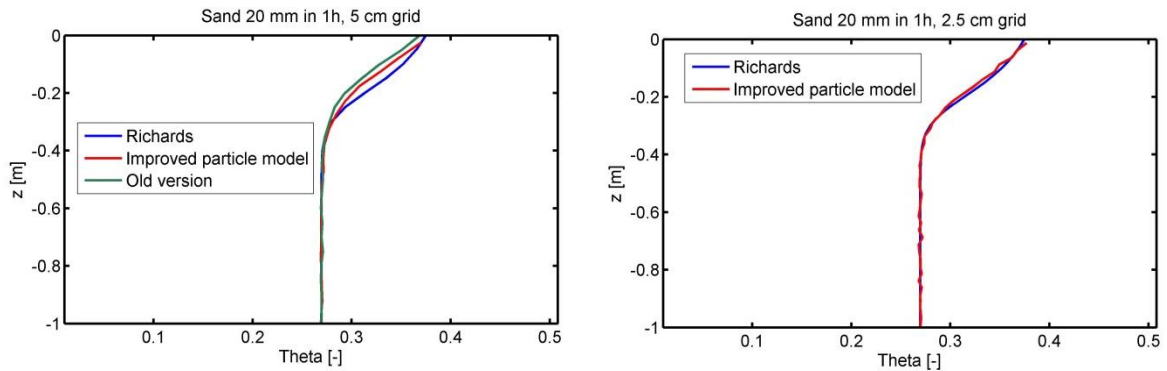


Figure 1: Comparison of the old particle model and the new one based on the corrected form of the Langevin Equation using $N(0,1)$ random numbers (simulation time step was 20 s).

Reviewer 3: Therefore, detailed numerical experiments are required. At least, the authors should also provide a long term simulation (over one year) with time varying boundary conditions (infiltration, evaporation) and compare the SE to a reference solution (fine time and space discretizations) obtained by Richards equation.

EZ: We agree that additional simulations with time varying boundary conditions will further illustrate the validity of the approach and will provide additional long term simulations with transient rainfall conditions using observed rainfall events at the two sites, which cover the entire range of short term convective events up to stratiform long term events of several days duration. Please note that the scope of the particle model is thus clearly on infiltration and soil water dynamics during rainfall driven conditions, because of its potential to cope with preferential flow. We do **not** recommend the use of this type of model during radiation driven/fair weather conditions, because it offers no

advantage here. We thus think that a one year simulation is inappropriate, because this is not the scope of the propose model.

We think and will better explain that the ultimate model is a hybrid, which uses the particle approach during rainfall driven conditions, when time stepping needs to be in the order of minutes, due to the characteristic time scale of changes in rainfall intensity. The primary variable is thus soil moisture/particle density. During radiation driven conditions when water flow is slow and in local equilibrium, it is favorable to switch to a Richards solver, because it works well and it is much more computationally efficient and treatment of for instance root water uptake is much more straightforward. We will stress this point in the revised manuscript.

Reviewer 3. It should be clearly stated that the chosen Richards formulation cannot be applied to heterogeneous domains.

EZ: We will do so in the revised manuscript. Please note that the particle model is not compared against a solution of the Richards equation in the theta based form. We use a the Richards equation in the mixed form, which is to our notion the still the mass conservative benchmark for simulating soil water dynamics in the unsaturated zone in the absence of preferential flow. We will add additional test for system with different soil horizons.

Reviewer 3: The time varying parameters are handled using a predictor-corrector scheme which consists in computing the parameters at time $t+0.5 \Delta t$. The parameters values at $t+0.5 \Delta t$ are not representative of the equivalent parameters defined over Δt due to the strong non linearity of the parameters with respect to the variables. It may be a good approximation for small time steps. Since the optimal time step is not known a priori, this makes the scheme tricky from a numerical point of view.

EZ: This is a very good point. Figure 2 show simulations with the corrected particle model for the sandy soil using different constant time steps (20 s, 50 s, 100 s, 200 s, 500 s). Deviations for time steps of up to 100s are of order 0.5 % VOL, one may observe clear oscillations for time steps larger or equal than 200 s.

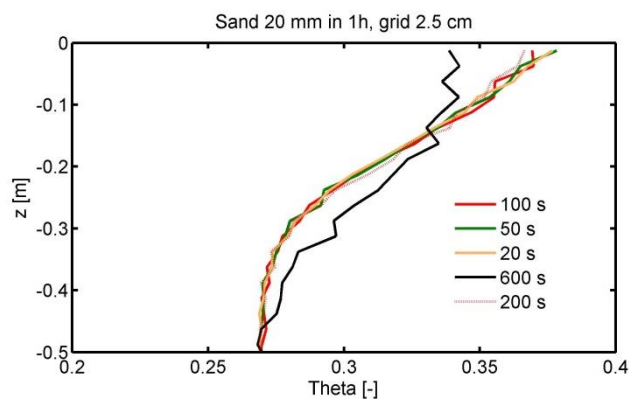


Figure 2: Simulations with the improved model using the the corrected form of the Langevin Equation for different constant time steps.

This implies that the particle model in fast draining soils needs to be operated at time steps of maximum 50 s (while larger time steps up to 600s are feasible for slower draining cohesive soils.). While thus appears as a drawback at first site time steps of 50s seconds are appropriate for the proposed model scope. Rainfall intensity changes at the scale of 1 – 6 minutes in case of convective rainfall events, which implies that the temporal resolution of highly resolved rainfall data does restrict the possible time to values smaller than 6 min (when treating with 6 minutes rainfall).

My personal experience is that a numerical accurate simulation of infiltration does, even when using a mass conservative implicit picard solver proposed by Celia et al. (1990) time step in the order of 10 seconds to assure convergence of the iteration. We will add similar tests by running the model at different time steps and discuss the issue restriction of the selected predictor corrector scheme per se, and in the light of the model scope.

Reviewer 3: Both approaches are also compared on a set of field data. Since the full mixing particle model differs from the Richards model by its formulation only, one model cannot be better than the other as it is stated lines 359 to 366 (p. 12). The difference is only due to the mathematical and numerical approximations made to establish the SE and to solve it.

EZ: This is absolutely correct we will reformulate this passage.

We again thank Reviewer 3 for her/his very much for the insightful comments that will surely help us to improve the quality of our study.

Erwin Zehe