

Reviewer 1: The paper proposes a stochastic random walk particle tracking based approach for the modeling of soil-water dynamics in the unsaturated zone. First, I find this an interesting approach that is conceptually relatively simply despite the non-linearity (saturation dependence) of the drift and diffusion coefficients, which impacts on the numerical efficiency of the scheme. The proposed approach is validated against numerical (finite difference?) solutions of the Richards equation and to a real-world benchmark.

Erwin Zehe (EZ): We sincerely thank the anonymous reviewer for his encouraging comments and helpful comments. Yes we use centered finite differences and will mention this in the revised manuscript.

Reviewer 1: The random walk model does not fit perfectly with the finite-difference solution of the Richards equation, the possible conceptual reasons of which will be discussed in the following. The stochastic approach describes the motion of fluid particles by a stochastic differential equation, whose equivalent Fokker-Planck equation resembles (but is not equal to) the Richards equation. Equation (2) of the paper is not equivalent to Eq. (1). Rather, it corresponds to the Fokker-Planck equation (or convection-diffusion equation due to the diffusion correction), see, for example the textbook by Risken (The Fokker-Planck Equation), $(*) \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} [k(\theta) \theta] + \frac{\partial}{\partial z} [D(\theta) \frac{\partial \theta}{\partial z}]$. The difference between this Fokker-Planck equation and the Richards equation (1) in the manuscript is the advection term. In equation (1) it is $k(\theta)$ while in the above Fokker-Planck equation it is $-\frac{\partial}{\partial z} [k(\theta) \theta]$ (Note that by convention the advection term in the Fokker-Planck equation is as in $\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z} [u \theta] + \dots$). If we use the following more standard form (e.g., Eq. [9.4.100] in Bear, Dynamics of Fluids in Porous Media) of the Richards equation $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} [K(\theta) + D(\theta) \frac{\partial \theta}{\partial z}]$, we can identify an advection term if we write it as $(**) \frac{\partial \theta}{\partial t} = K'(\theta) \frac{\partial \theta}{\partial z} + \frac{\partial}{\partial z} [D(\theta) \frac{\partial \theta}{\partial z}]$ where K' is the derivative of K with respect to θ .

(EZ): We sincerely thank for this important point. Eq. 1 in the first manuscript version is in fact wrong, as it is not the correct form of the Richards equation:

$$\frac{\partial \theta}{\partial t} = -k(\theta) + \frac{\partial}{\partial z} \left(D(\theta) \frac{\partial \theta}{\partial z} \right)$$

$$D(\theta) = k(\theta) \frac{\partial \psi}{\partial \theta}$$

The correct version of the Richards equation in the soil moisture base form is, as correctly pointed out by the Reviewer:

$$\frac{\partial \theta}{\partial t} = \frac{\partial k(\theta)}{\partial z} + \frac{\partial}{\partial z} \left(D(\theta) \frac{\partial \theta}{\partial z} \right)$$

$$D(\theta) = k(\theta) \frac{\partial \psi}{\partial \theta}$$

We sincerely apologize for that and will revise the manuscript accordingly.

Reviewer 1: The Richards equation in this form, however, is not a Fokker-Planck equation because it is not in a divergence-form (compare the advection terms in (*) and (**)). In order to achieve this

- one could (i) consider the new variable $\theta' = \frac{\partial \theta}{\partial z}$, which satisfies the Fokker-Planck equation $\frac{\partial \theta'}{\partial z} = \frac{\partial}{\partial z} [K'(\theta) \theta'] + \frac{\partial^2}{\partial z^2} D(\theta) \theta'$. The

corresponding Langevin equation is given by $dz = -K'(\theta) dt + Z \sqrt{6 D(\theta) dt}$, where z now is adjoint to the saturation gradient.

- Or (ii) define the velocity $u(\theta) = K(\theta)/\theta$ (see also the paper by Zoia et al., 2010). This may be the more suitable definition for the purpose of the manuscript. Using this formulation, the Richards equation becomes the Fokker-Planck equation. $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} [u(\theta) \theta + D(\theta) \frac{\partial \theta}{\partial z}]$ or equivalently $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} [u(\theta) \theta - D'(\theta) \theta + \frac{\partial}{\partial z} D(\theta) \theta]$ with $D'(\theta) = d D(\theta)/d \theta$. This Fokker-Planck equation is equivalent to the Langevin equation $dz = -u(\theta)dt - D'(\theta)dt + Z \sqrt{6 D(\theta)dt}$

EZ: Again we thank for this important point. We prefer the second formulation (ii) and will revise the manuscript accordingly.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[\frac{k(\theta)}{\theta} \theta \right] + \frac{\partial}{\partial z} \left(D(\theta) \frac{\partial \theta}{\partial z} \right)$$

Or equivalently

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[\frac{k(\theta)}{\theta} \theta - \frac{\partial D(\theta)}{\partial z} \theta \right] + \frac{\partial^2}{\partial z^2} (D(\theta) \theta).$$

Reviewer 1: In summary, my main concern arises from Eq. (1) and (2), which are not equivalent. If the implementation of the random walk method is indeed based on Eq. (2) with the drift terms specified as $-k(\theta)$, the random walk method is not equivalent to solving the Richards equation, but another problem, whose physical meaning is not clear. Thus, I encourage the authors to revise the manuscript along the lines detailed above.

EZ: We already implemented the corrected version of the Langevin Equation.

$$z(t + \Delta t) = \left(-\frac{k(\theta(t))}{\theta(t)} - \frac{\partial D(\theta(t))}{\partial z} \right) \cdot \Delta t + Z \sqrt{6 \cdot D(\theta(t)) \cdot \Delta t}$$

It yields simulations in much better accordance with the Richards solver as the former version, particularly when it is operated using normally distributed random numbers in case of uniformly distributed ones.

$$z(t + \Delta t) = \left(-\frac{k(\theta(t))}{\theta(t)} - \frac{\partial D(\theta(t))}{\partial z} \right) \cdot \Delta t + \xi \sqrt{2 \cdot D(\theta(t)) \cdot \Delta t}$$

$$\xi \in N(0,1)$$

Figure 1 corroborates for the sandy soil that the improved particle model matches the Richards solver much better than the old version, particularly for a reduced grid size of 2.5 cm the matching is almost perfect.

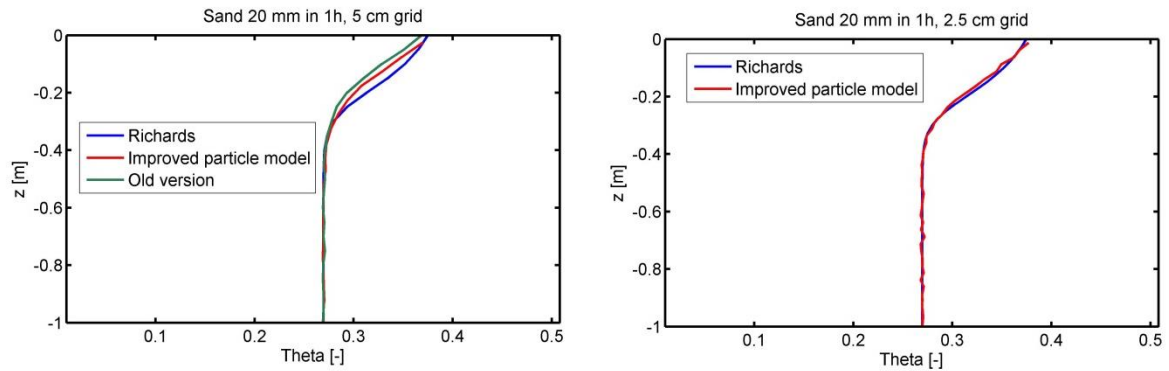


Figure 1: Comparison of the old particle model and the new one based on the corrected form of the Langevin Equation using $N(0,1)$ random numbers.

With respect to the physical meaning the proposed particle model remains of course an effective approach, as water does in fact flow as a continuum through the pore space. The main asset is however, that it still makes use of established soil physics, and that it allows for a straight forward implementation of non-equilibrium preferential flow. In fact the core objective of the study is to propose an alternative to the Richards equation for the case of rainfall driven conditions, which may cope with preferential flow and in an alternative manner and which avoids challenges of continuum approaches as for instance the dealing with partly wetted macropores or with macropore flow that hits the closed end of a macropore. We think the ultimate model is a hybrid, which uses the particle approach during rainfall driven conditions, when time stepping needs to be in the order of minutes, due to the characteristic time scale of changes in rainfall intensity. The primary variable is thus soil moisture/particle density. During radiation driven conditions when water flow is slow and in local equilibrium, it is favorable to switch to a Richards solver, because it works well and it is much more computationally efficient and treatment of for instance root water uptake is much more straightforward. We will stress this point in the revised manuscript.

We again thank Reviewer 1 for her/his insightful comments that clearly helped us to improve the propose model.

Erwin Zehe