Ordinary kriging as a tool to estimate historical daily streamflow records

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Abstract. Efficient and responsible management of water resources relies on accurate streamflow records. However, many watersheds are ungaged, limiting the ability to assess and understand local hydrology. Several tools have been developed to alleviate this data scarcity, but few provide continuous daily streamflow records at individual streamgages within an entire region. Building on the history of hydrologic mapping, ordinary kriging was extended to predict daily streamflow time series on a regional basis. Pooling parameters to estimate a single, time-invariant characterization of spatial covariance structure is shown to produce accurate reproduction of streamflow. This approach is contrasted with a time-varying series of variograms, representing the temporal evolution and behavior of the spatial covariance structure. Furthermore, the ordinary kriging approach is shown to produce more accurate time series than more-common, single-index hydrologic transfers. A comparison between topological kriging and ordinary kriging is less definitive, showing the ordinary kriging approach to be significantly inferior in terms of Nash-Sutcliffe model efficiencies while maintaining significantly superior performance measured by root mean squared errors. Given the similarity of performance and the computational efficiency of ordinary kriging, it is concluded that ordinary kriging is useful for first-order approximation of daily streamflow time series in ungaged watersheds.

1 Introduction

One of the most fundamental problems confronting the fields of hydrology and water resources management is the prediction of hydrologic responses in ungaged basins (PUB) (Sivapalan et al., 2003). While streamgages have long provided point measurements of the daily time series of streamflow, there are many regions of the globe that remain sparsely gaged, and thus, there are many completely ungaged locations (for an example in the US, see Kiang et al., 2013). Building on the long history of hand-drawn maps showing the spatial variation of hydrologic and climatic variables, geostatistical techniques are proposed as a means of leveraging the information content of streamgage networks to produce spatially and temporally continuous predictions of historical daily streamflow. The primary goal of this work is to demonstrate that simple geostatistical techniques can provide predictions of daily streamflow time series at ungaged sites that are superior to those produced by the single-index, transfer-based techniques. It is also hypothesized that simple geostatistical techniques produce estimates nearly as good as those produced by more advanced geostatistical tools.

Techniques for the reproduction of historical records of streamflow largely fall into two main categories: process-based models and transfer-based, statistical techniques. This work is concerned with the latter, which rely on transferring information

from an index site or set of index sites to an ungaged site by the means of a statistical relationship. These techniques include ungaged applications of record reconstruction techniques like the drainage-area ratio method (see Asquith et al., 2006), the maintenance of variance extension (Hirsch, 1979, 1982), and nonlinear spatial interpolation using flow duration curves (Fennessey, 1994; Hughes and Smakhtin, 1996). A portion of this work is dedicated to contextualizing geostatistical techniques

5 within these traditional approaches.

The prediction of daily streamflow records in ungaged basins, especially for statistical transfer methods, has largely been dominated by one-to-one transfers from an index streamgage to an ungaged site (as in Archfield and Vogel, 2010; Farmer et al., 2014). In some cases, information from a few neighboring streamgages has been blended to predict values at an ungaged site (Andréassian et al., 2012; Shu and Ouarda, 2012). Since not all streamgages are used to produce predictions, these approaches

10 neglect some of the information content of the streamgage network. Alternatively, regional hydrologic methods have sought to incorporate information from all the gaged sites to produce regression equations (Vogel et al., 1999) or contour maps (Sauquet, 2006) describing the spatial variation of hydrologic variables of interest. It is postulated that predictions of daily streamflow time series can be improved by incorporating regional information beyond the information available at single index streamgages and that, building on previous hydrologic time series analysis (Solow and Gorelick, 1986; Skøien and Blöschl, 2007), this can

15 be achieved by utilizing the geostatistical method known as kriging.

Geostatistical tools have been used to develop regional maps of measured and predicted hydrologic and climatic variables for decades. The U.S. Geological Survey has developed contour or isoline maps of runoff in the United States as far back as 1894 (Langbein, 1949). Langbein (1949) provide a summary of early hydrologic mapping efforts in the United States and elsewhere dating back to 1873. Such efforts produced largely hand-drawn maps of runoff, precipitation, and evapotranspiration that relied

20 heavily on expert judgment rather than algorithmic geostatistics (Langbein, 1949; Busby, 1963). As researchers gained access to higher-powered computers, efforts were made to automate the development of maps of mean annual runoff (Langbein and Slack, 1982). In both Europe and the United States, maps of mean annual runoff generated by geostatistical techniques were found to be as accurate as their hand-drawn predecessors (Rochelle et al., 1989; Domokos and Sass, 1990; Bishop and Church, 1992, 1995). Mapping techniques have also been used to explore other streamflow statistics (Gottschalk et al., 2006; Archfield

25 et al., 2013) and to assess the accuracy and performance of hydrologic models (Sauquet and Leblois, 2001).

Geostatistical maps of runoff and other variables are usually based on kriging, a technique developed in the mining industry (as described by Skøien et al., 2006). In kriging, the predicted variable is considered to be spatially continuous and predictions are based only on geospatial locations. This is generally valid for variables like precipitation and temperature, but runoff is different. Streamflows are organized hierarchically along a stream network and typically conserve mass (Sauquet et al., 2000;

30 Sauquet, 2006; Skøien and Blöschl, 2007). For this reason, topological kriging (top-kriging) was developed to incorporate the river network and its geographic extent into kriging estimates (Bishop et al., 1998; Sauquet et al., 2000; Sauquet, 2006; Skøien et al., 2006). In studies exploring the prediction of mean annual runoff (Skøien et al., 2006), percentiles, and other indices of the streamflow distribution (Castiglioni et al., 2011; Archfield et al., 2013) and streamflow signatures (Viglione et al., 2013), top-kriging has been shown to outperform many other techniques, including ordinary kriging. However, ordinary kriging is

35 better understood than top-kriging and, according to Sauquet (2006), may provide a competitive first-order approximation.

Despite its wide application for the prediction of streamflow statistics, kriging, top-kriging, and mapping in general have not widely been used to predict time series of streamflow and related variables. Despite the need for sub-monthly predictions of streamflow statistics, the prediction of sub-monthly variables was originally thought to be computationally prohibitive (Arnell, 1995). Previous work (Solow and Gorelick, 1986) showed kriging could be used for monthly time series prediction. With advances in computer technology, Skøien and Blöschl (2007) applied top-kriging to the prediction of hourly time series of runoff in Austria. Though they did not compare their techniques to ordinary kriging, they found that the embedded network structure of top-kriging produces good estimates of the runoff time series, but that additional spatial and temporal improvements, such as the inclusion of complex river routing and lag times, yielded diminishing returns. Aggregating their hourly model to daily estimates, they showed that top-kriging was superior to a mechanistic rainfall-runoff model. Their work emphasizes the need to explore and contrast the potential of ordinary kriging and top-kriging to predict streamflow time series in ungaged sites.

This work explores the potential of ordinary kriging to produce spatially and temporally continuous predictions of historical daily streamflow in the southeast region of the United States. Streamflow is a volumetric quantity that typically accumulates along a river network; as it is not reasonable to consider the regionalization of a volumetric quantity, a transformation is needed. This has been the rationale for the prediction of unit runoff values (Skøien and Blöschl, 2007), where unit runoff is defined as the ratio of streamflow to drainage area. Here, kriging is used to predict a time series of the same variable, as it is both spatially continuous and can be back-transformed to produce volumetric streamflow predictions.

Spatial interpolation driven by covariance – kriging – among daily streamflows is not new. Skøien and Blöschl (2007) used a single, temporally-aggregated representation of spatial correlation to predict all daily values. Similarly Archfield and Vogel (2010), in their map correlation method, leverage the spatial correlation structure of hydrographs in streamgage networks to identify ideal index streamgages. This work presents a different approach exploring the temporal evolution of daily variograms and seeking to characterize the spatial correlation of daily streamflows in a region. We evaluate the ability to estimate daily streamflow series at ungaged sites. Using a leave-one-out validation procedure, the predicted time series of daily flows at gaged-but-omitted sites are assessed across a range of goodness of fit metrics. Furthermore, the temporal evolution and stationarity of the spatial covariance structure of daily streamflow is explored through time-series analysis. It is shown that ordinary kriging of the logarithms of runoff can provide accurate streamflow predictions at ungaged sites, significantly outperforming more traditional approaches that employ a single index streamgage for transfer. The work presented in this manuscript is an extension of the material presented by Farmer (2015).

2 Data and Methodology

2.1 Study Area and Streamflow Data

Using a data set identical to that used by Farmer et al. (2014), this analysis was conducted with data from 182 streamgages in the southeastern United States. Basin characteristics are summarized in Table 1 of Farmer et al. (2014), but drainage areas averaged 979 km^2. The range of drainage areas was from 14 to 38 849 km^2, with a median of 417 km^2 and first and third quartiles of 150 and 886 km^2. Because the basins are free of major regulation or development, all of the streamgages were

considered near reference-quality because of their designation in the GAGES-II classification (Falcone, 2011) or their local approval and utilization in previous flood-frequency studies (Gotvald et al., 2009). (The two sources provide a more thorough description of their criteria.) Figure 1 shows the geographic extent of the study area and streamgage locations, which are defined by the Albers projection, in meters, of the latitude and longitude of basin outlet with respect to the North American Datum of

5   1983. As described by Farmer et al. (2014), the 355 000-km^2 study area, covering portions of seven Southern states, is warm, humid, temperate, and nearly 50% forested. Only 9% of the landscape is categorized as developed, while 18% is occupied by agricultural uses.

Daily streamflow records were obtained from the U.S. Geological Survey National Water Information System (http://waterdata.usgs.gov) for the period from 01 October 1980, through 30 September 2010. As documented by Farmer et al. (2014), very small portions

10   of the streamflow records – on the order of one to 33 days – were reconstructed using standard techniques. To avoid the complications of zero values, zero-valued streamflows were assigned a value of 0.00003 cubic meters per second, a value smaller than the smallest streamflow reported by the U.S. Geological Survey. (Farmer (2015) found that this substitution had only a minimal effect on the interpretation of results.)

2.2   Ordinary Kriging

15   Ordinary kriging is a geostatistical tool by which the distance between two points is used to predict the covariance of some dependent variable. The inter-site covariances of data from a measured network can be used to create a system of linear equations relating data from a site to data at another by a linear weighted average. For an unmonitored site, this allows for the derivation of linear weights between the unmonitored site and all monitored sites in the network. If all the assumptions of ordinary kriging are valid, this tool provides the best linear unbiased estimate.

20   Skøien et al. (2006) and many others provided an elegant and simple description of the mathematics of kriging; only a brief summary is provided here. Consider a network of measurements $z(\boldsymbol{x}_i)$ for $i = [1, n]$, where $\boldsymbol{x}_i$ is the Euclidean location of the measurement. Ordinary kriging allows for the prediction of an unmeasured value at location $\boldsymbol{x}_0$, $z(\boldsymbol{x}_0)$, by calculating a weighted sum of the observations $\hat{z}(\boldsymbol{x}_0) = \sum_{i=1}^{n} \lambda_{i,0} z(\boldsymbol{x}_i)$. The kriging weights, $\lambda_{i,0}$, for a particular ungaged location are determined by solving the linear system

25   $$\mathbf{CW} = \mathbf{D} \tag{1}$$

for the vector of weights, $\mathbf{W}$, where

$$\mathbf{C} \equiv \{c_{i,j} = cov(z(\boldsymbol{x}_i), z(\boldsymbol{x}_j)) \text{ for } i, j \leq n;$$

$$c_{i,n+1} = c_{n+1,j} = 1;$$

$$\text{and } c_{n+1,n+1} = 0\} \tag{2}$$

$$\mathbf{W} \equiv \{w_i = \lambda_{i,0} \text{ for } i \leq n;$$

$$\text{and } w_{n+1} = \mu\} \tag{3}$$

$$\mathbf{D} \equiv \{d_i = cov(z(\boldsymbol{x}_i), z(\boldsymbol{x}_0)) \text{ for } i \leq n;$$

$$\text{and } d_{n+1} = 1\} \tag{4}$$

This system ensures that all the weights sum to one and then the unknown mean of the z, μ, is maintained.

In practice, the elements of $\mathbf{D}$ cannot be calculated explicitly. Furthermore, the system can only be solved if it is positive definite. For these reasons, it is necessary to approximate the elements of $\mathbf{C}$ and $\mathbf{D}$ using a model of the covariance as a function of distance, a variogram. There are many models which ensure positive definiteness. These models are fit by assuming that covariance in the network varies as a function of distance and fitting an empirical variogram. Once a variogram model is selected, the system becomes

$$\hat{\mathbf{C}}\mathbf{W} = \hat{\mathbf{D}} \tag{5}$$

which is solvable.

Applying ordinary kriging to streamflow prediction is a simple extension of previous hydrologic geostatistics. The dependent variable of interest is the logarithm of the measured streamflow per unit drainage area, $z = \ln \frac{Q}{A}$. Previous work (Farmer, 2015) found that this dependent variable was the most stable predictand. Furthermore, based on initial exploration by the same, the spherical variogram model was selected. Alternatives are available, but Farmer (2015) and initial testing done here found the results to be generally insensitive to the theoretical variogram model. The spherical model has been used previously for hydrologic phenomenon (Archfield and Vogel, 2010). Finally, in building the empirical variogram, the covariances were stratified into ten groups based on distance, as suggested by Archfield and Vogel (2010).

The model of ordinary kriging presented above assumes a global neighborhood. That is, all observations are assigned a weight for the prediction of the ungaged site. In other hydrologic applications (Pugliese et al., 2014), some advantages have been gained by restricting the number of sites permitted to influence predictions. The neighborhood can be restricted to include only the k nearest neighbors. This approach was considered, but results were found to be generally insensitive to the number of neighbors. As a result, the global neighborhood was used, allowing the kriging algorithm to minimize the weights of far-distant sites if they are unimportant for estimation.

While there are many considerations in the development of a kriging system, this work is mainly focused on the temporal considerations of kriging time series. As such, the temporal evolution and behavior of variogram parameters was of most interest. As discussed above, there are many considerations in the development of a kriging system. Several were explored,

including the binning of empirical variograms, the number of contributing neighbors, and the maximum range of the variogram, but none were found to have only a marginal impact on the resulting estimates. Accordingly, the remainder of this manuscript considers the unique problems of temporal calibration and prediction.

2.3 Variogram Parameters

5   The variogram can be characterized by three parameters: The nugget value, partial sill, and the range. The nugget value is the covariance of collocated points, the partial sill represents the regional covariance, and the range represents the separation distance beyond which the inter-site covariance is best approximated by the regional covariance. In some previous hydrologic applications of kriging, the dependent variable, the covariance structure of which is modeled by the variogram, has been assumed to be temporally constant and thus only a single variogram model need be fit. This is clearly not the case for the

10   reconstruction of historical time series of streamflow. It is therefore important to consider the temporal evolution, or lack thereof, in the spatial covariance structure, as characterized by variogram parameters, of daily streamflows.

    The initial development by Farmer (2015) modeled each day of the streamflow record independently with unique variogram parameters. While this proved useful, it is not intuitive. With the temporal dependence of streamflows, it seems reasonable to consider some corresponding temporal dependence in variogram parameters. In the most extreme, Farmer (2015) showed that

15   assuming stationarity in variogram parameters resulted in barely any degradation of performance: The average of the daily variogram parameters, which are not identical to the pooled variogram parameters (described below), performed nearly as well as the independent daily models.

    Here, we consider the temporal evolution of variogram parameters more formally. We contrast the streamflow models based on independent daily variogram models with a pooled variogram model. The latter model requires the fitting of only a single

20   variogram, while the former requires the computation of as many variograms as there are days to be simulated. If the spatial co-variance structure exhibits some stability whereby a single variogram parameter set remains applicable, then the computational efficiency of the pooled model is highly advantageous for operational prediction.

    The pooled variogram is described by Gräler et al. (2011). For a daily variogram, the covariances for each site pair are plotted against distance, binned, and averaged to fit a theoretical variogram model; the process is repeated independently for

25   each day. For pooled variograms, the covariances calculated on each day are pooled into a single empirical variogram to which the variogram model is calibrated. The covariance is calculated spatially, as described above, but covariances between sites are not computed across time steps. That is, $cov(z(\boldsymbol{x}_{i,t_1}), z(\boldsymbol{x}_{j,t_1}))$ and $cov(z(\boldsymbol{x}_{i,t_2}), z(\boldsymbol{x}_{j,t_2}))$ are both considered and pooled into the empirical variogram cloud, but $cov(z(\boldsymbol{x}_{i,t_1}), z(\boldsymbol{x}_{j,t_2}))$, where the time $t_1 \neq t_2$, is never considered. In section 2.4 and elsewhere, Gräler et al. (2011) describe and contrast the performance of the pooled method and the averaging method. The

30   similarities identified by Gräler et al. (2011) suggest that the averaged model considered by Farmer (2015) can be represented much more efficiently by the pooled model. (Note that if all streamgages are operational on all days, then the average model is identical to the pooled model.)

2.4 Relative Performance

In addition to contrasting temporally independent variograms and pooled variograms, this paper also contrasts these methods with two standard, transfer-based statistical tools: the drainage area ratio (DAR)(Asquith et al., 2006) method and nonlinear spatial interpolation using flow duration curves (QPPQ) (Hughes and Smakhtin, 1996). The former scales index streamflows by

5 drainage areas, while the latter scales the entire flow duration curve of an index site. Both of these methods were implemented following the methodology of Farmer et al. (2014). The time series prediction methods were assessed by computing the Nash-Sutcliffe model efficiency (Nash and Sutcliffe, 1970) of the streamflow values and the logarithms of streamflow values. Nash-Sutcliffe model efficiencies range from one to negative infinity; values of one indicate a perfect model fit, while lower values indicate an increasingly poor fit; a value of zero indicates that the estimate is no better than a regional average. Correlations

10 between observed and simulated streamflows, root mean squared errors and average biases were also considered.

Using the same metrics, ordinary kriging was contrasted with top-kriging as defined by Skøien and Blöschl (2007). Here, top-kriging was applied with a minimum spatial resolution of 100 points per basin and a maximum of 5 neighboring basins per prediction. Furthermore, the daily covariances were pooled to create a pooled top-kriging model of the spatial covariance structure. This comparison of ordinary and top-kriging serves as only an initial comparison. It does not address deeper levels

15 of discrepancy between the two methods, a topic which, given the similarity of results, may warrant further research. This comparison also does not explicitly address questions of computational efficiency, a difference in which may favor one method over another.

3 Results

3.1 Optimal Variogram Parameters

20 In a leave-one-out validation procedure, both the daily and pooled parameter sets reproduce historical daily streamflow records quite well. Table 1 summarizes several common performance metrics calculated on the complete water years of observed daily streamflow. (A water year is the 12-month period 01 October through 30 September designated by the calendar year in which it ends.) For all metrics, the performances are very similar, but the pooled parameter set produced slightly better results. A two-sided Wilcoxon signed-rank test for each performance metric showed this difference to be significant in all cases except

25 median bias. Figure 2 shows a one-year example of the predicted and observed streamflows for a single site; this site and year were selected because the results are representative of median performance. This example highlights the similarity between estimates made with the daily and pooled variograms, but also demonstrates the poor performance during low-flow periods.

In addition to having similar point performance metrics, the daily and pooled variograms produced nearly identical distributions of absolute percent errors (Fig. 3). This summary plot shows the cross-site median cumulative distribution of absolute

30 percent errors. Both daily and pooled variograms perform well, with more than half of the estimates within 30% of the observed streamflows. Where the individual curves are distinguishable, the cumulative frequency of estimates at or below the indicated

error level was higher for estimates developed form the pooled variogram than from the daily variogram. This shows that the pooled variogram produces estimates with slightly fewer large percent errors.

Figure 4, binning with a width of one percentage point, plots the cross-site median percent error against observed exceedance probability. The result shows a concerning limitation of the kriging approach. From Table 1, both sets of estimates produced only a slight upward bias overall – 4.5% for the daily variograms and 2.5% for the pooled variogram – but the overall statistics do not capture the poor performance in the tails of the streamflow distribution (Fig. 4). Estimates appear to be nearly unbiased, plus or minus 5%, for streamflows between the 5th and 76th percentiles. For low streamflows, below the 5th percentile, both variogram methods consistently overestimate streamflows between the 5% and 15% non-exceedance probability. For high streamflows, beyond the 75th percentile, streamflows are actually underestimated; the underestimate approaches -40% for some of the greatest streamflows.

Finally, with the use of time-varying and time-invariant variograms, it is useful to consider how well the temporal structure of the daily streamflows is reproduced. Figure 5 summarizes the median observed autocorrelation of streamflows and how well it is reproduced. Again, both variogram methods produce similar results. For lags within 120 days, the error for each variogram method is below 60%. Beyond 120 days, the median observed absolute autocorrelation is 0.04%, ranging between plus or minus 0.1%, and each method produces an absolute error of approximately 65%. While the trend appears to be downward, this is a marked upward step beyond 120 days. However, because the autocorrelations are small, the simulations only produce and median absolute autocorrelation of 0.07%, which is not overly concerning when compared with 0.04%. Because of the dependent structure of daily time series, it is not surprising that simulated results would produce some aberrant residual correlation at long time lags. In general, the reproduction of the autocorrelation structure suggests that the temporal structure of the streamflow time series is reproduced tolerably well. Not surprisingly, the daily parameter set, which varies in time, more accurately reproduces the temporal structure. Interestingly, the difference is not as large as might be expected.

3.2 Temporal Evolution of Variogram Parameters

Because the pooled variogram parameters produce results fairly similar to the daily parameter sets, it is important to understand how the pooled parameters relate to their daily counterparts and how the daily counterparts evolved over time. Figure 6, described below, illustrates the temporal structure and seasonal nature of the daily parameters and contextualizes the pooled parameter sets. For each parameter, we consider its autocorrelation structure out to 365 days and its 31-day moving median. The temporal variability in variogram parameters is a reflection of the temporal and regional variability in streamflow and the factors producing streamflow.

As mentioned previously, the nugget value can be thought of as the covariance of collocated points. In the context of basins and daily parameters, the nugget on each day, because the covariance of co-located points is akin to a variance, is an approximation of the average of all at-site variances for that day. The autocorrelation structure of the nugget time series suggests that there is a substantial seasonal trend. This seasonal behavior is shown more clearly in the 31-day moving median. The nugget, or regional variability, and the variability thereof, are fairly constant from the beginning of January through May and rise to a peak in September and October. The pooled parameter, which can be thought of as a time-averaged variability of

an average site, is closer to the peak of the moving-median nugget than to the lower stable January-May values. The pooled parameter is greater than the median of the daily values. This suggests that, for much of the year, the pooled nugget, being greater than the daily values, introduces more daily variability than would be expected.

From the autocorrelation function, the partial sill, a limit on the regional covariance, shows a much weaker seasonal signal. The 31-day moving median shows a nearly binary structure of two values. The partial sill is small from January through March, transitions quickly in April, remains high through October and then returns towards January values. Again, the pooled parameter plots closer to the higher plateau of the moving median. This means that for parts of the year, the pooled parameter assumes the more distant neighbors hold appreciably less information than they really contain. For a smaller portion of the year, the pooled parameter, being greater than the daily values, assumes the more-distant neighbors hold slightly more information than they really do. However, the pooled partial sill remains within the inter-decile range of the daily parameter values for the majority of year.

The range parameter shows the least complex temporal structure. The autocorrelation function for the range decayed faster than it did for the other two parameters and does not suggest strong seasonal fluctuation. The 31-day moving median shows that the range varies over an order of magnitude, and year-to-year variability, as shown by the inter-decile range, is consistently large. There is a slight depression in the summer months, which indicates decreased regional homogeneity in that the regional covariance (partial sill) is reached at shorter distances. The pooled parameter is quite similar in magnitude to the median daily value and is almost completely contained by the daily inter-decile ranges.

It is clear that there is substantial temporal structure and seasonal variation in the spatial covariance structure of daily streamflows. Given the strong temporal dependence and seasonality of daily streamflows, this is not surprising. As with streamflow, it is extremely difficult to identify causal factors resulting in these patterns. Though not explicitly explored here, it is probable that the temporal structure is driven by climatic processes. The greater nugget value in the latter half of the year indicates increased streamflow variability, year to year, during the late Fall and early Winter. The partial sill and range interact strongly with each other, one being the threshold and the other being a sort of "time to threshold". The decreased Summer range suggests that climatic response is more homogeneous in Summer months, while the Winter and Spring rises are emblematic of increased regional heterogeneity (i.e. more localized climatic drivers of streamflow.) The partial sill demonstrates an increase regional variability, beyond the range, from late Spring through Fall; otherwise the sill is smaller, suggesting that, even beyond the range, variability is lower across the region in Winter months.

3.3 Relative Performance

In presenting a new model for daily streamflow reconstructions, it is useful to contextualize performance by comparing against previous methods. To this end, two common statistical, transfer-based tools for the prediction of daily time series are considered: the drainage-area-ratio (DAR) (Asquith et al., 2006) method and nonlinear spatial interpolation using flow duration curves (QPPQ) (Hughes and Smakhtin, 1996). Both are applied with index sites defined by spatial proximity. The methods and regional regressions used here are identical to those reported by Farmer et al. (2014), though a leave-one-out validation scheme

is used here. DAR is a single-index analog to the kriging approach, while QPPQ represents the optimal method for this region (Farmer et al., 2014).

The performance metrics of both DAR and QPPQ are outlined in Table 1. As concluded by Farmer et al. (2014), the QPPQ methodology performed better than the DAR technique. In this analysis, both DAR and QPPQ performed inferior to the

5   kriging approaches. As determined by individual Wilcoxon signed-rank tests of each performance metric for estimates from each method against the estimates from pooled variograms, the pooled variograms produce results with significantly better predictive power than both DAR and QPPQ individually for all performance metrics except median bias. The estimates from QPPQ were not shown to be significantly more biased than the estimates from the pooled variograms, on average. Note that the Wilcoxon test on biases was conducted on absolute values, indicating the significance of either method being closer to the

10   optimal level of zero bias regardless of the sign of the bias.

The comparison of the pooled ordinary kriging approach and the pooled top-kriging approach does not provide as definitive of a conclusion. The top-kriging approach provides a significantly greater Nash-Sutcliffe efficiency at the 5% significance level. However, the ordinary kriging approach yielded significantly smaller root mean squared errors. In terms of bias, top-kriging provides a significantly smaller absolute bias, but the median signed bias is slightly larger; the average bias is greater, but

15   the average deviation from unbiasedness is smaller. There is no significant difference between ordinary and top-kriging with respect to the correlations between observed and simulated streamflows and the Nash-Sutcliffe efficiencies of the logarithms of streamflow.

4   Discussion

The results of this analysis demonstrate that the computationally efficient routine of pooled variogram estimation can be used

20   to fit an ordinary kriging system that produces plausible estimates of daily time series at ungaged sites. The pooled parameter estimation, which ignores temporal variation of the spatial covariance structure, was able to reproduce observed hydrographs more accurately than other non-kriging methods considered. Both daily and pooled kriging approaches outperformed single-index transfers. It is intriguing that accounting for temporal variation in the variograms resulted in relatively minor changes in the kriging estimates and the performance thereof. Additionally, it is somewhat concerning that the kriging techniques show a

25   general inaccuracy in the tails of the distribution of streamflow. The comparison of ordinary and top-kriging was inconclusive, with some metrics favoring top-kriging while others favored ordinary kriging and still others were not significantly different.

It was clearly shown that the variogram parameters, characteristic of the spatial covariance structure, exhibit seasonal and other temporal patterns. However, the averaging that occurs when pooling daily covariance information actually resulted in a marginal improvement in the accuracy (as measured by several metrics) of resultant streamflow time series. In initial work

30   (Farmer, 2015), it was shown that pure averaging of variogram parameters, rather than pooling, similarly produced estimates similarly competitive with estimates from daily variograms. It is counterintuitive that ignoring temporal variation in spatial covariance structure would not appreciably degrade performance. Still, ignoring the temporal variation of variogram parameters produced some small degradation in the autocorrelation structure of estimated streamflows at long time lags.

Although ignoring the temporal variation in the variogram parameters did not appreciably degrade performance, it may be possible to gain some improvements while retaining computational efficiency by preserving some remnants of the observed temporal variability in variogram parameters. A clear avenue for future research is to evaluate the possibility of constructing a temporal model of variogram parameters. One could easily imagine monthly parameter sets or parameter sets reproduced by an autoregressive integrated moving average (ARIMA) (Box and Jenkins, 1970) model. Previous work has found only marginal advantage to incorporating complex temporal structures like streamflow travel times into hydrologic geostatistics (Skøien and Blöschl, 2007), but the temporal evolution of spatial covariance structure was not explicitly considered. As this manuscript serves as a general introduction of ordinary kriging to time-series prediction, this work was not explored further here.

However, contextualizing ordinary kriging in the context of other hydrologic applications of geostatistics, a brief comparison of ordinary and top-kriging was presented here. Skøien et al. (2006) introduced topological kriging to the hydrologic sciences and Skøien and Blöschl (2007) applied it to streamflow time series. Following the methods of Skøien and Blöschl (2007), a pooled top-kriging model of daily streamflows was developed and compared with the ordinary kriging approach. The comparison of ordinary and top-kriging does not provide strong evidence to favor one approach over the other. Subsequent analyses may elucidate further strengths and weaknesses, but it is not possible to dismiss either method based on the evidence presented here. The pooled top-kriging model was developed using the package provided by Skøien (2015). The need to spatially discretize the network at each time step substantially increased the computation time compared with ordinary kriging (depending on processor speeds, top-kriging required just under three days per site, while ordinary kriging required only hours per site); there is not currently a call in the package by Skøien (2015) to estimate the pooled variogram directly. Once the computation time is brought into parity with ordinary kriging, the marginal improvements of top-kriging may be more worthwhile. However, it may be that the relatively simpler formulation of ordinary kriging provides the majority of the added-value of hydrologic geostatistics.

The pooling of covariance to produce a single set of variogram parameters implicitly assumes that the spatial covariance structure is constant in time. While a seasonal fluctuation may be present, that same fluctuation may occur every year with no systematic change. For the study period, water years 1981 through 2010, the time series of daily variogram parameters were indeed stationary. Following the procedures of Hirsch et al. (1982) a block bootstrapping procedure with a fixed block width of one year (365 days) and 1 000 replicates of thirty-year time series was applied to approximate the probability distribution of a seasonal Mann Kendall trend test. For all three parameters, the null hypothesis of stationarity could not be rejected. The nugget had an approximate two-side-alternative p-value of 0.286; the partial sill, 0.184; the range, 0.178. While stationarity appears valid in this instance, it does raise an interesting question in the face of changing hydrology. Will changes in human populations, land uses, and climate significantly affect the spatial covariance structure of daily streamflows? The daily parameter sets may be an appropriate means of testing for changing hydrology and identifying dominant processes in a region.

Pooled variogram estimation and ordinary kriging allow for the efficient and, according to broad metrics, accurate prediction of daily streamflow at ungaged sites. Being able to regionally characterize networks of streamflow may provide additional advantages. Though not explored here, kriging algorithms also allow for the quantification of variances around estimates. This can serve two purposes: (1) It shows where in the network uncertainties are likely to be greatest, which might be a

means to identify optimal locations for additional monitoring. (2) It may be able to explicitly provide confidence intervals for estimated daily streamflows. Future studies will explore the accuracy of so-derived intervals. In any case, the theoretically-derived structure of the kriging system promises a more "closed-form" interpretation of predictive uncertainty than more traditional single-index hydrologic transfers, which require an ad-hoc procedure for uncertainty quantification.

5   One limitation of the kriging approach, as documented here, is the overestimation of the lower tail of the streamflow distribution and the underestimation of the upper tail. Similar results were documented by Skøien and Blöschl (2007). This is effectively a compression of the distribution of streamflows, resulting in estimated streamflows that are less variable than the observed streamflows. Less variability means that the estimated time series will not be able to faithfully reproduce the frequency and magnitude of the most extreme events. As the most extreme events tend to have the greatest impact on human

10 populations, the failure to accurately reproduce them may prove problematic for operational hydrology. Interestingly, this result may not be a product of the kriging system. Instead, it is an expected result of deterministic modeling. If sources of error or uncertainty are neglected in order to produce a deterministic estimate, this expectation of the conditional mean is less variable than the observed quantity. Stochastic simulation may be the only solution if the estimated time series are to be made useful in the context of operational hydrology.

5   Summary and Conclusions

The estimation of daily streamflow records at ungaged sites is a fundamental problem of water resources management and assessment. Many tools exist to aid in quantifying resources, but this paper discusses a statistical tool that is capable of combining time series at multiple sites for regional prediction. Building on the work of hand-drawn discharge maps, ordinary kriging is proposed as an efficient technique for reproduction of historical streamflow time series at ungaged sites. Using a leave-one-

20 out validation and daily streamflow data from 182 minimally-impacted and minimally-regulated watersheds, geostatistical techniques are shown to have advantages over other, common statistical approaches.

     Ordinary kriging is demonstrated to produce more accurate streamflow time series estimates than the drainage-area ratio method and nonlinear spatial interpolations using flow duration curves. In addition, using pooled variogram parameters with ordinary kriging produced marginally better performance than using parameters determined at a daily time step. This is sur-

25 prising, as pooling effectively averages out temporal variation. Though significant improvements are unlikely, it is observed that the variogram parameters, characterizing the spatial covariance structure, show clear seasonal patterns that may be reproducible in part without requiring the computation of daily variograms. However, in an initial exploration, the advantages of moving towards a more complex kriging system such as that provided by top-kriging are, at best, minimal. Further research may improve the computational parity of top-kriging and continue to elucidate the advantages and disadvantages of ordinary

30 and top-kriging for spatio-temporal hydrologic geostatistics.

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Table 1. Summary statistics of several performance metrics for different streamflow record prediction techniques. Summary statistics include (a) the median, (b) the 10th and 90th percentiles, and (c) the Wilcoxon signed-rank probability of a difference between pooled kriging and each other method equal to or more extreme than observed (commonly referred to as a p-value). Naturally, this is not applicable (NA) for the comparison of pooled kriging against itself. Nash-Sutcliffe efficiency values of 1 indicate perfect agreement between observed and predicted values, so values closer to 1 indicate better model performance. Root mean squared error is reported in cubic meters per second. The Wilcoxon test on bias was performed on the absolute value of the bias.

Performance Metric		Daily Kriging	Pooled Kriging	Pooled Top-Kriging	DAR	QPPQ
Nash-Sutcliffe Efficiency	a	0.7006	0.7040	0.7106	0.5529	0.5980
	b	(0.3905, 0.8433)	(0.4005, 0.8522)	(0.3612, 0.8805)	(-0.2283, 0.8248)	(-0.0072, 0.8026)
	c	0.0068	NA	0.0248	<0.0001	<0.0001
Nash-Sutcliffe Efficiency of the Logarithms	a	0.7830	0.7961	0.7974	0.6663	0.6738
	b	(0.0960, 0.9088)	(0.0935, 0.9117)	(0.2547, 0.9327)	(-0.7771, 0.8948)	(-0.2811, 0.8580)
	c	0.0001	NA	0.0776	<0.0001	<0.0001
Pearson Correlation Coefficient	a	0.8562	0.8686	0.8628	0.8094	0.8148
	b	(0.7004, 0.9365)	(0.7143, 0.9354)	(0.6808, 0.9542)	(0.5621, 0.9298)	(0.5842, 0.9219)
	c	<0.0001	NA	0.3330	<0.0001	<0.0001
Root Mean Squared Error	a	6.2733	5.8775	6.2613	7.2752	7.7550
	b	(1.2685, 26.236)	(1.2775, 27.017)	(1.1633, 25.590)	(1.4270, 33.361)	(1.5690, 30.931)
	c	0.0016	NA	0.0009	<0.0001	<0.0001
Median Percent Bias, [Median Absolute Value]	a	4.4727, [18.397]	2.4686, [19.260]	4.5863, [16.848]	13.350, [25.252]	8.1408, [23.191]
	b	(-23.206, 135.40)	(-25.492, 123.14)	(-22.132, 127.46)	(-26.262, 171.18)	(-26.236, 113.26)
	c	0.3386	NA	0.0229	0.0020	0.2812
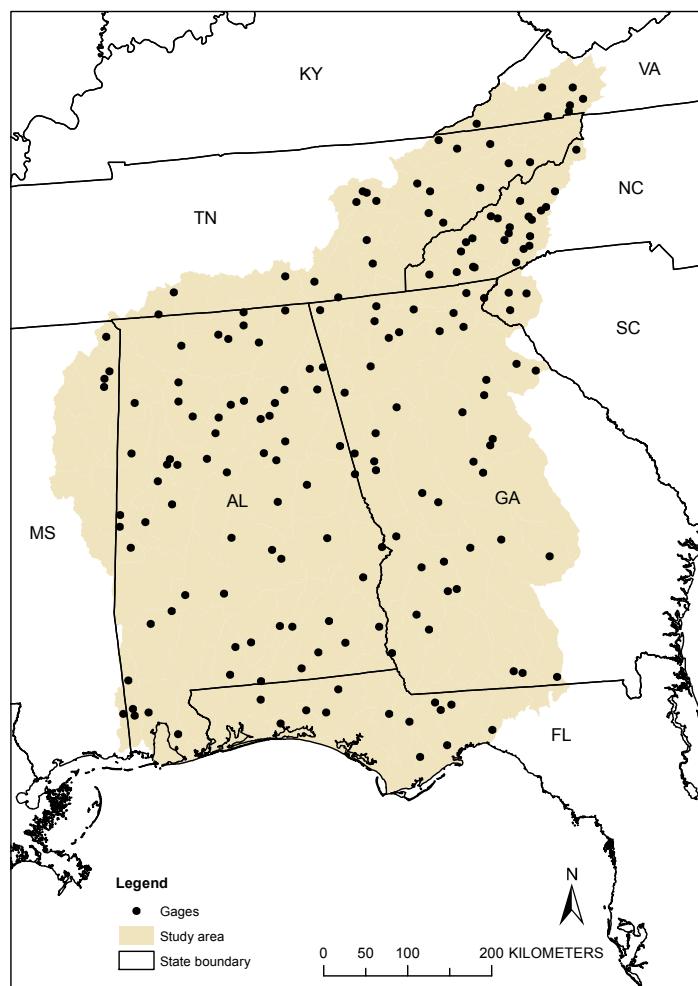
Figure 1. Map of study area showing the locations of 182 streamgages used for analysis and validation.
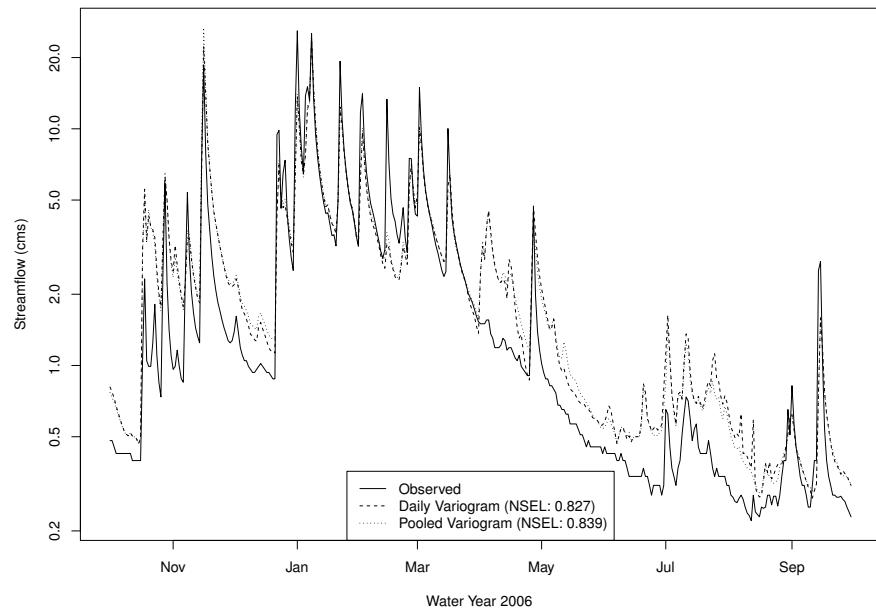
Figure 2. An example of the observed and simulated streamflows for a site and year selected to represent the median performance. The results are from site 02401390, with a drainage area of 365 km^2. Streamflow values are reported in cubic meters per second (cms).
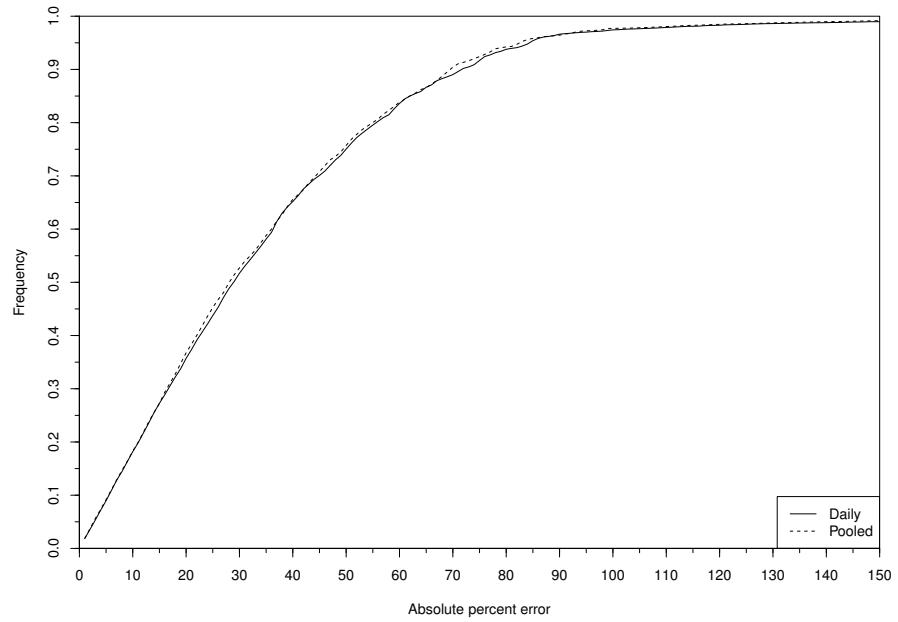
Figure 3. Median cumulative distribution of absolute percent errors in daily estimates for streamflow estimated from both daily and pooled variogram parameter sets.
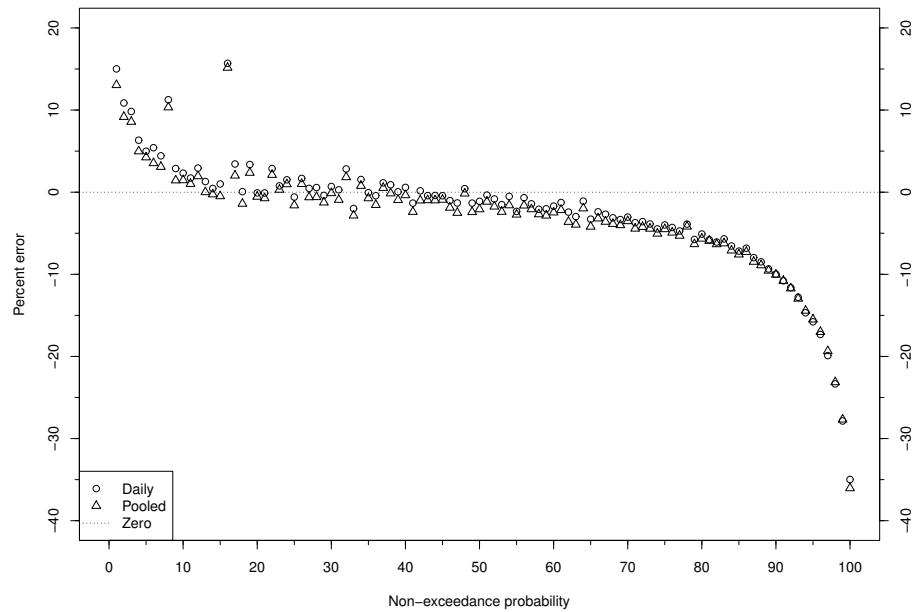
Figure 4. Median percent error for each non-exceedance probability, binned by single percentage points, for streamflow estimates from both daily and pooled variogram parameter sets.
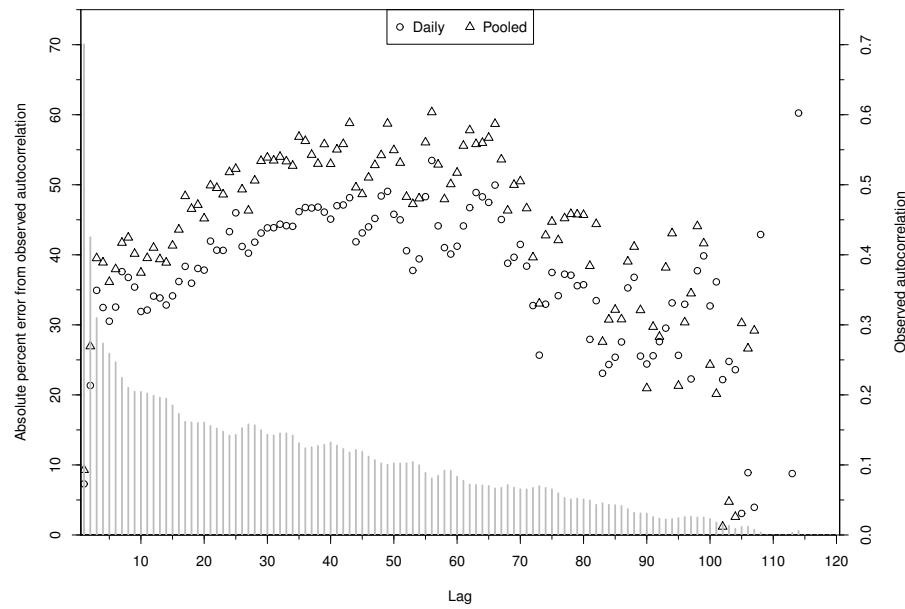
Figure 5. Absolute percent error between observed and estimated autocorrelation of daily streamflows for estimates from both daily and pooled variograms (left-hand, vertical axis). The vertical bars, quantified on the right-hand, vertical axis, describe the observed autocorrelation as a reference.
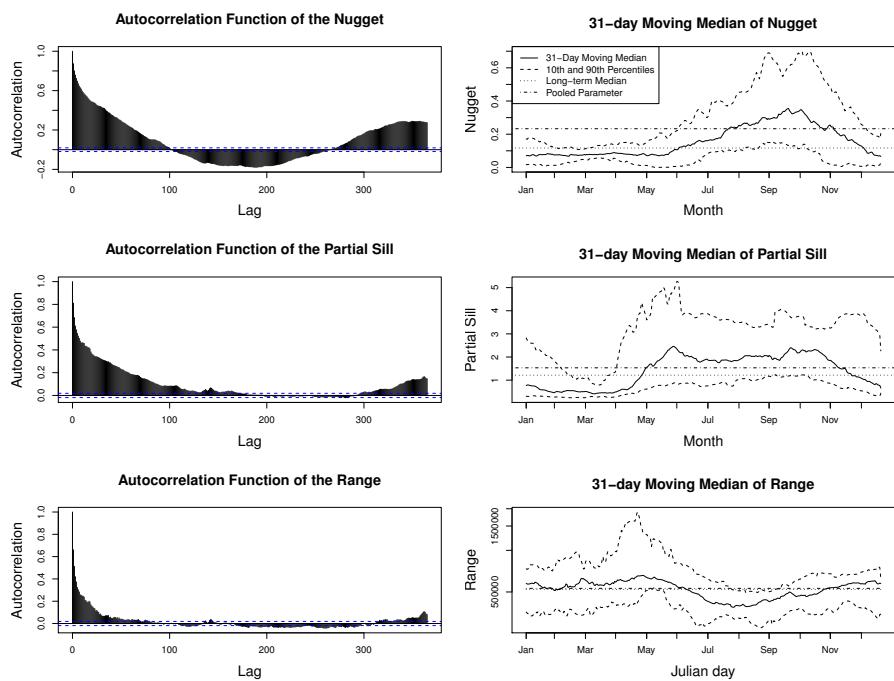
Figure 6. Autocorrelation function and 31-day moving median of daily variogram parameters.