We would like to thank the reviewer for the very thorough review and for the constructive comments which help us to improve the manuscript. Our replies are listed point-by-point below.

General comment

In general, the presented work is a novel and promising new method for small scale aquifer characterization. The manuscript is written in a well structured, easy understandable, clear and precise style, with a few exceptions (in sections 2 and 4, see below). The citation style - not distinguishing between "citep" and "citet" - is slightly confusing. Furthermore, the discussion on the limitation of the method could be outlined in a more elaborate way (spatial limitation of the method due to experimental conditions, "bias" of method to highly conductive material). Finally, given that the authors improve the few passages of their manuscript considering the remarks listed below, I highly recommend the manuscript for publication in HESS.

Specific Comments

line 38: aspect of conservativeness should be mentioned in this context

We include this aspect now in line 38 as follows:

"Main attributes of ideal tracers are their good detectability, their lack of influence on the flow regime, conservativeness, and nontoxicity to the environment."

line 144: The line integral appears from nowhere. A short introduction of the fundamental (transport) equation and a general/physical explanation of the line integral would be beneficial for the reader, who is not familiar with previous papers (e.g. Vasco and Gupta, 1999).

To keep our study focused we are hesitant in rephrasing the work by Vasco and Gupta, which provides a detailed description of the fundamentals underlying our approach. We add a detailed deduction of the eikonal equation into the appendix of the manuscript, which can be also found below in the appendix of this reply. For better comprehensibility, a short explanation is added to the presentation of the line integral in the manuscript:

"The line integral relates the tracer breakthrough time to the mean tracer velocity, and thus to the hydraulic conductivity along the transport trajectory."

line 155: The sentence is quite unspecific. The solution refers to what, the line integral? Is it the goal or a step of the method to find a solution? What exactly is determined, ttt(xr) or K?

The goal of the inversion is to find the K distribution. The revised sentence reads:

"With these assumptions and the use of standard tomography algorithms, the K distribution can be reconstructed on a pre-defined grid."

line 157: The sentence is hard to understand in this context: "The presented method" refers to what? To the calculation of the line integral or the experimental setup of the procedure ("step-function injection temperature signal"), which was not yet introduced. I cannot see the link between the content of the paragraph and the previous subsection on the line integral.

The introduction of a step-function signal is required in this section because the following data processing is built upon it. For clarification, we modified the text to:

"In this study, a step-function injection temperature signal is used for the active thermal tracer test."

line 176 ff: Most of the variables in Eq. 4 are not formally introduced: T, u, t, x, D. The same for T0 and erfc-function in Eq. 5. Please give short explanations.

Revised accordingly.

line 178-179: Please specify why the breakthrough time is associated with the peak in T 0 . Please state explicitly how tpeak is determined analytically from T 0 (respectively from T 00; is it T 00(x, tpeak) = 0?).

Reference added: (Vasco et al., 2000)

New equation added: $T''(x, t_{peak}) = 0$

line 179-181: Is the sentence a general statement or an announcement of experimental adaption to the analyzing procedure?

This is a general statement that during this section, the BTC is the first derivative of temperature. Travel times of step-function signals are generally calculated using the first derivative of the observed signal (Brauchler et al., 2003; He et al., 2006; Vasco et al., 2000; Ward et al., 1994).

lines 183-186: For the understanding of the derivation, it would be beneficial to introduce the proportionality factor α, the relative time to the peak time τα and the transformation factor fα at the beginning, give the reader an impression on their physical meaning and then derive the explicit expressions.

Revised accordingly.

"Early time characteristic values can be described proportionally to the peak value:

$$T'(x,t) = \alpha T'(x, t_{peak}) \tag{8}$$

which can be related to the relative peak time (τ_α) as:

$$\alpha = \frac{T'(x,t)}{T'(x,t_{peak})} = \frac{T'(x, \tau_\alpha t_{peak})}{T'(x, t_{peak})} \tag{9}$$

By relating these two expressions, the time of proportional value can be used to calculate the timing of any value of the signal."

line 187-190: This is a statement, which requires a certain proof. Please give a mathematical or visual argument for the validity of the simplification.

The insensitive parameters are either have higher orders, or multiplied by velocities at higher orders. By neglecting the terms with higher orders of velocities, all of these parameters are cancelled out.

"Although Eq. (10) has three additional parameters, velocity (u), distance (x) and dispersion coefficient (D), the function is not sensitive to these values because they are all at higher orders or multiplied with higher orders of velocity. So, by neglecting the terms with higher orders of velocity, they are cancelled out."

line 192 : At this stage there is no solution for τα presented, so the statement in brackets should be postponed to the according position.

The statement is postponed after Eq. 12.

line 194: Please introduce the Lambert Omega function and give reference how LambertW(..,..) is calculated in Eq. 12.

A short description and reference is added:

"The Lambert Omega function is the inverse function of $f(W) = We^W$ (Weisstein, 2002)."

Eq. 12: If the authors announce a solution for τα they should give it in line with fα or at least as 1/fα.

Revised accordingly.

line 201 - 212: the paragraph should be re-structured with respect to (i) the purpose of early time diagnostic, (ii) the procedure and (iii) the reasoning for the procedure.

For defining the reasoning for the procedure, the following sentence is added to the end of the paragraph:

"Step 3 allows the travel time to be related with the transport process, and equips the method to return a real and scaled K value instead of just information about the heterogeneity contrasts."

line 270: Please specify the "expansion" of the original data set (procedure of extension, new dimensions etc.)

Wrong wording in the manuscript. Expansion in this case did not mean spatial expansion, only the extension of the original dataset with thermal properties (Bayer et al., 2015). In the revised sentence "expansion" is replaced by "extension".

line 284 - 298: see comment to line 507 - 515.

Clarification is added to line 306:

"Note that independent of the dimensions of the reconstructed sections, the full 3-D analog model was always used to simulate the thermal tracer propagation and the travel times, considering buoyancy and viscosity effects."

line 362: Please specify the upscaling procedure.

For upscaling, arithmetic mean values of the cells were used. This is added to the revised text.

line 414-417: It should be shortly stated, why a factor of 2 is regarded as good match.

A factor of 2 fits in the same range as those found in several studies from hydraulic tomography (Brauchler et al., 2007; Cardiff et al., 2013; Jiménez et al., 2013).

line 498: Subtitel "Sensitivity Analysis" suggest a rather strict mathematical analysis of the methods parameters. See also the general statement on section 4.3 below.

After revision, the new section title reads: "Role of injection rate and temperature"

line 507-515 in combination with line 284 - 298: Simulating viscosity and density effects of heated water on flow requires a coupling of the flow and heat transport processes. It renders the system non-linear and makes simulations more complicated and error prone. The sentence in line 284 suggests, that these effects are taken into account, but I see a need for further explanations, especially a few more words on the density model used. It would be beneficial to convince the reader that the reported low sensitivity of the method on temperature differences is properly tested and not due to an incomplete simulation setup.

See the answer to your comment on line 284-298. A full physics finite element (FEFLOW) model was used, including variable density and viscosity effects in the simulations. The model was fully coupled.

line 564-570: Why was this quantitative analysis not already used in the previous sections 4.1 and 4.3. A separate introduction of the two analyzing strategies (visual inspection and quantitative analysis) and the use - especially in section 4.3 - would be beneficial to substantiate the sensitivity analysis.

In the previous sections, the reconstructed tomograms are visualized, and by this, direct visual comparison of the entire tomograms is facilitated. In our opinion, this is favorable to a quantitative analysis of the individual architectural elements and including such analysis already at this stage of the manuscript would not improve the readability. Instead, quantitative criteria are used when examining the applicability of the tomographic method under a broad range of alternative conditions, which however cannot be visualized in such detail.

line 569: The specification of the quantity for evaluating the result quality is quite unspecific. Maybe give a mathematical statement of how the difference between the connectivity time of the original model and the inverted results is used as measure. Are there thresholds defined or are the scenario results all compared relative to each other?

As a misfit, the root mean squared error between measured and predicted connectivity times was used. The quantification was only used to identify relative trends between the different results (see answer to line 584). In Figure 10, this relative trend is represented by the dashed line. The solid lines represent thresholds, where a) there is not enough breakthrough time to perform the inversion or b) where a zone on the tomogram is not properly reconstructed (see line 589-609).

571ff: What is the motivation to use these two parameters and not other? Why are they useful, especially with regard to the fact, that they are not independent?

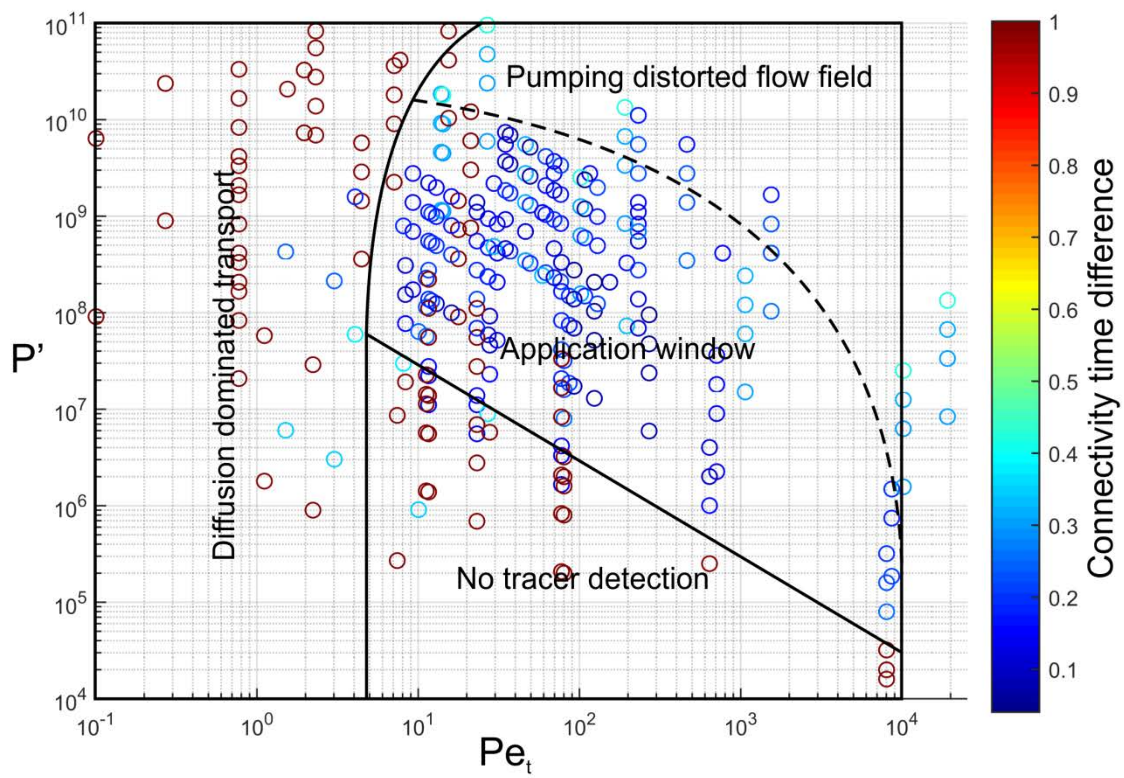
The effective injection power contains the technical parameters describing the experimental setup, whereas the thermal Péclet number is the standard parameter to describe the thermal transport behavior. These two parameters are independent if the assumption that the distribution of the hydraulic head gradient is uniform is valid. Using a high injection rate affects the head gradient distribution, increases the groundwater velocity, and increases the role of advection (see line 595).

Line 576: It would be helpful to state again what Cw is.

Revised accordingly.

Line 584-585: It is not clear to me, how the application window was constructed from connectivity time in combination with P et and P 0 . Furthermore, please specify what marks feasible and unfeasible regions and how boundaries between them were de- fined.

The following figure contains all the data points of the investigation.



The values are the normalized connectivity time differences between the original analog profile and the tomogram.

Red circles represent non reconstructed zones. Darker blue circles mean smaller differences. This comparison was only used to inspect the results relative to each other, and to identify trends related to the parameters.

The presented boundaries were defined arbitrarily through visual inspection of the resulting points. Solid lines show boundaries where the investigated zones are not reconstructed anymore. The dashed line represents trends where results start to weaken – the connectivity times start to move apart.

Line 589: How was the critical value for P et determined and what is the value/range (reference to Fig. 10)?

The critical value was defined where the zone is not reconstructed anymore. See previous figure.

General statement on section 4.3: After reading section 4.4, I wonder why the authors separate this two sections? The basic parameters tested in section 4.3 (injection rate and temperature difference) seem to mark the most important factors in section 4.4 as well. Furthermore, in section 4.4 qualitative and quantitative criteria are introduced, which would be beneficial to substantiate the sensitivity analysis in section

We found it important to emphasize the non-sensitiveness of the method with respect to the injection temperature, which is one of the most useful features of the method. Also, see our answer to your comment on line 498.

line 616: "of K" - Please, avoid or explain abbreviation in conclusion.

Revised accordingly.

line 623-625: The sentence is not fully clear: Do the three and five orders of magnitude for P et and P 0 refer to the tested parameters or the appropriate parameters for method application?

The application window refers to the tested parameters. Revised as:

“The presented application window of tested parameters of thermal tracer tomography is wide, and it covers three orders of magnitude for thermal Péclet numbers and five orders of magnitude for injection power.”

line 632: Specify "the values of K". State clearly what is "closely matched".

Figures and Tables

The figures should be at best comprehensible only with the aid of the legend and caption (without the running text). In this line, the following comments should be understand as advises for improving the readability.

Table 1: Superscript "1" and "2" for reference to Hyöng et al, 2014 and Bayer et al., 2015 might lead to confusion with exponents of units.

Revised accordingly.

Table 2: Leave the value of groundwater temperature out, since this parameter was not varied.

Revised accordingly.

Figure 1: State what ETD means or leave the abbreviation out.

Explanation of abbreviation is added to caption.

Figure 3, Caption: Specify "Distribution of hydraulic conductivity K", since K in the legend is currently not defined in the caption.

Revised accordingly.

Figure 5, Caption: The figure contains only to 50% reconstructed hydraulic conductivity profiles. Generally, hydraulic conductivity profiles are shown. The formulation "original" is misleading, better specify as "aquifer analogue" and "reconstructed tomograms".

Revised accordingly.

Figure 6, Caption, the same as in caption of Figure 5: Specify "3D distribution of hydraulic conductivity K: a)...b) reconstructed tomograms".

Revised accordingly.

Figure 7, Results are difficult to see due to figure size/visualization of results. Maybe chose different scale/range (e.g. broken y axis). Caption: Specify plot type as Histogram plot; state the total number of samples.

An additional scatter plot is added to this figure and the number of samples is stated.

Figure 8, Caption: Specify "injection temperature differences ΔT".

Revised accordingly.

Figure 9, Caption: Specify "injection rates Q".

Revised accordingly.

Figure 10, The caption description is not appropriate: Instead of generally stating what is seen an explanation of the figure construction is given. The figure shows the method performance with respect to the dimensionless parameters thermal Peclet number P et and effective injection power P 0 (state in word, not only using the abbreviations P 0 and P et), including the favourable application window. The explanations on how the figure was created and the other regions should be transferred to running text and omitted from the caption.

Revised accordingly.

Update reference of Doro et al., 2015

There are multiple typos as well as inconsistency in the use of upper and lower case letters in the references. Please check.

Revised accordingly.

Appendix - Transforming the transport equation into the eikonal equation

In the following, we present the mathematical procedure to transform the transport equation of a thermal tracer into the eikonal equation based on (Vasco and Datta-Gupta, 1999). First the solution of the transport equation is written as a series of wave functions. After neglecting the low frequency components, the transport equation is turned into the eikonal equation. Lastly, the travel time equation is presented as a solution to the eikonal problem. The procedure is presented on heat transport.

The **transport equation** of heat reads as follows (Stauffer et al., 2013):

$$\frac{\partial T(\boldsymbol{x},t)}{\partial t} = \nabla[D(\boldsymbol{x})\nabla T(\boldsymbol{x},t)] - \frac{C_w}{C_m}\nabla\big(\boldsymbol{q}T(\boldsymbol{x},t)\big) \qquad (1)$$

where $T(\boldsymbol{x},t)$ is the evolution of temperature distribution, $D(\boldsymbol{x})$ is the thermal diffusivity tensor, C_w and C_m are the heat capacity of the water and the aquifer matrix, $\boldsymbol{q}$ is the Darcy velocity and $\phi(\boldsymbol{x})$ is the porosity distribution. Assuming that $D(\boldsymbol{x})$ is a single scalar value and separating the velocity to a direction ($\boldsymbol{u}$) and a magnitude ($a(\boldsymbol{x})$) term, the equation simplifies to:

$$R_t\frac{\partial T(\boldsymbol{x},t)}{\partial t} = D\nabla^2 T(\boldsymbol{x},t) - a(\boldsymbol{x})\boldsymbol{u}\cdot\nabla T(\boldsymbol{x},t) \qquad (2)$$

where R_t is the thermal retardation coefficient. The solution to this equation can be formulated as a series of wave equations (Fatemi et al., 1995). Using the complex wave functions as an asymptotic expansion, the solution becomes:

$$T(\boldsymbol{x},t) = e^{i\omega\sigma(x,t)}\sum_{n=0}^{\infty}\tau_n(\boldsymbol{x},t)(i\omega)^{-n} \qquad (3)$$

where ω is the frequency and σ is the phase of the wave. Fast changes are represented in the initial terms of the series, and thus can be used ideally to describe tracer fronts. Keeping the first order terms and neglecting dispersion; after substitution Eq. (2) simplifies to:

$$R_t \tau_0(\boldsymbol{x},t)\sigma_t(\boldsymbol{x},t) = -\tau_0(\boldsymbol{x},t)[a(\boldsymbol{x})\boldsymbol{u}\nabla\sigma(\boldsymbol{x},t)] \tag{4}$$

This assumption is weaken if the dispersion is stronger. The equation for the thermal front, where $\tau_0(\boldsymbol{x},t) = 1$ reads:

$$R_t\ \sigma_t(\boldsymbol{x},t) = -[a(\boldsymbol{x})\boldsymbol{u}\nabla\sigma(\boldsymbol{x},t)] \tag{5}$$

Taking absolute values:

$$|R_t\ \sigma_t(\boldsymbol{x},t)| = |a(\boldsymbol{x})cos(\theta)||\nabla\sigma(\boldsymbol{x},t)| \tag{6}$$

where θ is the angle between the flow direction and $\nabla\sigma(\boldsymbol{x},t)$. By introducing $s(\boldsymbol{x}) = |a(\boldsymbol{x})cos(\theta)|^{-1}$ the velocity vector perpendicular to the tracer front, the Eq. (6) gives:

$$R_t\ s(\boldsymbol{x})|\sigma_t(\boldsymbol{x},t)| = |\nabla\sigma(\boldsymbol{x},t)| \tag{7}$$

Separating the temporal and spatial phase function the phase can be expressed as $\sigma(\boldsymbol{x},t) = \psi(\boldsymbol{x}) - t$ (Kline and Kay, 1965). After substitution and squaring Eq. (7) transforms to:

$$|\nabla\psi(\boldsymbol{x})|^2 = s^2(\boldsymbol{x})R_t^2 \tag{8}$$

where if we relate $s(\boldsymbol{x})$ to Darcy velocity:

$$s(\boldsymbol{x}) = \frac{\phi(\boldsymbol{x})}{R_t q} = \frac{\phi(\boldsymbol{x})}{(R_t K(\boldsymbol{x})|i(\boldsymbol{x})|)} \tag{9}$$

if the temperature gradient is perpendicular to the tracer front ($cos(\theta) = 1$). Equation 8 is known as the **eikonal equation** (Nolet, 1987). Solution methodologies for eikonal problems are available from seismic or electromagnetic wave propagation applications. $\psi(\boldsymbol{x}) = t$ describes the thermal front and because its gradient is parallel to the local transport direction, we can relate it to the transport trajectories:

$$\frac{dx_i}{dr} = \lambda \frac{\partial\psi(\boldsymbol{x})}{\partial x_i} \tag{10}$$

where r is the distance along the trajectory and λ is a scaling factor. The value of λ can be chosen arbitrarily, and if we choose $\lambda = s(\boldsymbol{x})^{-1}$, Eq. (10) returns the eikonal equation. With this substitution, Eq. (10) reads:

$$\nabla\psi(\boldsymbol{x}) = s(\boldsymbol{x})\frac{d\boldsymbol{x}}{dr} \tag{11}$$

Because $d\psi(\boldsymbol{x})$ is equal to dt, after integration, the total travel time of the thermal front along the trajectory can be written as:

$$t_{total} = \int dt = \int s(\boldsymbol{x})dr = \int \frac{\phi(\boldsymbol{x})}{(R_t\, K(\boldsymbol{x})|i(\boldsymbol{x})|)}dr \tag{12}$$

This is the **travel time equation** for a thermal tracer.

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