**Hydrology and
Earth System
Sciences
Discussions**

Interactive comment on "Kalman filters for assimilating near-surface observations in the Richards equation – Part 2: A dual filter approach for simultaneous retrieval of states and parameters" *by* H. Medina et al.

Anonymous Referee #2

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Kalman filters for assimilating near-surface observations in the Richards equation - Part 2: A dual filter approach for simultaneous retrieval of states and parameters

HESS, Medina et al. 2013

The paper discusses simultaneous state and parameter estimation in a synthetic setting, which would be an interesting topic for HESS. However, I have some concerns about the general conceptual validity and I would appreciate a more diligent finishing

of the paper.

1) General concern 1:

Why is the dual filter approach constructed with a SKF for the state update and a UKF for the parameters? I did read the authors' explanation on p.13332, lines 17-25, but this is not helpful.

Back to the drawing board:

- Non-linear state propagation model; (filter 1)

- Linear parameter propagation model; (filter 2)

- Non-linear or linear observation model; (applies to *both* filter 1 and 2)

Could you please explain this: why use a *linear* filter (SKF) to deal with a *non-linear* state model, and use a *non-linear* filter (UKF) to deal with a *linear* parameter model? This is *independent* of the observation model (see also below).

The UKF is in its core meant to deal with non-linearities in the propagation model, i.e. historically meant to get some good assessment of the dynamically evolving P-matrix, which seems to be somewhat understood by the authors in line 15-17. All 'special' Kalman filters (also the UKF) are really designed to "sample" the mean and error covariances of dynamically propagated variables that need to be updated. The "sampling" is mainly done because the calculation of the background (a priori) error covariance is cumbersome with non-linear propagation models. So, I don't see why the UKF would be more beneficial than the SKF for parameter estimation (constant propagation model).

Probably related to this problem, I could not follow section 5.4. re. the observation operator (btw, also check on inconsistent bold and italic H for non-linear H in this section). Line 15, p.13353: why think about inverting soil moisture to pressure? When assimilating soil moisture in the "\theta-h retrieving mode", one should convert the state h to soil

moisture observation predictions (with the forward observation operator), and calculate the soil moisture innovation: not invert soil moisture. The Kalman gain will transform the soil moisture innovations into pressure increments. Why consider a linearization of the observation operator here (p.13354, Line 1)?

2) General concern 2

Earlier publications have shown that simultaneous state and parameter estimation may not always be a good idea (Vrugt et al; Moradkhani et al) and in fact, the authors give a hint in the same direction by showing how Ks would not converge. The reason for these unsatisfactory parameter estimation results is the lack of 'observability' of the system to which the filter is applied. I would tone down the statement that the "dual filter is suitable for simultaneous retrieving of soil moisture ... and ... parameters" and instead recognize possible convergence issues. Another (secondary) issue is the underlying assumption of entirely independent state and parameter errors in a parallel dual formulation. Please mention this in the theoretical part and maybe provide a reflection on this. Unless these 2 issues are resolved (not for this paper; in general), I personally don't think that combined state and parameter estimation can be truly successful.

OTHER

1) Need correct mathematical expressions, and far more diligent explanations

a) The SKF is designed for linear systems with *additive* and *zero mean* noise.

Consequently:

- Eq.10-11: $R_{v0} = E[v0\ v0]$, etc. Also, what is the overline (as opposed to hat) in these equations? Please explain symbols when they are used for the first time.

- Eq. 12-13 are out of place for a SKF: v should be additive (while at is, turn the italic F into a linear matrix), and thus the second term in Eq. 13 should not have the $F.F^T$ around R_v.

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- Likewise: n should be additive for a SKF (take out in eq. 15, which BTW is an awkward mix of a linear H-matrix which receives non-linear arguments), and thus the $H.H^T$ around R_n in Eq. 14 are out of place.

b) Back to section 2.1:

Eq.1-2 + Eq.3-4: for clarity, maybe mention explicitly in the text that the posterior parameter estimate is passed into the Eq. 1-2 for state propagation and that the posterior state estimate is passed into the Eq. 4 to calculate the observation prediction. Line 7, p.13334: \hat-symbol does not show up between the quotes.

Eq. 7: the observation error covariance is missing in this equation? What is \tilde referring to? Line 19, p.13334 suggests that this $P_{\tilde{y}}$ would be the auto-covariance of the observation prediction error. Yet the observation predictions are $\hat{y}^-$ and do not include the observation error. Eq. 7 could only be right if $P_{\tilde{y}}$ is the innovation covariance, which is not mentioned as such, and needs explanation to be understood by an average reader.

Also note that the error covariances are meant to be *cross*-covariance (could include correlations between errors in different variables) and not *auto*-covariance. Along the same lines: throughout the entire paper, it is suggested that there are multi-dimensional observation (and observation prediction) error matrices, while I believe only scalar observations are assimilated. Either replace the matrices or state clear upfront that these are 1-dimensional entries.

c) Eq. 16: The Kalman filter *differs* very much from eq. 16. The Kalman filter includes in its very nature the information about the background or prior state (in this case: prior parameter estimate).

Line 5, p.13336: R_e: noise parameter covariance?? This should definitely be called something in observation space, *NOT* in parameter space. Yet, the parameter error covariance *should* show up in a second term that penalizes the background (the term

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that is forgotten in Eq. 16, as mentioned above). And, it will be artificial noise later in this paper, but up to this point, the artificial nature of the experiment is not mentioned in the state estimation, so please keep it general, as in previous sections.

Line 15, p.13337: R_r: this cannot be called innovation covariances (innovations are in obs space)?? R_r has to be in parameter space.

Eq. 24-29: the use of z in superscript, tildes and hats is not consistent, please check thoroughly.

2) P.13332: is the UKF still 'novel'? It is more than 10 years old?

3) Eq. 40 and eq. 41 should not use the same g-symbol for different things

4) Line 14, p13342, introduce acronym VGM before using it

5) P.13344: line 11: ... daily *precipitation* series

P.13344: line 18: not true: infiltration increases the state correlations – what you may want to say is that the correlations are made temporally variable

6) P. 13345: line 14: Montzka is the wrong reference for this. Please use Kerr et al 2010 if you refer to 3-day repeat for SMOS.

Line 28: I still understand that scalars are assimilated, so how could there be a diagonal matrix?

7) P. 13346 line 16: standard deviation across the profile?; say explicitly that N_{nod} = 27.

8) Please check the text thoroughly for grammar/spelling errors; e.g. L25, p13330, Soil water dynamic*s* *are*...; L 10, p. 13351: ...is able to f*i*nd...

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