MATLAB SCRIPT:

function StateFluxDemonstration

clc

% Time varying upper boundary condition:

t=0:100;

qT=zeros(size(t));

qT(t<25)=-1;

qT(t>50&t<75)=-1;

% Soil hydraulic properties:

thetaR=0;

thetaS=0.35;

alpha=-0.10;

n=1.4;

Ks=2;

neta=0.5;

% Numerical grid:

zN=500;

nz=100;

dz=zN/nz;

z=dz/2:dz:zN-dz/2;

% Plot hydraulic properties:

PlotHydProps(thetaR,thetaS,alpha,n,Ks,neta);

% Hydrostatic initial condition:

psi0=-z;

% Solve Richards equation using an ODE solver:

JPat=spdiags(ones(nz,3),[-1 0 1],nz,nz);

options=odeset('JPattern',JPat,'MaxStep',1);

[t,psi]=ode15s(@Richards,t,psi0,options,dz,nz,thetaR,thetaS,alpha,n,Ks,neta,t,qT);

% Get change in storage by integrating water content:

theta=thetaFun(psi,thetaR,thetaS,alpha,n,Ks,neta);

S=sum(theta\*dz,2);

S=S-S(1);

% Get recharge flux:

K=KFun((psi(:,1)+0)/2,thetaR,thetaS,alpha,n,Ks,neta);

rchg=K.\*((psi(:,1)-0)/(dz/2)+1);

figure

subplot(1,2,1)

hold on

plot(t,-qT,t,rchg,t,S)

legend('Influx at top of soil column','Discharge from base of soil column','Storage')

xlabel('Time [d]')

ylabel('Flux [m/d]/Storage [m]')

ylim([0 22])

box on

subplot(1,2,2)

plot(S,rchg)

xlabel('Storage [m]')

ylabel('Discharge flux [m/d]')

function [dpsidt]=Richards(t,psi,dz,nz,thetaR,thetaS,alpha,n,Ks,neta,tB,qT)

% Solve Richards' equation using a block centred solution

% with fluxes defined on each cell boundary

% Upper boundary flux:

q(nz+1,1)=interp1(tB,qT,t);

% Internal fluxes (based on Darcy-Buckingham Law):

Kn=KFun(psi,thetaR,thetaS,alpha,n,Ks,neta);

i=1:nz-1;

K(i+1,1)=(Kn(i+1)+Kn(i,1))/2;

dpsidz(i+1,1)=(psi(i+1)-psi(i))/dz+1;

q(i+1,1)=-K(i+1).\*dpsidz(i+1);

% Lower boundary (fixed water table) flux (based on Darcy-Buckingham Law):

KB=(Kn(1)+Ks)/2;

dpsidz(1)=(psi(1)-0)/dz\*2+1;

q(1)=-KB\*dpsidz(1);

%Apply continuity

dthetadt=-diff(q)/dz;

%Calculate specific capacity

C=CFun(psi,thetaR,thetaS,alpha,n,Ks,neta);

%Calculate pressure gradient:

dpsidt=dthetadt./C;

function th=thetaFun(psi,thetaR,thetaS,alpha,n,Ks,neta)

m=1-1/n;

Se=(1+abs(alpha\*psi).^n).^(-m);

Se(psi>0)=1;

th=thetaR+(thetaS-thetaR)\*Se;

function K=KFun(psi,thetaR,thetaS,alpha,n,Ks,neta)

m=1-1/n;

Se=(1+abs(alpha\*psi).^n).^(-m);

Se(psi>0)=1;

K=Ks\*Se.^neta;

function C=CFun(psi,thetaR,thetaS,alpha,n,Ks,neta)

m=1-1/n;

Se=(1+abs(alpha\*psi).^n).^(-m);

Se(psi>0)=1;

C=(thetaR-thetaS)\*(-m\*(1+abs(alpha\*psi).^n).^(-m-1)).\*(n\*abs(alpha).^n.\*abs(psi).^(n-1));

function PlotHydProps(thetaR,thetaS,alpha,n,Ks,neta);

psi=-logspace(-2,2);

th=thetaFun(psi,thetaR,thetaS,alpha,n,Ks,neta);

C=CFun(psi,thetaR,thetaS,alpha,n,Ks,neta);

K=KFun(psi,thetaR,thetaS,alpha,n,Ks,neta);

figure

subplot(3,1,1)

semilogx(psi,th,'linewidth',1.5);

ylabel('\theta (-)')

title('Hydraulic properties','fontsize',14)

subplot(3,1,2);

semilogx(psi,C,'linewidth',1.5);

ylabel('C (m^{-1})')

subplot(3,1,3);

semilogx(psi,K,'linewidth',1.5);

ylabel('K (m/d)')

xlabel('\psi (m)')