

## ***Interactive comment on “Kalman filters for assimilating near-surface observations in the Richards equation – Part 1: Retrieving state profiles with linear and nonlinear numerical schemes” by G. B. Chirico et al.***

**Anonymous Referee #2**

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Kalman filters for assimilating near-surface observations in the Richards equation: Part I: retrieving state profiles with linear and nonlinear numerical schemes.

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Summary: the paper repeats a synthetic assimilation study by Walker et al. 2001 with (i) different numerical methods to solve the Richards equations and (ii) different Kalman-filter-based assimilation schemes. The main goal is to evaluate different as-

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simulation techniques. Also discussed is the effect of non-linear observation operators in the assimilation of observations that are not directly mapped to the state variables.

There are several issues that require major revisions. Not the least of which is the lack of accuracy in the description of the assimilation techniques, which unfortunately puts this paper in a questionable corner and made reviewing rather tiresome. Also, the text could benefit from a revision of the English language in some parts.

1)– System description:

It would help if the system equations for each of the numerical schemes were written out upfront. That is, similar to Eq. 31 for the CN-scheme, please add the equations for the EX and NL schemes. Also, more explanation of the expected relative importance of the numerical scheme versus the chosen assimilation technique would be helpful. E.g.: if a forward solution converges faster with one numerical approach than another, then this should be indicated separately from the impact of assimilating.

2)– System representation and data assimilation:

please correct theory, maths, revise equations, description,... (the list below shows \*only\* obvious errors – please cross check)

P.13295

Eq. 2 and L20: Please correct the equation. What are the state system parameters  $w$  doing in the observation system? The parameters in the observation system should solely reflect the parameters of the observation operator. The model state  $x$  is itself already a function of state system parameters  $w$ .

L17: the forecasted state is a result of \*both\* the internal dynamics in  $F$  and the exogenous input  $u$ .

L24:  $Q_k$  reflects the model error variance, \*not\* the full uncertainty in the model predictions (the latter is  $P^-$ )

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P.13296

L7:  $p(u_k)$ : this  $u_k$  should probably read  $n_k$  for obs error.

L22:  $\hat{y} = E[]$ , remove the  $E[]$ , the observation predictions are straight deterministic forward simulations

Eq 4: No idea how this equation was obtained, but it cannot be right – the dimensions are wrong and the covariances do not make any sense. If you prefer a fancy equation different from a regular  $PH/[HPH+R]$ , please do provide a few lines to allow the reader to follow.

Eq. 5: This equation is also wrong in my eyes and I cannot trace back where it comes from: should it not be  $\hat{P} - K\hat{H}P^-$ ; please prove me wrong by giving a derivation. Besides, the corresponding Eq. for the EKF (Eq. 10) is right. . .

P.13298 Eq. 7-8 and 11-12: please correct: all derivatives are calculated at  $x_{k-1}$  (all 4 equations) and for the error term (either  $v$  or  $n$ ) set to 0 in the first of each pair of equations.

P.13299, L17, Eq.15:  $\hat{x}^a$  is an unfortunate choice as symbol for the augmented state. This  $\hat{x}^a$  is used as a symbol for the updated state ('analysis') in the DA community. Maybe choose another symbol?

P.13300: Eq. 19: second diagonal term should be  $Q_k$ , not  $R_v$ , for consistency

P13304, Eq. 13: why here  $\beta$ ?, P. 13310, Eq. 33: why here  $Q$ ? Please reserve  $Q$  for model error covariance.

P13310, Eq. 33 and 35: why is the time in superscripts, rather than subscripts? I thought that iterations are indicated in superscripts in this manuscript. What is  $f()$  in this equation 33?

P13306, L17 and Eq.13-14: the SKF is really designed with additive noise terms in both the state and observation system. Consequently, naming the SKF with a subscript

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SKF<sub>v</sub> is irrelevant: an SKF should not be applied with propagating errors through the dynamical state system (also, the <sub>v</sub> is not added everywhere, why? E.g. section 4.5). I suppose the real problem is that the SKF is not described per se in this paper. How about rewriting the section 2 on 'Kalman filtering' to describe the exact basic SKF instead of giving a general description?

Overall: unless I am mistaken, only scalar (one-dimensional) observations are assimilated in this paper. The use of boldface vectors for obs, and matrices for obs error covariances is thus not relevant. It may be an idea to simplify the notation to reflect the scalar nature of the obs.

3)– Kalman filters and other techniques

P.13293 L21: the Kalman filter is *not* a technique "to describe dynamic systems", but a technique to filter observations or to merge observations with dynamic systems. L23: the Kalman filter does *not* provide a prediction of the state system, but instead it provides an *analysis* (or posterior estimate or update). The system itself provides the prediction or forecast (or prior estimate).

P.13294 L2: EKF: 'but still widely used': where for example? Either insert a reference or delete. EK is still used in e.g. ECMWF-operations and Meteo-France, but in reality, all institutes have moved or are moving to EnKF. Also, P. 13297: I question if EKF is "undoubtedly the most widely used approach for dealing with nonlinearity".

L19: if it is important to think about DA techniques for operational settings, then it would be good to explain why the UKF is preferred in this study over the Ensemble KF (EnKF) or the particle filter (PF, admitted, the last one is no KF and may not fit in this paper): these are the most commonly used techniques in hydrologic DA. The UKF is not a commonly used KF-technique in hydrologic DA to deal with nonlinearities. What is the exact reasoning for trying the computationally more intensive UKF?

4)– Numerical experiment: Unless I missed it, please indicate in the text (not just in the

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table) how the synthetic observations are generated, both for the pressure heads and the soil moistures. I understand that the R (obs error variance) is defined in the table, but only for the pressure heads, not for soil moisture. Did the authors also perturb the observations themselves to generate them based on the truth? Or was the exact true value assimilated? It looks like almost perfect observations are assimilated, which is not too realistic.

5) P.13308, L7: ‘...propagated from the initial conditions’ → ‘...propagated from the initial \*uniform\* conditions’: were the profiles each propagated with their respective numerical scheme (CN, EX, NL)? See above comment: I would like to find out the relative effect of the numerical scheme versus that of the DA, i.e. the effect on convergence speeds.

6) Section 4.2, fig 1 and the subsequent figures: assimilation at different depths is shown in fig 1: please discuss in the text what you see and relate to Walker et al. 2001 in this paragraph. The next figures do not need the results for the different assimilation depths after having discussed them once in fig 1. Instead, the subsequent figures should merge information in the different panels for 1 assimilation depth, to focus on the differences caused by the different DA aspects or numerical schemes.

7) Section 4.3: could part of the success of the UKF be solely due to the iterative nature of the numerical scheme in the NL (as opposed to no iterations in the other schemes), rather than to the intrinsics of the UKF?

L21: LKF?? SKF instead??

8) Section 4.4: please indicate what is new here compared to what is already found in Walker et al. 2001?

9) Conclusions: that ‘general guideline’ is not generally useful as indicated later on: in general, we start with a model that has its own numerical scheme. Maybe think of a conclusion more in terms of suggestions for model development, rather than in terms

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of evaluating DA?

E.g. some examples on the first page where language could be improved (please move through the paper to clean up further)

- L2: ... assimilating observations in a ... → ... assimilating observations into a ...

- L15: retrieving algorithm → retrieval algorithm

- L18: The...filter reveals as the most... → The...filter appears to be the most...

- L21: This first sentence should be rewritten – the grammar is wrong. E.g. soil water dynamic\*s\*... \*are\*

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