Response to reviewer #1 comments

The authors would like to thank the reviewer for the meaningful comments on our study. As the reviewer points out some limitations in the paper, this will help us to improve comprehensibly our paper because it made us aware that some parts of the presentation of our methodology was omitted and the need to tailor the paper to hydro-meteorological view.

Response to reviewer #1 general comments. The paper is tailored to hydro-meteorological view by adding the following literatures; "Since the work of Hurst (1951) that detected the presence of long-term persistence in series of annual minima of the Nile River, a lot of studies have been carried out for testing and modelling long-memory in hydrological processes. For example, Montanari et al. (1997) studied the monthly and daily inflows of Lake Maggiore, Italy using fractional ARIMA model. Rao and Bhattacharya (1999) studied the memory of four hydrologic time series in Midwestern United States by testing the null hypothesis that there is only short-term memory in the series using a modified version of the rescaled range. The main conclusion from the study is that there is little evidence of long-term memory in monthly hydrologic series. However, for annual series, the evidence for lack of long-term memory is inconclusive, mainly because the number of observations is small and the power of the test based on modified rescaled range is low with small samples. Koutsoyiannis (2002) proposed a simple explanation of the Hurst phenomenon based on the fluctuation of a hydrological process upon different temporal scales. The stochastic process that was devised to represent the Hurst phenomenon, i.e. the fractional Gaussian noise, also studied. Based on its studied properties, three simple and fast methods to generate fractional Gaussian noise are proposed. Wang et al. (2005) analysed two daily stream flow series of the Yellow River in China, and found that both daily stream flow processes exhibit a strong long memory. Wang et al. (2007) applied four methods to the daily average discharge series recorded at 31 gauging stations with different drainage areas in eight river basins in Europe, Canada and USA to detect the existence of long-memory. The results show that 29 daily series exhibits long-memory as confirmed by three methods, whereas the other two series are indicated to have long-memory with two methods. Gil-Alana (2012) analysed the U.K. monthly rainfall data from a long-term persistence viewpoint using different modelling approaches, taking into account the strong dependence and the seasonality in the data. The results indicate that the most appropriate model is the one that presents cyclical long-run dependence with the order of integration being positive though small, and the cycles having a periodicity of about a year.

Other applications of long memory models to hydrological time series can be found in (Hosking, 1984; Koutsoyiannis, 2003; Koscielny-Bunde, 2006; Rybski et al. 2006; Mudelsee, 2007; e.t.c.).

Several authors made a reasonably good job in testing for long memory in hydro-meteorological time series; however, is it really a long memory process or a short memory with structural break? This is the main concern of this paper.

"Equation (5) is given as an alternative form for the expression of the logarithm of the spectral density in (p. 12277)". In this, we revised the methodology by assigning appropriate index of *j* and the reason for using the alternative for the expression of Eq. (5).

The equation of the spectral density given in line 6 of p. 12277 is corrected to:

$$f(\omega) = [2\sin(\omega/2)]^{-2d} f_u(\omega), \tag{4}$$

Where ω is the Fourier frequency, $f_u(\omega)$ is the spectral density corresponding to u_t and u_t is a stationary short memory noise with 0 mean. Consider the set of harmonic frequencies, $\omega_j = (2\pi_j / n)$, j = 0, 1, … … n/2, where n is the sample size. Taking the logarithm of Eq. (4) we have:

$$\ln f(\omega_j) = \ln f_u(0) - d\ln[4\sin^2(\omega_j/2)] \tag{5}$$

Eq. (5) can be re-written in an alternative form following (Wang et al. 2007) as:

$$\ln f(\omega_j) = \ln f_u(0) - d\ln[4\sin^2(\omega_j/2) + \ln\left[\frac{f_u(\omega_j)}{f_u(0)}\right] \tag{6}$$

The fractional differencing parameter *d* can be estimated by the regression equations constructed from (6). Using the periodogram estimate of $f(\omega_j)$, if the number of frequencies *m* used in Eq. (6) is a function *g(n)* (a positive integer) of the sample size n, where $m = g(n) = n^{\alpha}$ with $0 < \alpha < 1$, it can be demonstrated that the least squares estimate $\widehat{d}$ using the above regression is asymptotically normally distributed in large samples (Geweke & Porter-Hudak 1983).

$$\widehat{d} \sim N\left(d, \frac{\pi^2}{6\sum_{j=1}^{g(n)}(U_j - \bar{U})^2}\right)$$

Where $U_j = \ln[4\sin^2(\omega_j/2)]$ and $\bar{U}$ is the sample of U_j, j = 1 … … g(n).

Under the null hypothesis, of no long memory (*d* = 0), the t-statistic

$$t_{d=0} = \widehat{d}\left(\frac{\pi^2}{6\sum_{j=1}^{g(n)}(U_j - \bar{U})^2}\right)^{-1/2}$$

, has limiting standard normal distribution.

The value of the power factor α is the main determinant of the ordinates included in the regression. Traditionally the number of periodogram ordinates m is chosen from the interval $[T^{0.45}, T^{0.55}]$. However, Hurvich and Deo (1998) showed that the optimal m is of order O $(T^{0.8})$.

"The correlogram of first order differenced data not shown". Several plots were not shown in this paper, this is to reduce the bulkiness of the paper, and for example, the correlogram of the first difference series alone has 18 different plots and other plots like time plot, first difference series(s) plots etc. are having 9 plots each. These motivate us to reduce the number of plots to fewer factors. Nevertheless, the figure below shows the autocorrelation function of the first difference series of Kuantan and Kota Bahru. It is clearly shown from the figure that the autocorrelations at lag one exceeds -0.5 which is suggesting that taking the first difference is not appropriate to the series(s) under consideration.

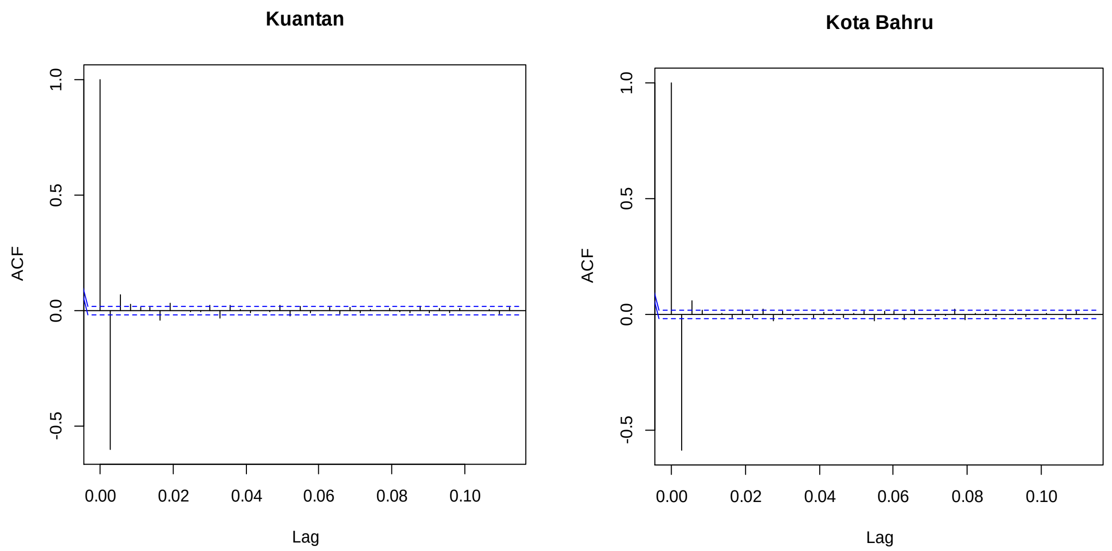

Figure 4 Autocorrelation functions of the first difference series

The reviewer also called our attention on the omitted CUSUM test for break detection which was not shown in the paper. Therefore we present the test as follows:

The CUSUM tests are concerned with testing against the alternative that an unknown coefficient vector varies over time (Zeileis, 2000). Ploberger and Kramer (1992) proposed a test based on the ordinary least squares residuals. The OLS-based CUSUM test uses the OLS residuals $\hat{u}_t = y_t - x_t^T \hat{\beta}$. The OLS-based CUSUM type empirical fluctuation process is defined as:

$$W_n^0(t) = \frac{1}{\hat{\sigma}\sqrt{n}} \sum_{i=1}^{[nt]} \hat{u}_i \,, \ (0 \le t \le 1)$$

Where, $\hat{\sigma} = \sqrt{\frac{1}{n-k}\sum_{t=1}^{n}\hat{u}_t^2}$, this path will always start in 0 at t = 0 and it also returns to 0 for t = 1, but if there is structural change at t_0 it should have a peak close to the break point t_0 . The null hypothesis H_0 is rejected if the path crosses either of the boundaries ($-\lambda, \lambda$) which is equivalent to rejecting when the test statistic

$$S^0 = \sup_{0 \le t \le 1} \left| W_n^0(t) \right|$$

is larger than λ which determines the significance level of the test. As $n \to \infty$,

$$W_n^0(t) \xrightarrow{d} B^0(t),$$

where $B^0(t)$ is the standard Brownian bridge.

The reviewer suggests for additional comments on how to detect over-differencing. The very simple way to detect over-differencing is to check if the first lag autocorrelation of the differenced series is -0.5 or more negative than -0.5, this is an indication that the series has been over-differenced. Another symptom of over-differencing is an increase in the standard deviation, rather than a reduction, when the order of differencing is increased.

"If detected breaks in the observations are real as opposed to spuriously generated by a long-memory process, this can be tested by comparing the statistical properties of the sub-series between the breaks". The authors tend to include the following as the outcome of the statistical properties of the sub-series comparison. "A time series which is generated by a true long memory process has a uniform data generating process (DGP) throughout the entire series. Thus if a structural break location method is mistakenly applied to the series it may report a number of breaks where no breaks exist. These spurious breaks will yield a number of partitions of differing lengths but this partition will only be subsamples of a single population. Thus the subsamples will have the same statistical properties as the full series. Table 2 (a-c) depicted a summary of the statistical properties for the full rainfall series, subseries before break and the subseries after the break. It could be observed from the tables that both series follows the same statistical patterns with standard deviation greater than the mean and high values for kurtosis and skewness.

Table 2a Statistics for the daily rainfall series

Station	Mean	Standard deviation	Skewness	Kurtosis
Alorsetar	5.54	12.76	4.05	24.06
Bayan Lepas	6.64	15.57	4.22	27.57
Ipoh	6.72	13.82	3.18	12.55
Kota baru	7.11	22.37	8.45	120.06

Kuantan	7.98	21.81	7.01	80.59
Malacca	5.45	12.96	4.37	32.69
Mersing	7.31	20.09	7.25	83.75
Sitiawan	4.83	11.76	4.13	23.49
Subang	6.70	13.84	3.45	16.41

Table 2b Statistics for the daily rainfall series before break

Station	Mean	Standard deviation	Skewness	Kurtosis
Alorsetar	6.25	13.92	3.91	25.15
Bayan Lepas	6.46	15.12	4.00	24.99
Ipoh	6.57	13.62	3.20	15.63
Kota baru	6.74	22.24	9.49	153.17
Kuantan	7.72	21.26	7.15	89.79
Malacca	5.21	12.84	5.29	58.96
Mersing	7.55	20.89	7.23	85.32
Sitiawan	4.61	11.56	4.42	31.14
Subang	6.13	12.89	3.34	19.08

Table 2c Statistics for the daily rainfall series after break

Station	Mean	Standard deviation	Skewness	Kurtosis
Alorsetar	5.31	12.35	4.09	27.55
Bayan Lepas	7.61	17.78	4.81	43.95
Ipoh	7.58	14.89	3.05	14.82
Kota baru	8.03	22.65	6.01	53.50
Kuantan	9.39	24.66	6.31	59.78
Malacca	5.59	13.04	3.85	22.98
Mersing	6.44	16.94	6.84	79.77
Sitiawan	5.08	11.99	3.83	21.70
Subang	7.38	14.89	3.40	18.86

Detailed comments

1. "If d is non-integer…

The authors revised the methodology of fractional integration as follows. "If a time series is non-stationary, one possibility for transforming the series into a stationary one is to take first differences of the series, such that:

$$(1-B)X_t = \mu_t \,, \quad t = 1, 2, \ldots \tag{2}$$

where B is the lag-operator ($B\,X_t = X_{t-1}$) and μ_t is I(0). In such a case, X_t is said to be integrated of order 1, denoted I(1). Likewise, if two differences are required, the series is integrated of order 2, denoted I(2). If the number of differences required to get I(0) stationary is not an integer value but a fractional one, the process is said to be fractionally integrated or I(d). Therefore, X_t is I(d) if

$$(1-B)^d X_t = \mu_t, \qquad t = 1, 2, \ldots \tag{3}$$

With μ_t equal to I(0). The expression in the left-hand-side in Eq. (3) can be presented in terms of Binomial expansion, such that, for all real d,

$$(1-B)^{\,d} = \sum_{r=0}^{d} \binom{d}{r} B^r (-1)^r$$

$$= 1 - dB + \frac{d(d-1)}{2} B^2 - \frac{d(d-1)(d-2)}{3!} B^3 + \ldots$$

Therefore, Eq. (3) can be written in the following form:

$$x_t = dx_{t-1} - \frac{d(d-1)}{2} x_{t-2} + \ldots + u_t \tag{4}$$

If d is a positive integer value, X_t will be a function of a finite number of past observations, while if d is not an integer, X_t depends strongly upon values of the time series far in the past (e.g., Granger and Ding, 1996; Dueker and Asea, 1998). Moreover, the higher the value of d, the higher will be the level of association between the observations (Gil-Alana, 2007).

The parameter d plays an important role from a statistical viewpoint. Thus, if $-0.5 < d < 0.5$, μ_t is a stationary and ergodic process with a bounded and positively valued spectrum at all frequencies. One important class of process occur when μ_t is *I(0)* and is covariance stationary. For $0 < d < ½$, the process exhibits long memory in the sense of Eq. (1), its autocorrelations are all positive and decay at a hyperbolic rate. For $-0.5 < d < 0$, the sum of absolute values of the process autocorrelations tends to constant, so that it has short memory according to Eq. (1). In this situation the ARFIMA (0, d, 0) process is said to be anti-persistent or to have intermediate memory and all its autocorrelations excluding lag zero are negative and decay hyperbolically to zero. As d increases beyond ½ and through 1 (the unit root case), X_t can be viewed as becoming "more nonstationary" in the sense, for example, that the variance of the partial sums increases in magnitude. This is also true for $d > 1$ (Gil-Alana, 2007).

2. "In the final paragraph of section 3", the fluctuation process and the F-statistics

Action taken:

In line 18 of p. 12280, "Figure 3" changed to Figure 5.

3. "The fluctuation process needs to be clearly defined".

   Action taken:

   The authors decided to remove the fluctuation process (Fig. (5)) and its content due to the fact that it is a repetition to the OLS-based CUSUM test for break dates presented in Table 4.

References

Gil-Alana, Luis A.: U.K. Rainfall Data: A Long-Term Persistence Approach. *J. Appl. Meteor. Climatol.*, **51**, 1904–1913, 2012.

Gil-Alana, Luis A.: Modelling Australian annual mean rainfall data: a new approach based on fractional integration. Australian Meteorological and Oceanographic Journal, 58:2, 120-128, 2009.

Hosking, J. R. M.: Modeling persistence in hydrological time series using fractional differencing, *Water Resour. Res.*, 20(12), 1898–1908, 1984, doi:10.1029/WR020i012p01898

Hurst, H. E.: Long term storage capacities of reservoirs. *Trans. Am. Soc. Civil Engrs* **116**, 776–808, 1951.

Koscielny-Bunde, E. Kantelhardt, J. W. Braun, P. Bunde, A. and S. Havlin, S.: Long-term persistence and multifractality of river runoff records: Detrended fluctuation studies, J. Hydrol., 322, 120– 137, 2006.

Koutsoyiannis, D.: The Hurst phenomenon and fractional Gaussian noise made easy, *Hydrological Sciences Journal*, 47 (4), 573–595, 2002.

Koutsoyiannis, D.: Climate change, the Hurst phenomenon, and hydrological statistics, *Hydrological Sciences Journal*, 48 (1), 3–24, 2003.

Montanari, A., R. Rosso, and M. S. Taqqu: Fractionally differenced ARIMA models applied to hydrologic time series: Identification, estimation, and simulation, *Water Resour. Res.*, 33(5), 1035–1044, 1997, doi:10.1029/97WR00043.

Mudelsee, M.: Long memory of rivers from spatial aggregation, *Water Resour. Res.*, 43, W01202, doi:10.1029/2006WR005721. 2007.

Rao, A. R. and Bhattacharya, D.: Hypothesis testing for long-term memory in hydrological series, J. Hydrol., 216(3–4), 183–196, 1999.

Rybski, D., Bunde, A., Havlin, S., and von Storch, H.: Long-term persistence in climate and the detection problem, Geophys. Res. Lett., 33, L06718, doi:10.1029/2005GL025591, 2006.

Wang, W., van Gelder, P. H. A. J. M., and Vrijling, J. K.: Long memory properties of streamflow processes of the Yellow River. Proceedings of the International Conference on Water Economics, Statistics and Finance, 1, Rethymno-Crete, Greece, 481–490, 8–10 July, 2005.

Wang, W., Van Gelder, P. H. A. J. M., Vrijling, J. K., and Chen, X.: Detecting long-memory: Monte Carlo simulations and application to daily streamflow processes, Hydrol. Earth Syst. Sci., 11, 851-862, doi:10.5194/hess-11-851-2007.

Ploberger, W. and Kramer, W.: The CUSUM test with OLS residuals. Econometrica, 60(2):271–285, 1992.

Zeileis, A.: *p* Values and Alternative Boundaries for CUSUM Tests. http://www.wu-wien.ac.at/am, 1-16, 2000 .