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Hydrology and Earth System Sciences Discussions

Interactive comment on "Thermodynamics, maximum power, and the dynamics of preferential river flow structures on continents" *by* A. Kleidon et al.

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1 Overview

The purpose of this note is to demonstrate that sediment transport from land to ocean results in the reduction of potential energy of continental crust material. To do so, we consider a simple configuration that is shown in Fig. 1 (The figure is attached to this comment as a supplement). A block of continental crust of length L_c with a density ρ_c rests within oceanic crust of a higher density ρ_o and length L. The vertical position of the block of continental crust is given by the vertical extent $\Delta z_c = \Delta z_{c,l}$ from a reference line (lower dashed line in Fig. 1a). The thickness of the oceanic crust is considered with regard to its vertical extent $\Delta z_l = \Delta z_{o,l}$ taken from the same reference line. The indices 'c' and 'o' refer to continental and oceanic crust, while the indices 'l' and 'g' refer to the local isostatic equilibrium shown in Fig. 1a and the global, stratigraphic equilibrium shown in Fig. 1c.

To show the reduction in potential energy due to lateral sediment transport, we consider the conservation of mass of the total mass of continental and oceanic crust, which set the constraints on the vertical extents, and the changes in potential energy within the system.

2 Mass balance constraints

We assume in this example that the mass of both, continental and oceanic crust, m_c and m_o, are being conserved.

The mass of continental crust, m_c, is given by the density ρ_c as well as the dimensions of the block. In the configuration shown in Fig. 1a, this mass is determined by:

$$m_c = \rho_c L_c \Delta z_{c,l} \tag{1}$$

where, for simplicity, we assume that the third dimension is included in the density ρ_c. For a given mass m_c, this translates into an expression for $\Delta z_{c,l}$ of

$$\Delta z_{c,l} = \frac{m_c}{\rho_c L_c} \tag{2}$$

Similarly, the mass of oceanic crust, m_o, is given by the density $\rho_o > \rho_c$ and the dimensions:

$$m_o = \rho_o (L - L_c) \Delta z_{o,l} \tag{3}$$

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For a given mass of oceanic crust, m_o, this yields the vertical extent of the crust, $\Delta z_{o,l}$:

$$\Delta z_{o,l} = \frac{m_o}{\rho_o(L - L_c)} \tag{4}$$

When continental crust is redistributed to the state shown in Fig. 1c, the mass of continental crust is given by:

$$m_c = \rho_c L \Delta z_{c,g} \tag{5}$$

and the vertical extent changes to $\Delta z_{c,g}$:

$$\Delta z_{c,g} = \frac{m_c}{\rho_c L} \tag{6}$$

Likewise, the mass of oceanic crust is given by

$$m_o = \rho_o L \Delta z_{o,g} \tag{7}$$

and the vertical extent changes to $\Delta z_{o,g}$:

$$\Delta z_{o,g} = \frac{m_o}{\rho_o L} \tag{8}$$

3 Potential energy in local, isostatic equilibrium

The potential energy of the configuration shown in Fig. 1a is given by the contributions by continental crust, $U_{pe,c,l}$, and by oceanic crust, $U_{pe,o,l}$. These contributions are given by:

$$U_{pe,c,l} = \int_0^{\Delta z_{c,l}} L_c \rho_c g z dz = \frac{L_c \rho_c g}{2} \Delta z_{c,l}^2 \tag{9}$$

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$$U_{pe,o,l} = \int_0^{\Delta z_{o,l}} (L - L_c)\rho_o g z dz = \frac{(L - L_c)\rho_o g}{2} \Delta z_{o,l}^2 \tag{10}$$

Using eqns. 2 and 4 to express $\Delta z_{c,l}$ and $\Delta z_{o,l}$ in terms of the masses m_c and m_o, the total potential energy is expressed by

$$U_{pe,tot,l} = U_{pe,o,l} + U_{pe,c,l} = \frac{g}{2L_c\rho_c} m_c^2 + \frac{g}{2(L - L_c)\rho_o} m_o^2 \tag{11}$$

4 Potential energy in global, stratigraphic equilibrium

The potential energy of the configuration shown in Fig. 1c is derived equivalently. The individual contributions by the continental and oceanic crust are given by:

$$U_{pe,c,g} = \int_0^{\Delta z_{c,g}} L\rho_c g z dz = \frac{L\rho_c g}{2} \left(\Delta z_{c,g}^2 - \Delta z_{o,g}^2\right) \tag{12}$$

and

$$U_{pe,o,g} = \int_0^{\Delta z_{o,g}} L\rho_o g z dz = \frac{L\rho_o g}{2} \Delta z_{o,g}^2 \tag{13}$$

Taken together, and using eqns. 6 and 8 as above to express $\Delta z_{c,g}$ and $\Delta z_{o,g}$ in terms of the masses m_c and m_o, we obtain:

$$\begin{aligned} U_{pe,tot,g} &= U_{pe,o,g} + U_{pe,c,g} \\ &= \frac{g}{2L\rho_c} m_c^2 + \frac{g}{2L\rho_o} m_o^2 \left(1 - \frac{\rho_c}{\rho_o}\right) \end{aligned} \tag{14}$$

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5 Difference in potential energy

We now consider the difference in potential energy, $\Delta U_{pe,tot}$, between the global equilibrium state shown in Fig. 1c to the local equilibrium state shown in Fig. 1a. We use eqns. 11 and 14 to get the following expression for $\Delta U_{pe,tot}$:

$$\begin{aligned}
\Delta U_{pe,tot} &= U_{pe,tot,g} - U_{pe,tot,l} \\
&= \frac{gm_c^2}{2\rho_c}\left(\frac{1}{L} - \frac{1}{L_c}\right) \\
&+ \frac{gm_o^2}{2\rho_o}\left(\frac{1}{L}\left(1 - \frac{\rho_c}{\rho_o}\right) - \frac{1}{L - L_c}\right) \\
&= -\frac{gm_c^2}{2\rho_c L_c}\left(1 - \frac{L_c}{L}\right) \\
&- \frac{gm_o^2}{2\rho_o(L - L_c)}\left(1 - \left(1 - \frac{L_c}{L}\right)\left(1 - \frac{\rho_c}{\rho_o}\right)\right)
\end{aligned} \tag{15}$$

What can be seen from eqn. 15 is that both terms are negative, that is, that the potential energy decreases from the state shown in Fig. 1a to Fig. 1c.

6 Summary

In summary, this note shows in relatively simple terms that the transition from a local isostatic equilibrium to a global stratigraphic equilibrium is accompanied by a reduction of potential energy in the overall system.

In principle, one could also show that the initial state of local isostatic equilibrium represents a state of minimum potential energy with respect to the vertical position of continental crust, $\Delta z_{c,j}$, and that the state of global equilibrium represents a state of

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minimum potential energy with respect to the horizontal extent of continental crust L_c. This would, however, require quite lengthly algebraic computations, which has been omitted here for reasons of brevity.

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