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Hydrology and Earth System Sciences Discussions

Interactive comment on "How extreme is extreme? An assessment of daily rainfall distribution tails" *by* S. M. Papalexiou et al.

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RC: Reviewer's comments
AC: Authors' comments

We honestly thank Francesco Laio for his useful comments.

RC1. *The Authors compare four probabilistic models, two with an heavy right tail (pareto and weibull) and two with exponentially decaying tails (lognormal and gamma). For each model, parameters are estimated based on the available sample, and a modified mean squared error (page 5763, eq. 3) is calculated to measure the distance between the hypothetical and empirical distribution function. The considered variable in the adopted error function (eq. 3) is the exceedance probability, which might be sen-*

*sible in a verification problem (e.g., determining the return period of a thunderstorm), but has some limitations in a design framework. In case of a design rainfall application, probably the best variable to be consider to judge the quality of a probabilistic model is rainfall itself; in fact, in design applications one fixes the probability level, and finds the design rainfall: as a consequence, discrepancies between data and models should be evaluated on the rainfall axis. I therefore believe the Authors should also consider in their analyses another (more standard) form of the error function, based on the squared differences between the empirical rainfall values, xi, and the corresponding design values x*i (one for each distribution), where x*i is found as the quantile corresponding to the probability level given by equation (2). The conclusions drawn about the better performances of the heavy-tailed distributions may be completely changed (or strongly supported) by using this other error function.*

AC1. To clarify, the ordering of the distributions, from heavier to lighter tail, is: Pareto, Lognormal, Weibull ($\gamma < 1$) and Gamma (for the various classes see e.g., Klüppelberg, 1989; Embrechts et al., 1997, pp.34–35; El Adlouni et al., 2008). Pareto is the only power type while the rest three are of exponential form. Note that the term "exponential tail" is used to describe the tail of the exponential distribution (in general a "heavier" tail than the exponential is characterized as subexponential and a "lighter" tail as hyperexponential or as superexponential). Lognormal is considered a heavy- or fat-tailed distribution (see e.g., Mitzenmacher, 2004); Weibull is sub-exponential for $\gamma < 1$ (see e.g., Goldie and Klüppelberg, 1998); Gamma is essentially exponential but not precisely (we will add an extensive note for the Gamma distribution to better clarify this issue).

Theoretically a different norm could potentially modify the conclusions, yet, this is not the issue. The issue is to use a norm with desired properties, or, the better one among candidates. We searched the literature and we did not find any study to verify that the norm the reviewer proposes is better than the one we use. If it is a matter of belief or a common practice, then this is not an argument against our approach. Indeed, the

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norm the reviewer is referring to is the most commonly used, but this does not prove its superiority.

For this reason, we implemented a Monte Carlo scheme, replicating the method we followed, that compares three different norms, i.e.,

$$\mathrm{N1} = \frac{1}{N} \sum_{i=n-N+1}^{n} (x_u - x_i)^2 \tag{1}$$

$$\mathrm{N2} = \frac{1}{N} \sum_{i=n-N+1}^{n} \left(\frac{x_u}{x_i} - 1 \right)^2 \tag{2}$$

$$\mathrm{N3} = \frac{1}{N} \sum_{i=n-N+1}^{n} \left(\frac{\bar{F}(x_i)}{\bar{F}_N(x_i)} - 1 \right)^2 \tag{3}$$

where $x_u = Q(u)$ and Q is the quantile function of the distribution under study and u the corresponding empirical probability of the x_i. The last (N3) is the one we used in the paper while the first (N1) is the one proposed by the reviewer, with N2 being a nondimensionalized version of N1. Our Monte Carlo scheme can be summarized in the following steps: (a) we generated 1000 random samples for each one of the four distributions we studied with sample size equal to 6600 values which is approximately the average number of nonzero daily rainfall values per record, (b) we selected the scale and shape parameter values approximately equal to the median values resulted from the analysis of the real world dataset (see Table 1 in the discussion manuscript) in order to be representative of the real data, and (c) we estimated the parameters of each distribution by applying our method, i.e., we minimized each of the three norms focusing only in the tail with *N* equal with 80, which is approximately the average number of years per record. The results are given in Fig. 1 and Fig. 2 of this reply. The whiskers of the box plots express the 95% CI of the parameters while the dashed lines

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show the true parameter values. It is clear that the norm N1 proposed by this and other reviewers (and indeed the one most commonly used in practice) is by far the worst and the norm N3 the best. The only exception is in the Gamma distribution, where N1 performs equally well as N3.

We especially thank the reviewer for this very useful comment which prompted us to make a more thorough study on the issue he posed. We will incorporate the above analysis in our revised manuscript and we hope the reviewer will agree that it suffices to justify our methodology. In any case, we believe the importance of this issue exceeds the scope of this paper and we hope to report further on this issue in the future.

RC2. *The Authors use in their comparison four models, each one with two parameters: using models with the same number of parameters is essential when comparing the performances of different models, because more parameterized models would be improperly favored by their higher adaptability in a comparison with more parsimonious models. A similar effect applies also when models have the same number of parameters, but different structure: one model can be improperly favored toward the others, except in very special cases (e.g., when all models belong to the position-scale family, or when the likelihood function is used to compare the model as in the Akaike Information Criterion or with the chi-squared test). Evidence for this effect is found, for example, in the fact that the acceptance limits for goodness-of-fit tests may be rather different in applications to testing different two-parameter distributions (with unknown parameters). A lower acceptance limit implies that the distribution of the test statistic (or, analogously, of the MSE norm as used in this paper) is shifted toward lower values under the null hypothesis (i.e., when the parent and hypothetical distribution are the same); this in turn implies that a distribution may tend to be favored toward another in a direct comparison, because, for example, the distribution has a greater adaptability due to the specific analytical form of the relation between the random variable and probability. One may be tempted to conclude that this more adaptable distribution is better than the others, but unfortunately a greater adaptability (lower estimation bias) frequently*

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entails more difficulty in parameter estimation (larger estimation variance). To summarize: the finding that heavy-tailed distributions have better performances may be an artifact related to the fact that heavy-tailed distributions have a power-law parameter, while exponentially decaying distributions have not. The presence of the power-law parameter may provide greater flexibility to the models, but on the other hand it may entail an increase in the estimation variance, which is also very important to be considered in design applications. To better support a claim about the superiority of the heavy-tailed distributions for use in engineering practice (last paragraph of the manuscript), this bias-variance tradeoff should be further explored in my opinion.

AC2. The distributions we compare here, clearly, are of similar structure: each comprises one scale parameter and one shape parameter. Among the various distributions with the same parameter structure, inevitably, some will be more flexible than the others, and one way to quantify this is by comparing the range of values of various shape measures (e.g., skewness, kurtosis, etc.) that they can describe. For example, the ranges of skewness for the Pareto, Lognormal, Weibull and Gamma, respectively, are $(2,\infty)$, $(0,\infty)$, $(-1.14,\infty)$ and $(0,\infty)$. So, according to the reviewer's argument the Weibull distribution would be the most adaptable and the Pareto the least. However, in reality the Pareto performs better. Additionally, and most importantly, this is not a study regarding the flexibility, or, the adaptability of distributions, but rather focuses only on the tails; the distributions under study here were chosen so that their tails are as simple as possible and representative of many other distributions.

Not all heavy-tailed distributions have a power-law parameter; only power-type distributions have. A power-law parameter does not provide greater flexibility to the tail of a power-type distribution than, e.g., the stretched-exponential parameter provides to the tail of the Weibull distribution. In each distribution studied here the "heaviness" of the tail is controlled by a single parameter; the only doubt regards the Gamma distribution where the tail behaviour is quickly dominated by the behaviour of the exponential function despite the fact that the distribution comprises a shape parameter. To summarize:

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four basic and simple types of tails are compared here and the analysis shows that in most of the cases the heavier tail performs better; this result is obtained, obviously, based on the methodology we followed and we described in detail, but we believe other possible methodologies would not contradict this result.

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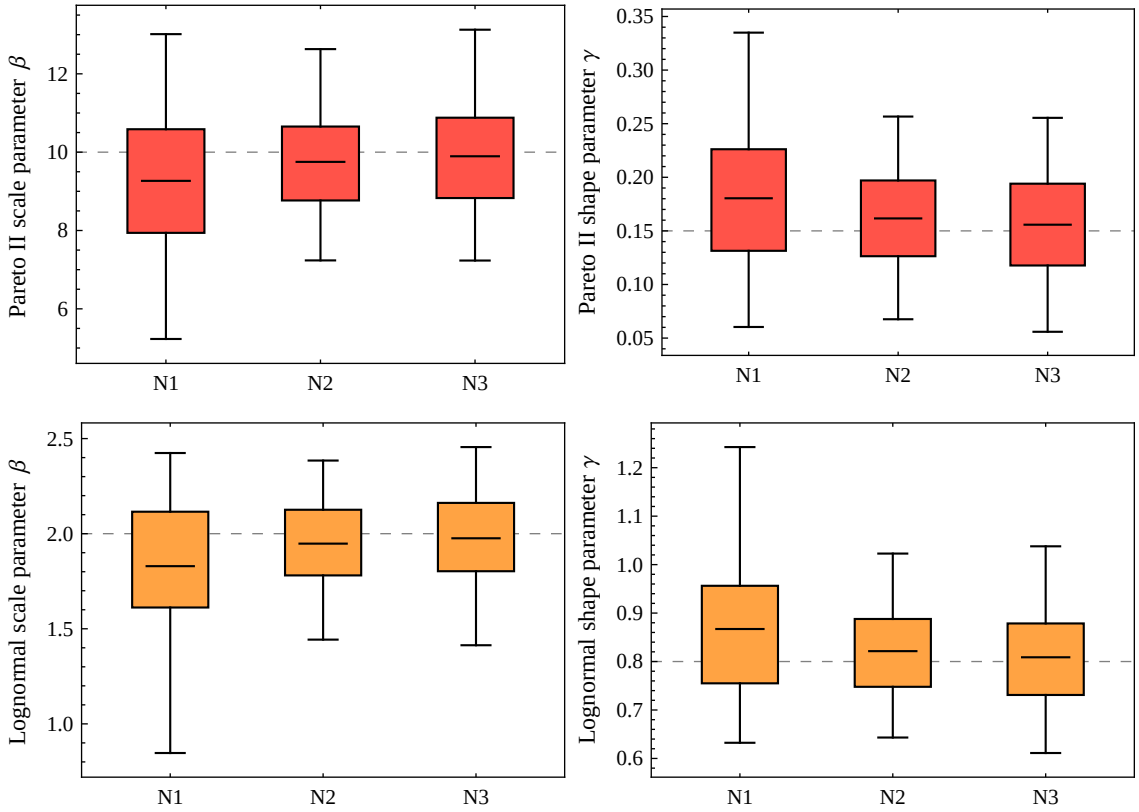

Fig. 1. Monte Carlo simulation results for the Pareto type II and the Lognormal distributions.

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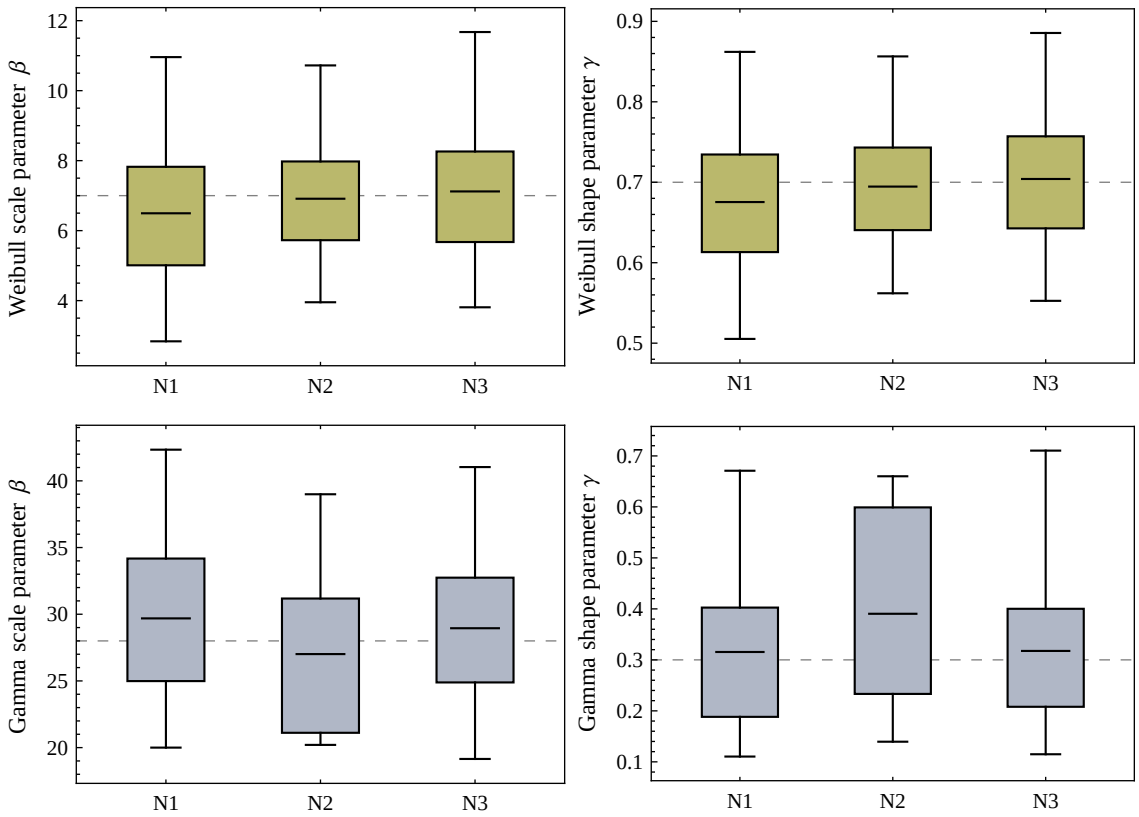

Fig. 2. Monte Carlo simulation results for the Weibull and the Gamma distributions.

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