

Responses to Specific Questions and Comments of Reviewer 2

1. We agree that the subscript t in the equations is confusing and not standard to indicate a dependence on time and removed the subscript in the equations. In addition we also simplified the subscripts for baseflow with “ B ” and Interflow with “ I ” (page 2129, line 6). We kept the t for tons although Mg would be the official metric unit but poorly understood by many. When introduced first we will explain the “ t ”.
2. The following responses relates to comments and questions relating to Table (2):
 - a. We agree that it is surprising that the half-life of the aquifer is twice as high for the relatively small Anjeni basin (@ 70 days) than for the great Blue Nile basin (@35 days). For other basins that we have tried the model and the half-life is much shorter (Bayabil 2009, Demisse, 2011 and Engda 2009). The springs in the Anjeni watershed are likely the cause of the long half-life for the baseflow. As indicated in the response to reviewer 1, half-life is directly derived from the hydrograph and we can only guess what the reason is. The interflow behaves as expected. For the short hill slopes of Anjeni, it takes only 10 day while for the Blue Nile with a km deep gorge for a major part of its travel to Sudan it is almost 5 months and includes the non-accounted water from Anjeni
 - b. We apologize for the misprints: S_{\max} in A_1 . The value in the table was correct.
 - c. $BS_{\max} = 100$ mm for Anjeni in the text was correct.
 - d. Thanks. This is corrected as well.
 - e. Indeed, the saturated areas for Anjeni and the Blue Nile are not equal and it was wrong in the text. Other watersheds that we tried (see thesis of Engda (2009), Bayabil (2009) and Demisse (2012) in <http://soilandwater.bee.cornell.edu/papers.htm>) have values much more equal to that of the Blue Nile). Anjeni has much deeper soils than the other watersheds and is the reason for the difference.
 - f. As discussed in the description of the watershed in the HESSD paper the soils are very deep and that could be the reason that the water flows under the gage. However,

in none of the head water watersheds that we tested our model, we could account for all the rain water. So it is likely that there are several types of regional subsurface flow paths. More research is needed on what they are.

g. The values of Q_B and Q_I for the sediment model were simulated by the hydrology model. In calibration the interflow can be recognized as the linear decrease in discharge after a large rainstorm. The base flow can be calibrated against the flow in the dry period after the interflow ceased.

3. We agree that the assumption, concentration was at the transport limit, is a strong assumption. It was made based on field observation that after agricultural fields are plowed, we observed that the water became very brown and without any calculation, that took it for granted that concentration should be at the transport limit.

First it is obvious that for the saturated area the concentration is not at the transport limit. Grass is growing in this area and it can and should not be at the transport limit. This is changed in the text. More research is here needed, because the area is small (2%) that any value for “ a ” *could* be used here.

For the degraded area, it is possible to calculate the transport limit. The value of “ a ” in table 2 for the source area is $3.2 \text{ (g l}^{-1}\text{)(mm day}^{-1}\text{)}^{-0.4}$. The value of “ a ” can be calculated from watershed characteristics as

$$a = \frac{F\sigma SL^{2/5}}{\left(\frac{\sigma}{\rho} - 1\right) \varphi_e} \left(\frac{\sqrt{S}}{n}\right)^{3/5}$$

where F is the fraction of the stream power effective in erosive processes, φ_e (m/s) is the effective sediment depositability. S (m/m) and L (m) is the slope and length of the respectively sediment generating area (terraces in this case), n is Manning’s coefficient of roughness and σ (kg/m^3) and ρ (kg/m^3) are soil particle and water density, respectively. By assuming that the terrace is 10 m long a slope of 1%, particle density of $2,500 \text{ kg/m}^3$, water density of $1,000 \text{ kg/m}^3$, $\varphi_e = 0.1 \text{ m/s}$; $n = 0.3$, a will be 3.4.

We hypothesize that the value for a for the Blue Nile in Table 2 of the HESSD P paper is smaller because there are forest in the Southern part of the basin that contributes sediment free water. It is obvious that we need to do more research what determines this “ a ” value. Nevertheless it is interesting that our simple approach works so well with the calibrated values and outperforming the much more complex WEPP model. Clearly as indicated before, we will need to do more research why.

4. We replaced Eqn. 4 on P.2130, line 14 with number 3. Thanks

References

- Demisse. B.A.: Discharge and Sediment Yield Modeling in Enkual Watershed, Lake Tana Region, Ethiopia. M.P.S Thesis. Cornell University, Ithaca, NY, USA, 2011.
- Engda, T.A. Modeling rainfall, runoff and soil loss relationships in the northeastern Highlands of Ethiopia, Andit Tid watershed. M.P.S Thesis. Cornell University, Ithaca, NY, USA, 2009.
- Legesse E.S.: Modeling Rainfall-Runoff relationships for the Anjeni watershed in the Blue Nile Basin M.P.S Thesis. Cornell University, Ithaca, NY, USA, 2009.

Addendum

“First order reservoir”, a “linear interflow reservoir”, and a “zero order reservoir”

The flux from an aquifer in general can be expressed as a function of flux from a reservoir (Brutsaert and Nieber, 1977)

$$\frac{dQ}{dt} = -aQ^b \quad (1)$$

Where Q is the flux (m^3/day), and a is constant. Usually groundwater outflow (springs) can be modeled as a “first order reservoir” with $b=1$, which is known as Maillet’s approach (Maillet, 1905). For details on the different usage of this type of reservoir equations in Hydrology see Rimmer and Hartmann (2012). The flow from hill slopes may be described as a “zero order reservoir” with $b=0$ in equation 1 (Steenhuis et al., 1999).

Both types of models are linear by their mathematical definition.

A “first order groundwater reservoir”

In a linear reservoir model, flux Q_{out} through the outlet is proportional with storage V .

$$V = KQ_{out} \quad (2)$$

K is known as the reservoir constant. The equation for the continuous water balance in this reservoir is:

$$\frac{dV(t)}{dt} = Q_{in}(t) - Q_{out}(t) \quad \text{st:} \quad Q(0) = Q_0 \quad (3)$$

And the linear reservoir equation is:

$$K \frac{dQ_{out}(t)}{dt} = Q_{in}(t) - Q_{out}(t) \quad ; \quad t_0 \geq t \geq 0 \quad (4)$$

If we assume $Q_{in}(t) = 0$, Equation 4 can be written in a different shape:

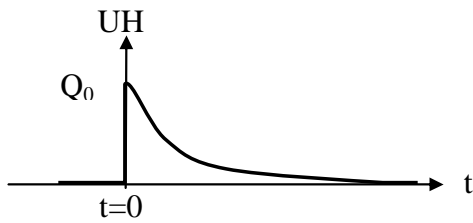
$$\frac{dQ_{out}(t)}{dt} = -\frac{Q_{out}(t)}{K} \quad ; \quad t_0 \geq t \geq 0 \quad (5)$$

Which is identical to Equation 1 with $a=1/K$ and $b=1$.

The solution to this problem is:

$$Q_{out}(t) = Q_0 \exp\left(-\frac{t}{K}\right) \quad (6)$$

This means that the reaction of the “first order groundwater reservoir” is exponential.



A “zero order groundwater reservoir”

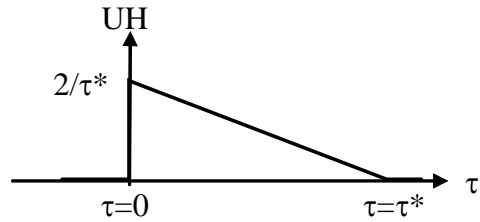
The flow from hill slopes may be described as a zero order reservoir with $b=0$ in equation 1 (Steenhuis et al., 1999). Here we look at the flux equation for a zero order reservoir, which for a single storm decreases linearly in time, i.e.:

$$\frac{dQ_t}{dt} = -a_0 \quad (2)$$

(Units of a_0 are $m^3 \times t^{-2}$). We can replace the time t in Eq. 2 with τ , defined as the time after the storm has occurred. If the added water to the hill slope is P_t^* the reaction of this type of reservoir is linear:

$$Q_t = UH \cdot P_t^* \quad ; \quad UH = \begin{cases} \frac{2}{\tau^{*2}} (\tau^* - \tau) & ; \quad 0 \leq \tau \leq \tau^* \\ 0 & ; \quad \tau \leq 0 \quad \text{and} \quad \tau \geq \tau^* \end{cases} \quad (6)$$

$$a_0 = \frac{2P_t^*}{\tau^{*2}}$$



References for the Addendum

Brutsaert, W., and Nieber J.L. 1977. Regionalized drought flow hydrographs from a mature glaciated plateau. *Water Resources Research* 13: 637-643

Maillet, E., 1905. *Essais d'hydraulique souterraine et fluviale*. In: Hermann, A. (Ed.), *Mécanique et Physique du Globe*, Paris.

Rimmer, A. and A. Hartmann. 2012. Simplified conceptual structures and analytical solutions for groundwater discharge using reservoir equations . Chapter 10 in *InTech Open Access book, "Water Resources Management and Modeling"*, ISBN 978-953-51-0246-5

Steenhuis T.S., Parlange J.-Y., Sanford W.E., Heilig A., Stagnitti F., Walter M.F. 1999. Can we distinguish Richards' and Boussinesq's equations for hillslopes?: The Coweeta experiment revisited. *Water Resources Research* 35: 589-593.