A few remarks on "Vandenberghe et al.: Joint return periods in hydrology: a critical and practical review focusing on synthetic design hydrograph estimation, Hydrol. Earth Syst. Sci. Discuss., 9, 6781-6828, doi:10.5194/hessd-9-6781-2012, 2012"

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The topic discussed in this paper is interesting; thus, I would like to take the opportunity offered by HESSD to share my personal opinion with the Authors and possibly contribute to the discussion. As stated in the title, the paper attempts to provide a critical (theoretical and practical) overview of the concepts related to the joint return period (RP). Based on the previous work by Vandenberghe et al [2011], the Authors are in good position to accomplish the task rather well; however, the manuscript under review seems to overlook to some extent the discussion reported Vandenberghe et al [2011], thus looking less detailed and formally correct. In my opinion, some sources of possible confusion are also introduced. In the following, I provide a few remarks on the topic.

1 Paying attention to data selection: physical relationships or MAX-SUM outcomes?

My first remark concerns the data selection and processing. I agree with the Authors when they stress the importance of a good copula fitting, but it is also worth mentioning the importance of an overall accurate inference on marginals along with a deep understanding of the theoretical and physical objects that one is dealing with.

In the context of a multivariate frequency analysis, the choice of the random variables used in the study is fundamental. Even though several papers consider Q_p, V_p and D as stochastically dependent variables, and, therefore, suitable to be modelled by multivariate distributions and copulas, actually the sampling procedure can introduce some subtle relationships that must be taken into account before performing the inference. In particular, a deterministic relationship emerges in Fig. 3 and 5 as a lower bound in the relationship between Q_p and V_p. Since $V_p = (Q_1 + ... + Q_p + ... + Q_n)\Delta t$ (where Δt is the time resolution properly scaled to obtain the required measure unit for the volume), it follows that $V_p = Q_p \Delta t + V_n$. As V_p cannot be smaller than $Q_p \Delta t$, this introduces a boundary condition that tends to be more prominent when the duration is short and $Q_p \Delta t$ is large compared to V_n. This aspect was already mentioned by Grimaldi and Serinaldi [2006] in a different context. Moreover, we can show that dependence structures similar to those shown in figure 3 of the manuscript can be obtained by a suitable combination of independent random variables with no relationships with physical processes. The toy model algorithm is as follows:

1. simulate N samples from an exponential distribution mimicking the hydrograph duration D. The values may be rounded to the first upper integer in order to obtain the discretization effect due to the time resolution;
2. for each simulated value of D_i, $i = 1, ..., N$, simulate a sample of length D_i from a skewed distribution defined in $(0, \infty)$, e.g., Weibull. These values mimic the discharges $Q_{t,i}$, with $t = 1, ..., D_i$ and $i = 1, ..., N$. Note that any temporal structure is introduced in the resulting pseudo-hydrographs and the parameters of the distributions were chosen to obtain skewed distribution with no link with physical variables;

3. select the maximum value for each pseudo-hydrograph $Q_{p,i} = \max\{Q_{1,i}, ..., Q_{D_i,i}\}$, for $i = 1, ..., N$;

4. compute the sum of the elements of each pseudo-hydrograph $V_{p,i} = \sum_{t=1}^{D_i} Q_{t,i}$ for $i = 1, ..., N$;

5. compute the rescaled ranks of $Q_{p,i}$, D_i and $V_{p,i}$ and draw the scatter plots.

The `R` code below does the job:

```
set.seed(666)
d <- ceiling(rexp(500, 0.07))
vp <- numeric()
qp <- numeric()

for(i in 1 : 500)  {
     q <- rweibull(d[i], 0.3, 2)
     vp[i] <- sum(q)
     qp[i] <- max(q)
}
res <- cbind(rank(qp), rank(d), rank(vp)) / (500 + 1)
colnames(res) <- c("Qp", "D", "Vp")
pairs(res)
```

Figure 1 shows that the algorithm is able to reproduce the key features of the scatter plots shown in figure 3 of the manuscript rather well, even though the distributions and parameters were chosen with no reference to the dataset analyzed by the Authors. Also the pairwise Kendall's correlation values are reproduced very well:

```
> cor(res,method="k")
          Qp         D        Vp
Qp 1.0000000 0.4814406 0.8662124
D  0.4814406 1.0000000 0.5777254
Vp 0.8662124 0.5777254 1.0000000
```

Playing with the distributions and their parameters, one can see that a variety of similar dependence structures can be obtained; however, the key point is that such structures only depend on SUM and MAX operators applied to independent random variables with suitable distributions irrespective of the internal structure of the pseudo-hydrographs. In other words, the driving process is the sum of random numbers $\sum Q_{t,i}$ over random durations D_i irrespective of the meaning of Q. Therefore, every copula that does not describe such a process as well as the emerging lower boundary of the (Q_p, V_p) relationships should be considered as just an approximation. In this sense, the vine copulas are not so superior to meta-Gaussian or other more or less exotic copulas that do not account for the generating mechanism. A brief discussion on the topic will be available soon in Serinaldi [2012, in press].

2 Intrinsic conditional sampling

This comment slightly extends a remark raised by a reviewer. For a multivariate frequency analysis focused on extreme events, a source of ambiguity is related to the method of selection of the variables used to describe the properties of a physical object, such as a hydrograph. The problem of selecting complex objects that are truly extreme is not of secondary importance. The problem was already recognized by Kao and Govindaraju [2007] who selected the extreme events as the events that exhibit the maximum joint probability of three variables X, Y and Z for each year. Based on the data selection method used by the Authors, only Q_p are extreme values (annual maxima), whereas the corresponding values of V_p and D are not extreme, or, at last, it is not guaranteed that they are annual maxima. Therefore, the good performance of the exponential distributions for V_p and D is reasonable and expected because the data are not truly extreme. The choice of the Weibull distribution for Q_p is coherent as well for this family is the penultimate approximation in the extreme value theory and often works well for annual maxima. Thus, the statement "In first instance, the marginal distribution functions of Q_p, V_p and D need to be estimated. As these variables are annual extreme values selected from the 500-year discharge series, the fit of several extreme value distributions is considered" is not strictly true from a conceptual point of view. As the extracted hydrographs are truly extreme only in terms of a single variable, Q_p, introducing the univariate marginal return period for the other quantities is computationally feasible, but theoretically questionable, because V_p and D are intrinsically conditioned on Q_p via the sample selection. This is the reason why Serinaldi and Grimaldi

[2011] mentioned that the dataset selection must be focused on the annual maxima of V_p (or D) when the focus of the analysis is on V_p (or D), thus accepting implicitly that the corresponding values of Q_p and D (Q_p and V_p, respectively), are not the most extreme values observed in the hydrograph dataset. Obviously, the data can be selected in several alternative ways that can be more or less elegant and effective; however, the chosen method must be kept in mind in order to interpret the result correctly. Moreover, the final aim of the project which one is working on must be the guide for choosing the method of selection and the type of return period of interest. The latter point is further discussed in the following, whereas a brief discussion on hydrograph sampling will be available soon in Strupczewski et al [2012, in press] and Serinaldi [2012, in press].

3 Comparing different return periods: statistics for engineering

Focusing on bivariate joint distributions and the corresponding RPs that can be derived from them, the literature on this subject is not so extensive, but Yue and Rasmussen [2002], Salvadori et al [2011] and Vandenberghe et al [2011] provided a rather good picture to start with in order to shed some light on the topic. In particular, Vandenberghe et al [2011] provided a comprehensive list of the state-of-the-art of the types of RP related to bivariate joint distributions by using a suitable notation that helps understanding the different meaning of each RP type. They also give an updated list of the mutual relationships (inequalities) that link the different RPs to each other.

In this context, I think that the paper under review introduces some confusion by merging concepts that were clearly distinguished in the previous paper, using an ambiguous notation, and missing the interpretation of the results in terms of theoretical relationships and engineering meaning, thus leading to misleading statements. In more details, Vandenberghe et al [2011] clearly recognized that, among the RPs derivable from a bivariate joint distribution, some of them are conditional return periods, whereas others are properly joint return periods. This important distinction is missed in the manuscript under review, which actually deals with only one type of conditional RP (MAR type in Vandenberghe et al [2011]) and two types of joint RPs (the OR type and the secondary RP, also known as Kendall's RP after

Salvadori et al [2011]). Therefore, the overview is partial, does not use the correct notation and does not consider the theoretical relationships detailed in Vandenberghe et al [2011], thus lacking a clear interpretation of the results. Using T to denote every type of return period does not help as well.

A separate remark must be devoted to the "joint RP based on regression analysis" introduced in section 3.1. This type of analysis was introduced by Serinaldi and Grimaldi [2011] and does not aim to provide any joint or marginal return period. When Serinaldi and Grimaldi [2011] thought about this methodology, the underlying idea was the following: as the selected sample is extreme only in terms of Q_p, it might be no strictly correct to assign a marginal return period to V_p and D because, as already discussed, they are not truly extreme (annual maximum) values and their values are intrinsically conditioned to Q_p (in light of the sampling procedure); therefore, the rationale was to provide a sound values of V_p and D by using simple approaches. Note that the small sample size also prevented more refined and possibly unreliable analyses. This derived values were denoted as the expected values corresponding to the value of Q_p for a given marginal return period of $Q_{p,T}$. This also explains why Serinaldi and Grimaldi [2011] referred to the derived variables as $E[V_p|Q_{p,T}]$ instead of e.g., $V_{p,T}$, which is incorrectly used in equation 6 of the manuscript. Moreover, it is worth noting that the expectation operator used by Serinaldi and Grimaldi [2011] must be interpreted in a broad sense; it refers to average values that can be obtained by a number of different techniques that do not provide necessarily the mathematical expectation. In the context of the copula framework, $E[V_p|Q_{p,T}]$ must be specialized as the expected value of the conditional distribution function described in section 3.2. Obviously, the comparison with the results provided by the conditional MAR RP (introduced in section 3.2) is rather trivial, as it is expected that the expectation of the conditional distribution is smaller than every (more or less) extreme quantile used in section 3.2. Thus, the results reported in table 3 can be promptly foreseen based on (1) the above discussion and (2) the chain of theoretical inequalities $T_{\mathrm{OR}} \leq \min T_X, T_Y \leq \max T_X, T_Y \leq T_{\mathrm{AND}} \leq T_{\mathrm{COND1}}$ reported by Yue and Rasmussen [2002] and Vandenberghe et al [2011].

Once recognized that the results are expected in light of the above mentioned relationships, and that the marginal, conditional and joint RPs can be always compared from numerical and statistical point of view, what really matters in hydrological engineering is the meaning of these quantities, their appropriate use and the related uncertainty. In my opinion, from a physical perspective, the information conveyed by the marginal, conditional and joint RPs cannot be compared because these distributions essentially provide answers to rather different physical problems. An example can help better understanding my point of view. In drought analyses, water managers are interested to assess, for instance, the probability of exceedance (or the RP) of the drought severity X given that an ongoing drought have been spanned Y months; in this context, the managers are interested to $P[X > x|Y \geq y]$, that is, the COND1 RP in Vandenberghe et al [2011]. It does not matter that $T_{\mathrm{OR}} \leq \min T_X, T_Y \leq \max T_X, T_Y \leq T_{\mathrm{AND}} \leq T_{\mathrm{COND1}}$ because the other RPs, or better, the other marginal, conditional and joint distributions do not provide the required information. Therefore, even though the above inequalities are analytically justified and allow for foreseeing and checking the correctness of the inference, I believe that from engineering perspective the comparison is unfeasible and substantially ill-posed because the different underlying probabilities refer to different physical conditions, design requirements and policies. A COND1 scenario can lead to an overestimation if the design requirements match with the OR condition and vice versa. Is one scenario more correct than the others? Say no. Which is the best one? Say all. All values are correct an might be used according to nature of the physical problem and the aim of the analysis. So, while I agree with the final recommendation that the practitioner should avoid a blind use of just one joint RP estimation method, on the other hand, I also think that the subsequent statement "Based on the available literature and the case study in this paper, the JRP method based on the Kendall distribution function is probably the most valuable in a multivariate context, when applied correctly" is a bit contradictory, does not synthesize the complexity of the topic, and gives a message that might be misleading: with no reference to a real world problem, we do not know what type of probability we are interested in (univariate, conditional or joint (and which type of conditional and joint probabilities)).

Eventually, it is worth mentioning the work by Ganguli and Reddy [2012] which shares several aspects with the paper under review; for instance, a rather detailed description of the inference procedures for multivariate analyses of the hydrograph properties (Q_p, V_p, D), and a discussion on joint and conditional multivariate joint return periods (OR, AND, Kendall, and joint-conditional) but using an appropriate notation that allows distinguishing the different cases. Unfortunately, also Ganguli and Reddy [2012] did not interpret the results in light of the above mentioned theoretical inequalities and limited their discussion referring to generic engineering problems that require multivariate approaches without specifying which RP type must be used for which problems.

4 Ensemble simulation and uncertainty

Section 6.2 focuses on the simulations along the isolines of the bivariate joint distribution related to the K_C RP. The same approach might be used in principle for the other joint RPs: for instance, simulating from the boundary lines corresponding to T_{OR} and T_{AND} can provide rather different scenarios that can be more extreme in terms of absolute values. The idea of considering the most probable event along the isolines could be discussed in more detail, as this choice contrasts to some extent with the rationale of the extreme events, or better, of the rare events: in other words, the most dangerous events should be the less probable among the less probable instead of the most probable among the less probable (whose definition is not unique in light of the different possibilities to define conditional and joint probabilities). The first class of events lies on the boundary of the clouds of data. In this sense, the concept of depth function that is behind diagnostic tools such as the bag plot, can provide useful information to identify the actual rare events and recognize the physical phenomena that generate them [e.g., Chebana and Ouarda, 2011a]. Moreover, it might be worth to set the discussion on the isolines in the context of the multivariate quantile curves discussed by Chebana and Ouarda [2009, 2011b].

As far as the uncertainty is of concern, it must be mentioned that the ensemble simulation proposed by the Authors does not allow taking into account the uncertainty, which can be classified as uncertainty related to data, inherent (or structural) uncertainty, and epistemic uncertainty [e.g., Montanari, 2011], the latter being the model and parameter uncertainty of marginals and copula. Loosely speaking, the uncertainty of the copula parameters leads commonly to stronger and weaker structures of dependence around the point estimates. In a bivariate case, this results in changes of the shape and curvature of the isolines. On the other hand, the marginal uncertainty commonly entails a shift of the isolines along the two axes. Both the sources of uncertainty must be carefully considered. In light of the high uncertainty that usually affects the univariate analysis of extreme values, it is likely even more prominent in a multivariate framework, and can be considered the main obstacle to the practical (effective and reliable) use of the quantiles resulting from a multivariate (extreme value) frequency analysis. In this context, the simulation of large samples via e.g. rainfall-runoff models can help only marginally because every model synthesizes the information contained in the original data, and the synthesis implies some loose of information, especially if one does not use a priori (general) knowledge but only the specific knowledge [Christakos, 2011] contained in the data, as in the present copula modelling. It follows that the simulations cannot contain more information than the original data as the inherent uncertainty is irreducible from an epistemological point of view [e.g., Popper, 1932]. The suggested ensemble scenarios do not describe any of the above mentioned types of uncertainty but only bivariate equi-probable quantiles that are similar to the point estimates in the univariate case: as the confidence intervals are one-dimensional objects that describe the uncertainty of the zero-dimensional univariate point estimates, the uncertainty of a one-dimensional object, such as the isolines of a bivariate distribution, is described by a two-dimensional object i.e., areas around the isolines (the same dimensionality ratio holds for higher dimensional distributions). In other words, exploring the isolines do not enable to incorporate any uncertainty, but only the (joint) equi-probable point estimates of an higher dimensional probability distribution. Without an appropriate evaluation of the epistemic uncertainty (at least), it is not possible to infer about the significance of the difference of two (multivariate) point estimates.

I think that the Authors should carefully reconsider the content of the manuscript. As several theoretical results on RPs are already provided by mathematicians and statisticians, a critical overview in hydrologic perspective cannot overlook the physical meaning of the variables and their accurate choice and interpretation as well as a thorough understanding, a detailed description, and a correct interpretation of the statistical tools in light of different real-world engineering problems in an integrative problem solving context [Christakos, 2011].

Editing note: Perhaps, sections 4.2 and 5.1 can be shortened and properly referenced or moved in an Appendix as they refer to already published materials that do not support the discussion on the RPs, which in turn must be extended. The inference procedure for copulas could be also shortened since quite a standard procedure is applied and vine copulas are already known and applied in hydrology [e.g., Gyasi-Agyei, 2011]. Some typos must be fixed throughout the text.

5 Some additional remarks on the concept of return period and its use: A (not so) subjective point of view

The ambiguity of definition and notation of T reported in the manuscript suggests to give the basic assumptions of the concept of RP careful consideration. This appears to be more and more important as multivariate/nonstationary frequency analyses become more and more widespread. The return period is usually preferred to the underlying values of the probability of exceedance as it seems to be (apparently) more friendly than the concept of probability. However, the experience tells us that this feeling is generally ill-posed and often leads to misleading statements. The policy makers are commonly the first victims of the tricky nature of the return period when they experience that extreme events with mid-high return periods, say e.g. 100-200 years, can occur (not) surprisingly more often that they think during their term. To avoid misleading statements some basic definitions must be kept in mind. Let we assume that a phenomenon is synthesized by a random variable X and we observe a realization of the phenomenon at fixed time intervals, say daily or annual time scales. Under the hypotheses that the phenomenon is stationary (the distribution function of X, $F_X(X < x)$, is independent of time) and each realization is independent from the previous ones (i.e., the realizations represent the outcomes of a series of independent experiments under the same (controlled) conditions), the return period T is defined as the expected value of the number of realizations (observed at fixed time steps) that one has to wait before observing an event whose magnitude exceeds a fixed value x_T. In spite of this rather simple definition, the analytical derivation relies on the summation of the power series expansion involving the probability of exceedance $P = 1 - F(X \leq x)$ [Chow et al, 1988, p. 382]:

$$
\begin{aligned}
\mathrm{E}[\tau] &= \sum_{\tau=1}^{\infty} \tau(1-P)^{\tau-1}P \\
&= P[1 + 2(1-P) + 3(1-P)^2 + 4(1-P)^3 + ...] \\
&= \frac{1}{P} = T
\end{aligned} \tag{1}
$$

where τ denotes the duration of the recurrence intervals between to exceedances. The measure unit of the final value of T can be easily set up by multiplying the numerator by the average number of occurrences in the desired time scale (e.g. the mean number of exceedances per year). As the recurrence intervals τ and their expectation T can be always expressed in years, the return period is deemed a rather friendly measure of the degree of rarity of an event, which, however, leads to statements such as "This event is expected to occur on average once each T years". This statement is formally correct but also possibly misleading because the underlying probability P actually provides another type of information: it tells us that there is a probability P to observe the so called T-year event "each year". A toy example can help clarify the point. Let us suppose that a gambler named Mr. Cat(astrophe) tosses an unbalanced coin each year (without reminding what he did the previous years), and the coin has a probability P for the tail (e.g., we are flooded... and lost!) and a probability $1-P$ for the head (i.e., we are not flooded... and safe!): when a structure/plan is designed with a T-year return period for X, engineers, decision makers and politicians accept to play the game N times, where N is the design life of the structure expressed in years, hoping that Mr. Cat is forgetful and regular in its habits (and the human activities do not change the environmental conditions). To account for this game, Chow et al [1988, p. 382] introduced the probability P_N to observe a T-year event at least once in N years:

$$
P_N = 1 - (1-P)^N = 1 - \left(1 - \frac{1}{T}\right)^N ; \tag{2}
$$

such a probability can be rather high and is $\approx 65\%$ when $N = T$. Thus, there is a rather high probability for a city major to be criticized more than once during his term, for instance, for the failures of a sewer system that has been designed with 5-, 10-year RP pipe diameters.

It is worth noting that the above remarks can be overlooked in other fields of research and industry. For example, in the insurance and reinsurance industry, the design life of a contract is usually one year, meaning that the insurers play with Mr. Cat only once (if they bet on annual maxima), then they reassess the fairness of the game and decide to play again or not (each year).

Therefore, even though the probability P seems to be less friendly than T, it provides a clearer picture of the phenomena under study (i.e., the occurrence of extreme values) and its iterative nature (i.e., the repetition of independent experiments along the time axis). On the other hand, the return period provides a derived variable that (1) does not add much information with respect to P, (2) does not allow for a direct computation of derived variables such as P_N (which has to be expressed again in terms of probabilities), and (3) can hide the actual meaning of the underlying (marginal, conditional or joint) probabilities when an ambiguous "T" symbol is used, thus leading to misleading comparisons of alternative design scenarios. The concept of return period shows definitely its nature and possible shortcomings when we move from stationary to nonstationary conditions and from a univariate to a multivariate framework. In a nonstationary context, Equations 1 and 2 become [Olsen et al, 1998; Sivapalan and Samuel, 2009]:

$$\mathrm{E}_t[\tau] = \sum_{\tau=1}^{\infty} \left\{ \tau \left(\prod_{i=t}^{\tau+t-2} [F_{X,i}(x)] \right) [1 - F_{X,\tau+1}(x)] \right\} \quad t = 1, 2, ... \tag{3}$$

where t denotes the start time, and

$$P_N = 1 - \prod_{i=1}^{N} [1 - P_i]; \tag{4}$$

where P_i is the probability that the annual maximum X is greater than or equal to x in any given year, under given climate/environmental state i. Equations 3 and 4 highlight that the return period results from a combination of locally stationary probabilities, thus introducing additional and unnecessary complexity in the representation of the probability of occurrence. The above remarks along with the possible confusion resulting from merging marginal, conditional and joint RPs should lead to reflect on the suitability of reasoning (and communicating results) in terms of probabilities instead of (easily misleading) RPs. In this direction, Theiling and Burant [2012] used for instance a better communication strategy by reporting both RPs and "annual" exceedance probabilities.

References

Chebana F, Ouarda TBMJ (2009) Index floodbased multivariate regional frequency analysis. Water Resour Res 45:W10,435, DOI 10.1029/2008WR007490

Chebana F, Ouarda TBMJ (2011a) Multivariate extreme value identification using depth functions. Environmetrics DOI 10.1002/env.1089

Chebana F, Ouarda TBMJ (2011b) Multivariate quantiles in hydrological frequency analysis. Environmetrics 22(1):63–78, DOI 10.1002/env.1027

Chow VT, Maidment DR, Mays LW (1988) Applied Hydrology. McGraw-Hill, New York

Christakos G (2011) Integrative Problem-Solving in a Time of Decadence. Springer, London, UK

Ganguli P, Reddy M (2012) Probabilistic assessment of flood risks using trivariate copulas. Theoretical and Applied Climatology DOI 10.1007/s00704-012-0664-4

Grimaldi S, Serinaldi F (2006) Design hyetograph analysis with 3-copula function. Hydrological Sciences Journal 51(2):223–238, DOI 10.1623/hysj.51.2.223

Gyasi-Agyei Y (2011) Copula-based daily rainfall disaggregation model. Water Resour Res 47:W07,535, DOI 10.1029/2011WR010519

Kao S, Govindaraju RS (2007) A bivariate frequency analysis of extreme rainfall with implications for design. Journal of Geophysical Research 112:D13,119, DOI 10.1029/2007JD008522

Montanari A (2011) Uncertainty of hydrological predictions. In: Wilderer P (ed) Treatise on Water Science, vol. 2, Oxford: Academic Press, Oxford, pp 459–478

Olsen JR, Lambert JH, Haimes YY (1998) Risk of extreme events under nonstationary conditions. Risk Analysis 18(4):497–510, DOI 10.1111/j.1539-6924.1998.tb00364.x

Popper K (1932) Die beiden Grundprobleme der Erkenntnistheorie (The Two Fundamental Problems of the Theory of Knowledge), 1st edn. Routledge

Salvadori G, De Michele C, Durante F (2011) On the return period and design in a multivariate framework. Hydrol Earth Syst Sci 15:3293–3305, DOI 10.5194/hess-15-3293-2011

Serinaldi F (2012, in press) Closure to "Synthetic Design Hydrographs Based on Distribution Functions with Finite Support by F. Serinaldi and S. Grimaldi, J. Hydrol. Eng. 16(5), 434-446.". Journal of Hydrologic Engineering DOI 10.1061/(asce)he.1943-5584.0000692

Serinaldi F, Grimaldi S (2011) Synthetic design hydrographs based on distribution functions with finite support. Journal of Hydrologic Engineering 16:434–446, DOI 10.1061/(ASCE)HE.1943-5584.0000339

Sivapalan M, Samuel JM (2009) Transcending limitations of stationarity and the return period: process-based approach to flood estimation and risk assessment. Hydrological Processes 23(11):1671–1675, DOI 10.1002/hyp.7292

Strupczewski WG, Bogdanowicz E, Kochanek K (2012, in press) Discussion on "Synthetic Design Hydrographs Based on Distribution Functions with Finite Support by F. Serinaldi and S. Grimaldi, J. Hydrol. Eng. 16(5), 434-446.". Journal of Hydrologic Engineering

Theiling CH, Burant JT (2012) Flood inundation mapping for integrated floodplain management: Upper Mississippi River system. River Research and Applications DOI 10.1002/rra.2583

Vandenberghe S, Verhoest NEC, Onof C, De Baets B (2011) Evaluation of Bartlett-Lewis rainfall models through a comparative copula-based bivariate frequency analysis of observed and simulated rainfall time series. Water Resour Res 47:W07,529, DOI 10.1029/2009WR008388

Yue S, Rasmussen P (2002) Bivariate frequency analysis: discussion of some useful concepts in hydrological application. Hydrol Process 16:2881–2898

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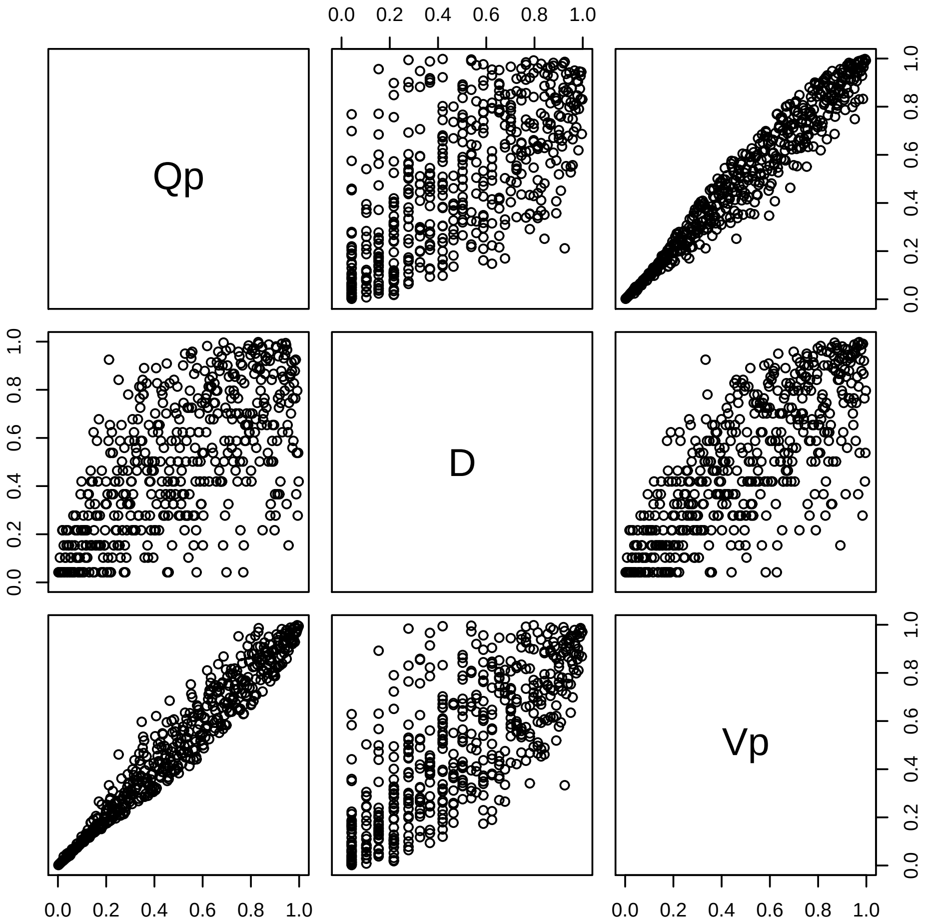

Figure 1: Pairwise scatter plots of the standardized rank for the three pairs of variables (Q_p, D), (Q_p, V_p) and (D, V_p)