

## ***Interactive comment on “Thermodynamics, maximum power, and the dynamics of preferential river flow structures on continents” by A. Kleidon et al.***

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We thank Hubert Savenije for the very thorough reading of the manuscript, for finding several typing mistakes as well as for the few errors in interpretation. Most of his points address minor suggestions, comments and pointing out errors in the equations that we will address in our reply and the revision of the manuscript. In the following, we focus on providing some clarifications on the derivation of Eqn. 4, as indicated in his review.

The first and second law of classical thermodynamics are typically expressed as:

$$dU = dQ - dW \quad (1)$$

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and

$$dS \geq 0 \quad (2)$$

where  $dU$  is the change in total energy of the system,  $dQ$  is the heat added to, or removed from the system,  $dW$  is the performed work done by the system, and  $dS = dQ/T$  is the change in entropy of the system. The change in entropy  $dS$  is formulated in terms of a small addition (or removal) of heat,  $dQ$ , for which a change in temperature  $T$  of the system can be neglected.

When we consider these changes in time  $t$ , we can rewrite eqns. 1 and 2 as

$$\frac{dU}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} \quad (3)$$

and

$$\frac{dS}{dt} \geq 0 \quad (4)$$

The term  $dQ/dt$  expresses the net heating or cooling rate of the system, and the term  $dW/dt$  expresses the work performed through time (or power  $P = dW/dt$ ) by the system. The total change in the entropy of the system in time,  $dS/dt$ , consists of the entropy produced by irreversible processes within the system,  $dS_i/dt = \sigma$ , and the entropy exchange across the system boundary,  $dS_e/dt = NEE$ , that is associated with the heating and cooling fluxes. Hence, we obtain an entropy budget of the system, expressed by (see also eqn. 2 in the manuscript):

$$\frac{dS}{dt} = \frac{dS_i}{dt} + \frac{dS_e}{dt} = \sigma + NEE \quad (5)$$

In the context of the entropy budget, the second law demands that the processes within the system follow the second law, that is,  $dS_i/dt = \sigma \geq 0$ .

We can now consider a steady state, in which the total energy  $U$  and the total entropy  $S$  of the system do not change in time, i.e.  $dU/dt = 0$  and  $dS/dt = 0$ . We then have

$$\frac{dU}{dt} = 0 = (J_{h,in} - J_{h,out}) - P \quad (6)$$

in which the net heating,  $dQ/dt$  is expressed by  $(J_{h,in} - J_{h,out})$ , and

$$\frac{dS}{dt} = 0 = \sigma + \left( \frac{J_{h,in}}{T_h} - \frac{J_{h,out}}{T_c} \right) \quad (7)$$

The latter term in the brackets in eqn. 7 expresses the net entropy exchange, that is, the addition of entropy to the system due to the addition of heat by  $J_{h,in}$  to the hot reservoir (with a fixed temperature  $T_h$ ), and the removal of entropy from the system by the removal of heat by  $J_{h,out}$  from the cold reservoir (with a fixed temperature  $T_c$ ).

Since the second law demands that  $\sigma \geq 0$ , we can use these two equations to constrain the power that can maximally be derived from the heating of the system in steady state. The first law, as expressed by eqn. 6, then states that (see also eqn. 3 in the manuscript):

$$P = J_{h,in} - J_{h,out} \quad (8)$$

The second law requires that  $\sigma \geq 0$ , which constrains the value of  $J_{h,out}$  in this expression. We can see this directly by rearranging eqn. 7 to

$$\frac{J_{h,out}}{T_c} - \frac{J_{h,in}}{T_h} = \sigma \geq 0 \quad (9)$$

which is the same as eqn. 4 of the manuscript (except for an erroneously missing sign in equation 4 of the manuscript, in which the left hand side of the equation should read  $-NEE$ ). When we consider the best case of  $\sigma = 0$ , we can use eqn. 9 to express  $J_{h,out}$  in terms of  $J_{h,in}$ ,  $T_h$  and  $T_c$ :

$$J_{h,out} = \frac{T_c}{T_h} J_{h,in} \quad (10)$$

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Combined with eqn. 8 we obtain the well-known expression for the Carnot limit:

$$P_{max} = J_{h,in} \frac{T_h - T_c}{T_h} \quad (11)$$

which is eqn. 5 of the manuscript.

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