# Bench scale laboratory tests to analyze non linear flow in fractured media

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**Abstract**

The knowledge of flow phenomena in fractured rocks is very important for groundwater resources management in hydrogeologic engineering.

A critical emerging issue for fractured aquifers is the validity of the Darcian-type “local cubic law” which assumes a linear relationship between flow rate and pressure gradient to accurately describe flow patterns.

Experimental data obtained under controlled conditions such as in a laboratory increase our understanding of the fundamental physics of fracture flow and allow to investigate the presence of non linear flow inside the fractures which generates a substantial deviation from Darcy law.

In this study the presence of non linear flow in a fractured rock formation has been analyzed at bench scale in laboratory tests. The effects of non linearity in flow have been investigated by analyzing hydraulic tests on artificially created fractured rock samples of parallelepiped (0.60×0.40×0.8 m) shape.

The volumes of water passing through different paths across the fractured sample for various hydraulic head differences have been measured, and the results of the experiments have been reported as flow rate/specific discharge vs. head gradient. The experimental results closely match the Forchheimer equation and describe a strong inertial regime. The results of the test have been interpreted by means of numerical simulations. For each pair of ports several steady-state simulations have been carried out varying the hydraulic head difference between inlet and outlet ports. The estimated linear and non linear Forchheimer coefficients have been correlated to each other and respectively to the tortuosity of the flow paths. A correlation among the linear and non linear Forchheimer coefficients is evident. Moreover, a tortuosity factor has been determined that influences flow dynamics.

1. Introduction

The aim of present work is to experimentally investigate the behavior of high velocity flow regime in fractured network at bench scale.

High velocity flow dynamics can have significant impact in diverse fields such as radioactive waste disposal, geothermal engineering, environmental risk assessment and remediation, reservoir engineering, and groundwater hydrology (Cherubini et al., 2010)

In most studies examining hydrodynamic processes in saturated porous and fractured media, it is assumed that flow is described by the Darcy’s law which expresses a linear relationship between pressure gradient and flow rate. Darcy law has been demonstrated to be valid at low flow regimes (Re <<1). For Re>1 a non linear flow behavior is likely to occur.

Ing & Xiaoyan (2002) have showed how non-Darcian flow has a significant impact on consolidation rate in geotechnics.

Basak & Rajagopalan (1982) have demonstrated that seawater intrusion length increases when the flow deviates from Darcian linearity.

Exposure and risk assessment of chemical pollution based on the applicability of Darcy’s law are sometimes inappropriate when applied to fractured aquifers because of the possible occurrence of non-laminar flow regimes. In fact, considerable evidence exists to refute their reliability in assessing solute-migration rates and for determining downgradient concentrations (Field, 1997).In solute transport, non-Darcian flow might give rise to tailing in contaminant’s breakthrough curves that show a non-Fickian behavior (Boutt et al, 2006, Cardenas et. al, 2007).

A non-Fickian behavior is also valid in case of high-concentration brine transport, where the assumption that the linear Darcy law holds as shown in Watson et. al.(2002). On the other hand, Tenchine & Gouze (2002) carried out density driven flow simulations in a rough walled natural fracture extracted from a limestone quarry and observed measurable non-linearity between the velocity growth rate and the velocity indicating that the N–S formulation must be used.

The mathematical representation of fluid dynamics in fractured rock aquifers is of a great concern for environmental and petroleum engineering and in geological sciences.

The local Cubic Law adapts Darcy Law to flow through fractures under the assumption of ideal fractures with flat and smooth and parallel walls and infinite lengths, together with laminar flow, incompressible fluid and confined aquifer configuration.

Real rock fractures instead, are characterized by rough walls, variable surfaces and geometry and apertures. The presence of asperities and obstructions or sharp changes in fracture profile is the reason for microscopic inertial phenomena that cause an extra macroscopic pressure loss which deviates flow from linearity.

Roughness has a large influence in fluid flow and transport through tight, rough- walled fractures, (Boutt et al, 2006) where non-Darcian flow is particularly easy to occur (e.g. Lomize, 1951; Louis, 1969; Qian et al., 2005, 2007, 2010; Wen et al., 2006).

Tortuosity, that usually characterizes the ratio of the effective path length connecting two locations in porous media to the geometric distance (Tenchine & Gouze (2002) has been found to affect significantly fluid flow in fractured media under certain conditions (Tsang Y.W. 1984, Wang & Narasimhan, 1985).

Yeo & Ge (2005) identified a criterion parameter function of the roughness and tortuosity for the applicability of the Reynolds equation to fluid flow in rock fractures.

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Flow regimes and non-linear behavior of fluid flow through fractures have been investigated, empirically (Lomize, 1951; Louis, 1969), experimentally (Witherspoon et al. 1980, Qian et al., 2005, Chin et al. 2009), and numerically (Zimmerman et al., 2004; Kolditz, 2001, Brush & Thomson, 2003).

Zimmerman et al. (2004) studied non-linear flow regimes both with experimental and numerical simulation of Navier Stokes equations and suggested the critical Reynolds number of 10 for practical purposes.

Witherspoon et al. (1980) conducted experiments in rock fractures to study the hydraulic behavior. They demonstrated that if fractures are rough or flow velocities are high, Darcy's law is not applicable.

Chin et al (2009) investigated the variation of effective hydraulic conductivity as a function of specific discharge in several 0.2-m and 0.3-m cubes of Key Largo Limestone. The experimental results closely match the Forchheimer equation.

Qian et al (2005) carried out laboratory experiments to study groundwater flow in a single fracture under different conditions of fracture aperture, surface roughness, and water pressure. The experimental results show that the average flow velocity (V) could be approximated by an empirical exponential function of the hydraulic gradient (I), which varies in the range of 0.003–0.02. The power index of the exponential function is close to 0.5. Such a V–I relationship indicates a non-Darcian turbulent flow even though the Reynolds number is relatively low.

Javadi et al. (2010) performed both laminar and turbulent flow simulations for a wide range of flow rates in an artificial three-dimensional fracture. They developed a new geometrical model for non-linear fluid flow through rough fractures, which suggests a polynomial expression, like Forchheimer law, to describe the dependence of pressure drop on flow rate. Finally, this model has been evaluated with experimental results of a fracture with different geometries. A good accuracy was found between the proposed model and turbulent flow simulation results.

Brush and Thomson (2003) carried out simulations of fluid flow through single rough-walled fractures with various apertures using Navier–Stokes, Stokes, and Local Cubic Law simulations. They observed that several Navier-Stokes velocity profiles have flatter peaks or noses that indicate the formation of an inertial core between the walls. They demonstrated that inertial forces can significantly influence the internal flow field within a fracture so that the forces driving the flow field are reduced and the overall flow rate is decreased. As the mean aperture increases, the effect of surface roughness diminishes (Boutt et al 2006).

In the present paper, several fracture network configurations are studied in order to observe non-linear behavior of the flow regime. The study is aimed at determining the relationship between the average velocity and the hydraulic gradient and will serve as first exploratory step of further investigation of solute transport in fracture systems under the non-Darcian flow.

1. Experimental setup
   1. Characterization and preparation of block sample

The experiments have been performed on a limestone block with parallelepiped shape (0.6×0.4×0.08 m3) recovered from the ‘Calcare di Altamura’ formation which is located in Apulia region in south-eastern part of Italy.

In Table 1 are returned the bulk hydraulic parameters of limestone block. The fracture network has been made artificially through 5 kg mallet blows. The fissured system and the fracture aperture on the block surfaces have been recorded with high resolution digital camera. Subsequently the images have been scaled and rectified using *Perspective Rectifier* ([www.rectifiersoft.com](http://www.rectifiersoft.com)) (Figure 1). Profiles of discontinuities and aperture measurements have been extracted from the recorded images using *edge* function with *‘canny’* filterbuilt-in *Scilab Image Processing Toolbox* ([www.scilab.org](http://www.scilab.org)). For each discontinuity, the median profile, aperture distribution and the fractal dimension using box counting method have been determined (Table 2).

The surface of block sample has been sealed with transparent epoxy resin (Leven et al., 2004). A hole of 1 cm diameter has been opened for each discontinuity in correspondence of the boundary of the block.

The spatial position of opened ports (Figure 2) and the illustration of construction details (Figure 3) have been reported.

* 1. Materials and methods

In Figure 4 is shown the diagram of experimental set up. The sealed block sample is connected with a hydraulic circuit. Water moves from the upstream to the downstream tank and returns to the upstream tank by means of a transfer pump. A flow cell is connected to the outlet port. The sealed block and the tubes of hydraulic circuits are completely saturated. Initially, the valves ‘*a*’ and ‘*b*’ are closed and the hydrostatic head in flow cell is equal to *h0*. The ultrasonic flow velocimeter measures the snapshot flow rates that enter the sealed block. The experiment begins with the opening of the valve ‘*a*’ and it is reclosed when the hydraulic head in the flow cell is equal to *h1*. Finally the hydraulic head in the flow cell is reported to *h0* through the opening of the valve ‘*b*’.

The average flow rate through the sealed block can be estimated by means of the volumetric method:



Where *S*1 [L2] represents the cross-sectional area of the flow cell. During the experiments, these values are compared with the snapshot flow rates measured by ultrasonic velocimeter in order to check the absence of hydraulic loss due obstructions and leaks in the hydraulic system.

The experiment is repeated changing the hydraulic head  of the upstream tank and for each configuration of inlet – outlet ports. For different values of  the time  required to fill the flow cell from  to  has been registered.

The storage property of the upstream tank (*S*2) is much higher than downstream flow cell () therefore during the experiment the upstream hydraulic head can be considered constant. Given that the sealed block sample and the hydraulic circuit are very rigid, their compressibility can be neglected.

On the basis of these assumptions the drainage process is governed by the following equation:



Where: [L] is the hydraulic head of the downstream flow cell; (L) is the hydraulic head of upstream tank;  (L2T-1) is the hydraulic conductance term (Harbough, 2005) representative of both hydraulic circuit and the active fracture network configuration.

Hydraulic loss within the single hydraulic circuit can be expressed according to Chezy law as:



Where *C* is a characteristic coefficient related to the roughness, section and length of the tubes of the hydraulic circuit.

Whereas, only for the sealed block, *Δh – Q* relationship can be represented through the following polynomial expression:



Where *A* and *B* are the linear and non-linear hydraulic loss coefficients respectively and are related to the roughness, aperture, lengths and shape of fractures.

Combining Equation (11) and Equation (12) the conductance term representative of the whole hydraulic system assumes the following expression:



Substituting Equation (13) in Equation (10) and integrating the latter from  to  with the initial condition  the following equation is obtained:



**Then, fitting experimental relationship between the time  and hydraulic head of downstream tank** **, an estimate of parameters *A*, *B* and *C* can be made.** Parameter *C* is estimated conducting the mentioned experiments without the sealed block (*A*=0; *B*=0).

1. ****Experimental results****

Several experiments have been conducted for each in–out port configuration. Control head *hc* varies in the range of 0.17 – 1.37 m and the average flow rates observes are the range of 3.08×10-7 – 2.99×10-5 m3/s. All the experiments carried out show a non-linear  relationship that can be well described by equation (12). Figure 5 shows the fitting method described in previous section to estimate the linear (*A*) and non-linear (*B*) terms. The double entry Table 3 show the estimated of *A* and *B* for each pair of ports.

If the experiments were carried out on a single fracture, *af* and *bf* coefficients of Forchheimer Equation (5) could be derived in analytical way from Equation (12). In the present case, in order to obtain Forchheimer terms for each path a numerical model has been implemented.

Starting from fracture profiles (figure 1) three-dimensional geometry of fissured system has been carried out using *GMSH* pre-processing tool (Geuzaine and Remacle, 2010). Geometry has been imported in *Comsol Multiphysics*® v 4.0a (Comsol AB, 2010) using STL exchange file format (Vinciguerra and Bernabè, 2010). Furthermore the geometry of the port holes has been modeled.

COMSOL uses the finite element scheme to solve generic partial differential equations. In particular “*Weak Form Boundary PDE Interfaces*” included in “*Mathematics Module*” has been used to solve the following continuity equation:



Figure (7a) shows the mesh of finite element model used for numerical simulations. *af* and *bf* are constant for the whole domain and represent the equivalent parameters for linear and non linear hydraulic losses.

The aim of numerical model is to estimate *af* and *bf* for each ports configuration. By means of Equation (9) an estimate of flow rate *Qobs* for steady-state condition can be obtained in correspondence of a hydraulic head difference imposed between the inlet and outlet ports. In a similar way through numerical simulations a relationship between hydraulic head difference imposed between inlet and outlet ports and the simulated flow rate *Qsim* in correspondence of outlet ports can be obtained varying *af* and *bf*. The idea is to find the equivalent parameters for each paths *af* and *bf* that minimize the difference between *Qobs* evaluated through Equation (9) and *Qsim* evaluated by numerical model for each imposed.hydraulic differences. The double entry table 4 shows *af* and *bf* estimatedfor each pair of ports.

1. Discussion

In order to analyze the experimental results two dimensionless numbers have been evaluated: the Reynolds number and the Forchheimer number.

Reynolds number is defined as the ratio of inertial forces to viscous forces:



Where  represents the average velocity evaluated on the active path and *Dh* represents the characteristic dimension. For fracture having small aperture with respect to its height, the characteristic dimension radius is equal to . Assuming that the average specific discharge is equal to  the Reynolds number becomes:



The Forchheimer number is the ratio of the non linear and linear pressure loss:



According to equation (8) the Forchheimer number can be reformulated as:



According to Zeng & Grigg (2006) the Forchheimer number is recommended as a criterion for identifying non-Darcy flow because it has the advantage of clear meaning. It equals the ratio of pressure drop caused by liquid-solid interactions to that by viscous resistance and it is directly related to the non-Darcy effect. Inertial effects dominate over viscous effects at the critical Forchheimer number (Fo > 1) (Ruth & Ma, 1992).

Reynolds number instead is a dimensionless number that indicates when microscopic inertial effects become important. It is inappropriate on the macroscopic level because microscopic inertial effects do not directly lead to macroscopic inertial effects. In fact, high microscopic Reynolds number does not necessarily imply non-Darcian flow. Instead Fo indicates precisely the onset of non-Darcian flow (Ruth & Ma, 1992): it accounts for both velocity (v) and structure of the medium because β is structure dependent. The term β inherently contains information on the tortuosity of the flow paths that leads to changes in the microscopic inertial terms. In fact, if the structure of the medium is such that microscopic inertial effects are rare, then *β* will be small and Fo will remain small until v (i.e. Re) is large. Instead, both *β* and Fo will be large if the structure of the medium is such that microscopic inertial effects can be expected.

In the Figure (8) the relationship between Re and Fo is graphed for each set of experiments. Under different ports configurations the Reynolds numbers are in the range 20 – 350 whereas the Forchheimer numbers are in the range 0.1 – 3.9.

The results showed in Figure (8) are consistent with the previous considerations. For example even if the paths (1-4 and 4-6) reach relatively high values of Re they present values of Fo lower than unity.

The slope of the straight lines represents the ratio between Re and Fo that is equal to the ratio of their respective characteristic dimensions:



This dimensionless group ζ is characteristic of the flow path. Relatively high values of this parameter correspond to a more linear flow behavior because the inertial effects dominate viscous ones at higher Reynolds numbers. Therefore it permits to distinguish a different behavior of the experiments carried out varying configurations of ports.

In particular way in Figure (8) it is possible to distinguish a set with more shallow slope. This set has in common the outlet port 7. In Figure (9) is showed the shape of fracture in correspondence of the port 7. The particular shape of this fracture gives rise to a greater contact between fracture surfaces compared to the others. In fact the path that contains this fracture presents a very high hydraulic loss.

For each path the equivalent aperture *weq* has been estimated from the linear term assuming the cubic law is valid:



This term has been compared with the average measured aperture of each path *w*(figure 9). Though *weq* underestimates *w*, they are of the same order of magnitude.

A power law has been observed between the terms *a* and *b* (figure 11):



This correlation between inertial and viscous coefficient is customary used in petroleum production engineering in order to predict high velocity well performance. Geertsma (1974) and Skjetne et al. (1999) found similar relationship for high velocity flow in porous media and fractures.

Zimmerman (1996) analyzed flow in two-dimensional rough walled rock fractures and found a factor equal to the ratio of the cubes of *weq* and *w* that reflects the tortuosity induced into the streamlines by the obstacles.

In the case of a single rock fracture this factor depends only on the area of the asperity region. Whereas in the case study it depends on several parameters such as roughness, the areas where the rock faces are in contact with each other, fracture intersection, position and shape of the inlet and outlet ports:



This parameter measures how much each path deviates from the parallel plate model.

The pressure drops depend by the morphology of the fracture wall surfaces and on the tortuosity of the flow paths. Significant head losses may be envisaged to occur adjacent to sharp corners of fracture where sudden change of aperture occurs (Javadi e. al 2010).

Figure (12) and Figure (13) show the *τ* - *b* and *τ* - *a* relationships respectively. *τ* results correlated with *b* and *a* by means of a power function.

1. Conclusions

In this paper non-linear fluid flow through rock fractures was studied by means of laboratory tests and numerical modeling.

The Forchheimer equation has been proved to explain reasonably the relationship between flow rate and pressure drop, which depicts a strong inertial regime where the viscous and inertial pressure drop are controlled respectively by *v* and *v*2 term.

The equivalent linear and non linear terms of Forchheimer’s law have been estimated by numerical modeling.

The equivalent aperture of each flow path is determined assuming the cubic law is valid. Though it underestimates the mean measured aperture it keeps its same order of magnitude.

A tortuosity factor *τ* has been determined as the ratio of the cube of the equivalent aperture and the cube of the mean measured aperture. This factor measures how the flow path deviates from the parallel plate model. In other words it measures the effects of different factors such as roughness, the contact area between fracture surfaces, fracture intersections and the position and the form of the inlet and outlet ports.

A power law between the Forchheimer terms and *τ* has been detected. In complex fracture networks the tortuosity factor plays an important role in fluid flow dynamics.

The experimental results showed that the dependence of hydraulic conductivity on specific discharge cannot be neglected in fractured media. For instance during pumping tests a linear flow model can cause errors in the determination of transmissivity in fractured rock aquifers because much of the data collected can be non-linear due to flow occurring in transition between linear and fully turbulent flow. On the other hand, potential errors induced by non-linear flow model in constant pressure tests have also been recognized in the engineering literature (Louis & Maini, 1970)

Elsworth and Doe (1986) used mathematical modeling of packer tests in fractured rock to show that calculation of transmissivity using non-Darcian constant head data can lead to underestimation errors as much as an order of magnitude.

In our study the effective hydraulic transmissivity proves to be less than 46.59% (average value) of the Darcian (linear) flow hydraulic transmissivity. In particular way in correspondence of path 3-4 the variation reaches the maximum value equal to 59.38%.

In pumping tests multiple pressure steps (i.e. higher flow rates resulting in a much greater differential pressure) should be used for a more accurate identification of the Darcian range and the quantification of the linear to nonlinear flow relations resulting in better transimmivity estimates as non-linear function of gradient.

This concept has to be taken into account in cases of anthropogenic stresses in the aquifer that might give rise to high hydraulic gradients.

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| --- | --- |
| Bulk density (g/cm2) | 2.21 |
| Porosity (%) | 0.20 |
| Moisture content (%) | 2.44 |
| Hydraulic conductivity (m/s) | 1.63e-8 |

Table 1. Properties of the limestone block.

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| --- | --- | --- | --- | --- |
| **Fracture number** | **Mean aperture**  **bm**  **(mm)** | **Aperture range**  **(mm)** | **Roughness**  ***ε***  **(mm)** | **Length**  **(mm)** |
| *1* | 1.496 | 0.811÷2.907 | 0.180 | 119 |
| *2* | 1.069 | 0.541÷1.977 | 0.187 | 88 |
| *3* | 1.076 | 0.573÷2.784 | 0.187 | 172 |
| *4* | 1.100 | 0.271÷3.900 | 0.101 | 171 |
| *5* | 1.040 | 0.811÷2.194 | 0.181 | 141 |
| *6* | 0.917 | 0.272÷1.794 | 0.141 | 107 |
| *7* | 0.906 | 0.540÷1.510 | 0.010 | 69 |
| *8* | 1.484 | 0.540÷3.097 | 0.150 | 62 |
| *9* | 0.703 | 0.540÷3.577 | 0.450 | 155 |
| *10* | 0.875 | 0.810÷1.300 | 0.063 | 65 |
| *11* | 1.075 | 0.270÷2.901 | 0.152 | 78 |
| *12* | 0.921 | 0.544÷1.912 | 0.135 | 246 |
| *13* | 1.130 | 0.541÷1.9097 | 0.096 | 258 |
| *14* | 1.576 | 0.344÷3.050 | 0.222 | 143 |

Table 2 Characteristic Parameters of the discontinuities.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
| **1** | − | 1.54e4 | 4.16e4 | 1.77e4 | 3.06e4 | 1.63e4 | 1.47e5 |
| **2** | 1.53e9 | − | 3.71e4 | 2.81e4 | 4.11e4 | 2.42e4 | 2.06e5 |
| **3** | 2.79e9 | 3.75e9 | − | 5.78e4 | 7.32e4 | 5.62e4 | 2.50e5 |
| **4** | 7.22e8 | 3.00e9 | 3.94e9 | − | 3.42e4 | 1.45e4 | 1.64e5 |
| **5** | 4.01e9 | 6.61e9 | 7.80e9 | 3.58e9 | − | 2.90e4 | 1.83e5 |
| **6** | 1.11Ee9 | 2.73e9 | 4.22e9 | 9.18e8 | 6.71e9 | − | 2.28e5 |
| **7** | 3.22e11 | 2.66e11 | 2.66e11 | 2.79e11 | 2.95e11 | 2.97e11 | − |

Table 3 Double entry table. Upper and lower triangular matrix represents the linear coefficient (A) and the non linear coefficient (B) respectively obtained for each pair of the ports.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
| **1** | − | 2.96e3 | 6.34e3 | 2.64e3 | 5.29e3 | 3.11e3 | 3.02e4 |
| **2** | 1.94e7 | − | 8.04e3 | 3.83e3 | 6.30e3 | 3.88e3 | 2.99e4 |
| **3** | 5.77e7 | 1.02e8 | − | 5.86e3 | 8.43e3 | 6.97e3 | 3.00e4 |
| **4** | 9.32e6 | 2.91e7 | 6.33e7 | − | 6.10e03 | 2.97e3 | 2.06e4 |
| **5** | 5.69e7 | 8.34e7 | 1.42e8 | 7.15e7 | − | 1.33e4 | 2.62e4 |
| **6** | 1.66e7 | 3.39e7 | 8.92e7 | 1.51e7 | 3.80e8 | − | 3.91e4 |
| **7** | 4.97e9 | 3.91e9 | 3.74e9 | 2.47e9 | 3.53e9 | 5.67e9 | − |

Table 4. Double entry table. Upper and lower triangular matrix represents the linear coefficient (*af*) and the non linear coefficient (*bf*) respectively obtained for each pair of the ports.

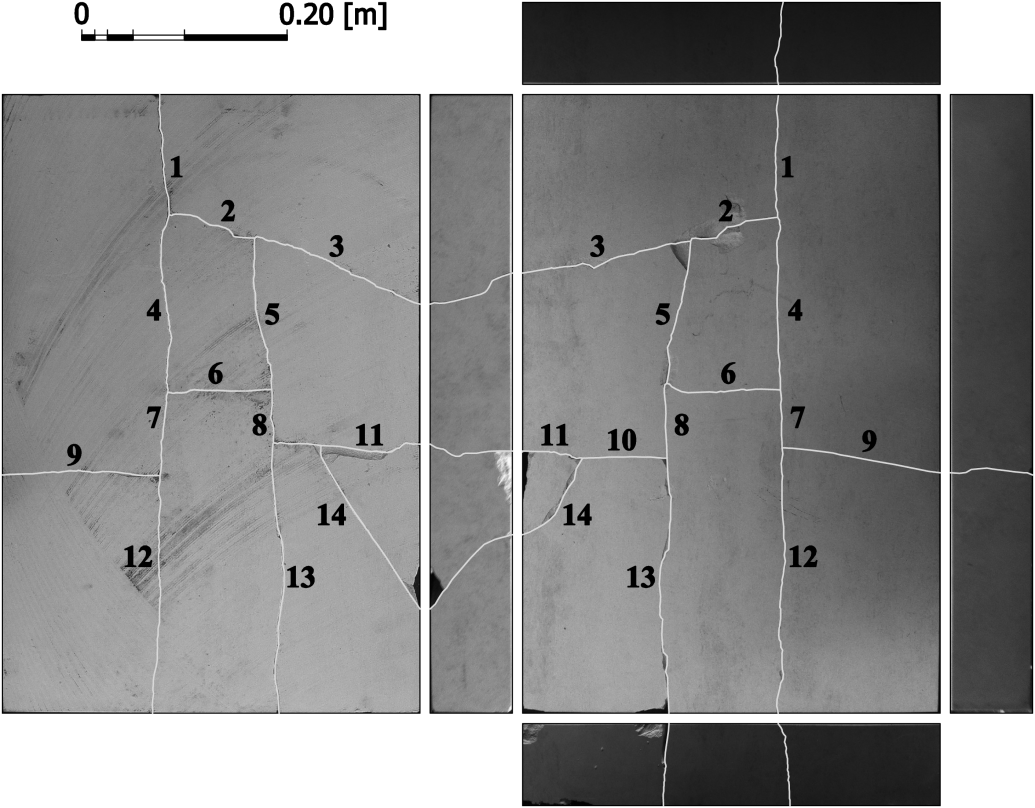


Figure 1. Block sample rectified images (0.6×0.4×0.08 m3). White traces indicate the median profiles.

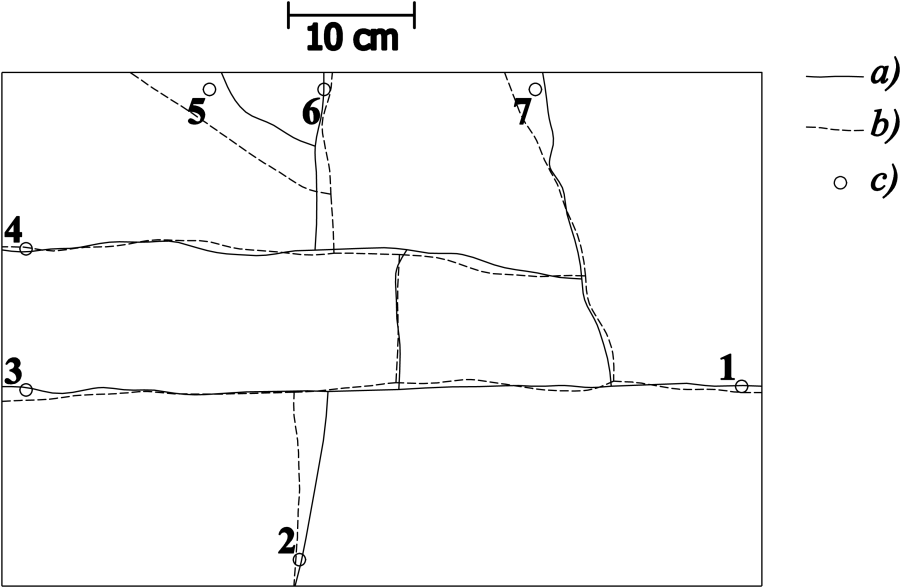


Figure 2. Localization of the ports on the top surface of block, a) top fracture traces b) bottom fracture traces c) ports.

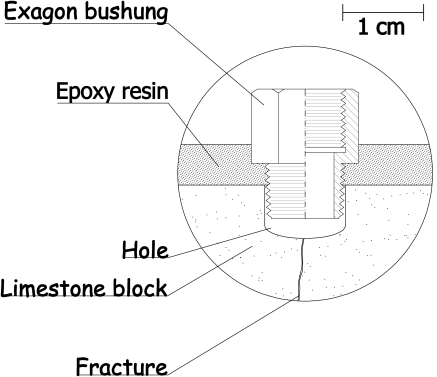


Figure 3. Illustration of a ‘port’ construction details

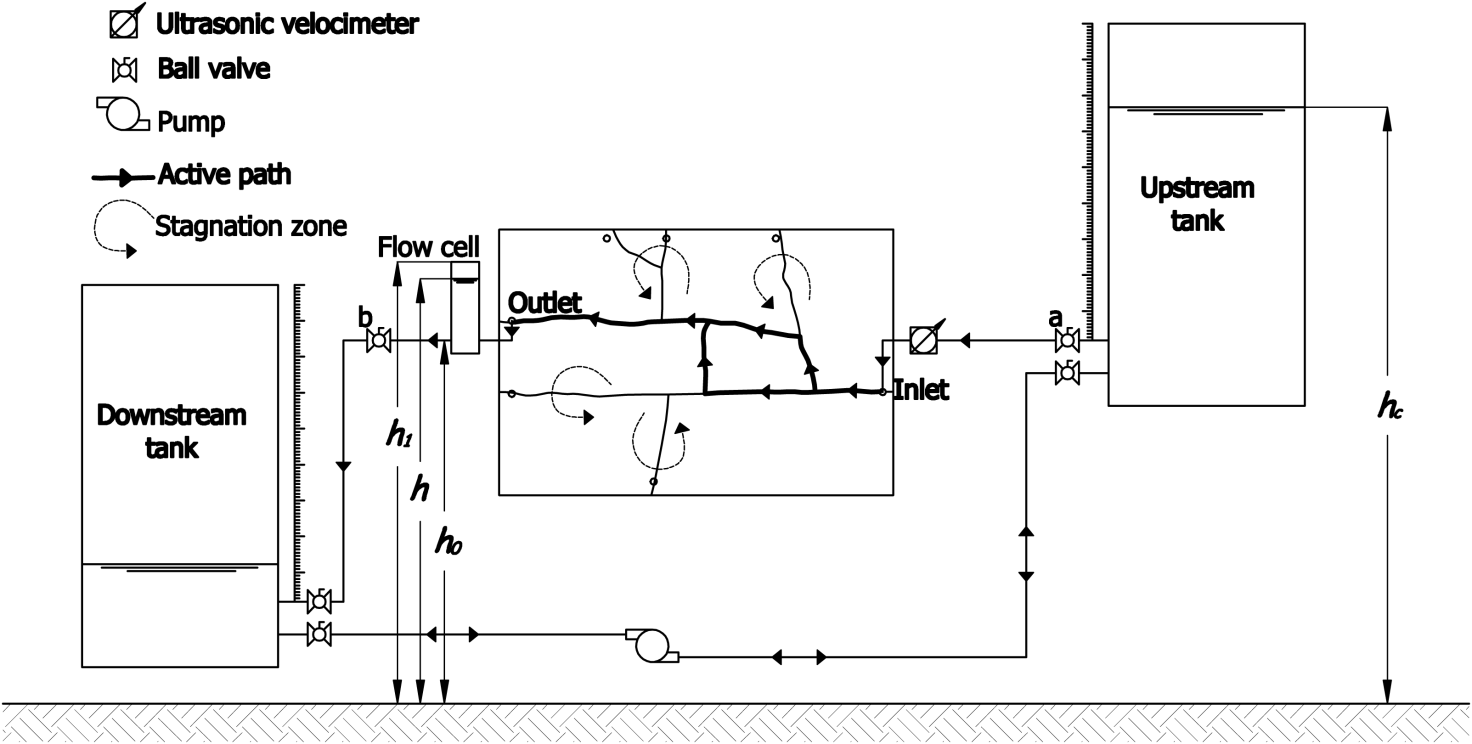


Figure 4. Schematic diagram of the experimental setup (picture is not to scale).

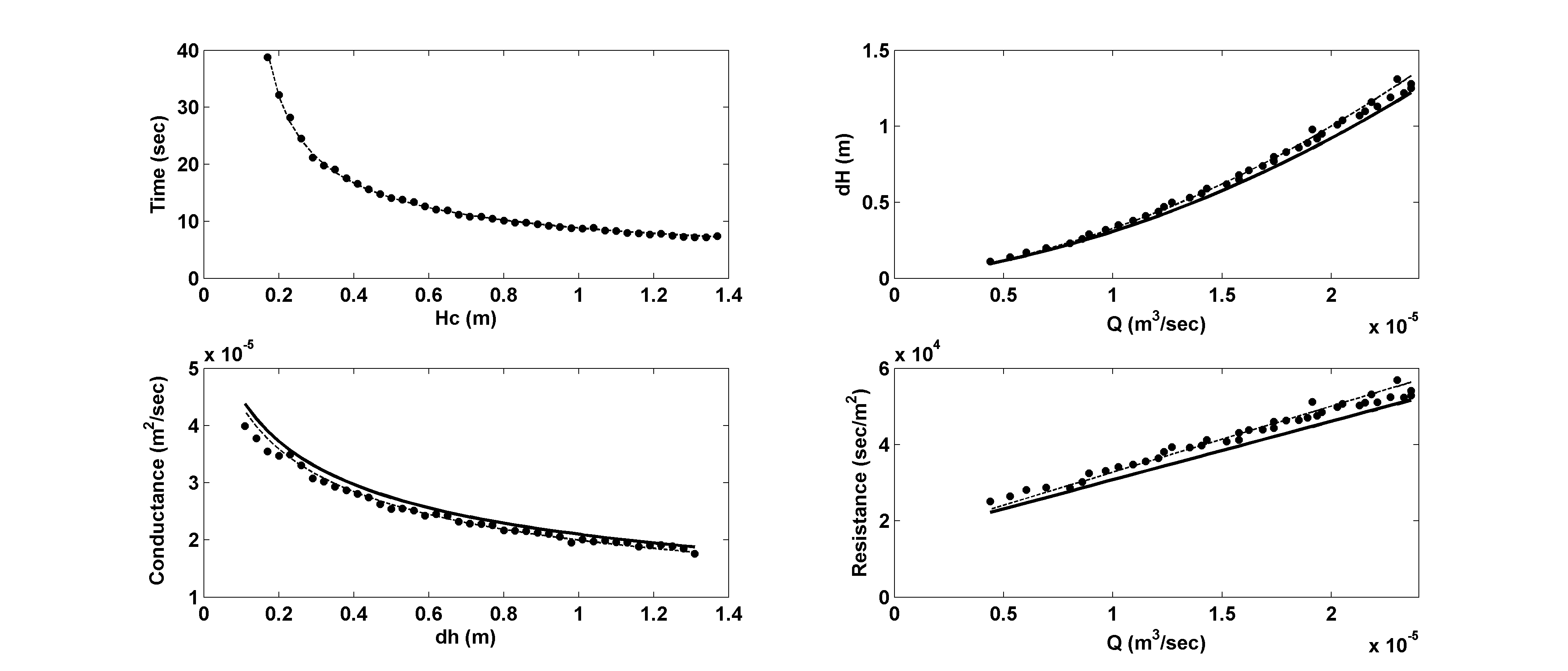


Figure 5. Experimental results obtained for 1 – 2 configuration ports. a) Control head Hc versus time. b) average flow rate *Q* versus difference head evaluated as dh= Hc+(h0 +h1)/2 c) difference head versus conductance term evaluated as Equation 13 d) average flow rate versus resistance term evaluated as the inverse of conductance. Dots represents the experimental values, dashed line represents the fitting of experimental values, marked line represents the functions without the effect of circuit (C=0).

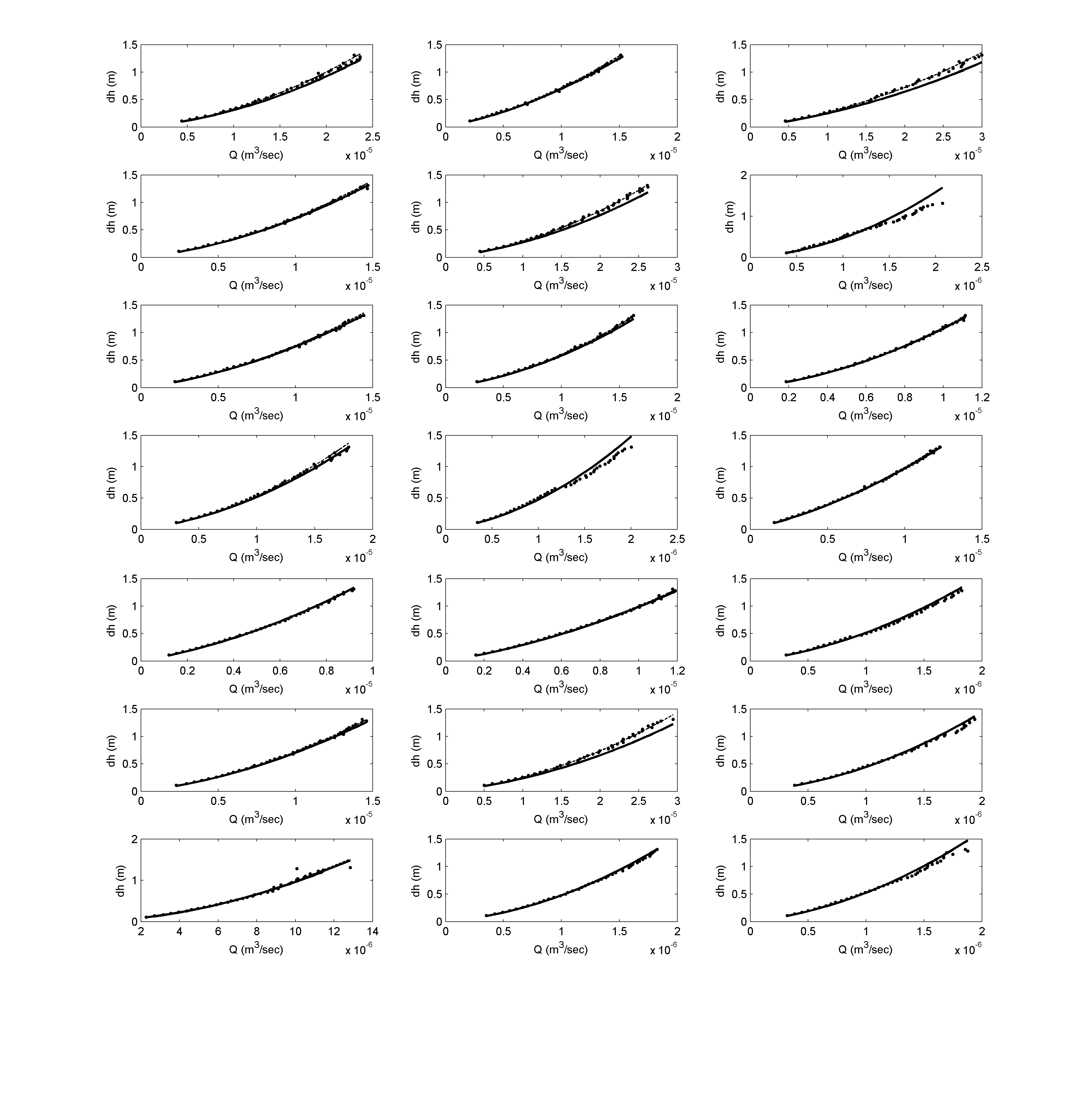


Figure 6 Flow rate *Q* versus hydraulic head difference dh for all the experiments. Dots represents the experimental values, dashed line represents the fitting of experimental values, marked line represents the relationship without the effect of circuit (C=0).

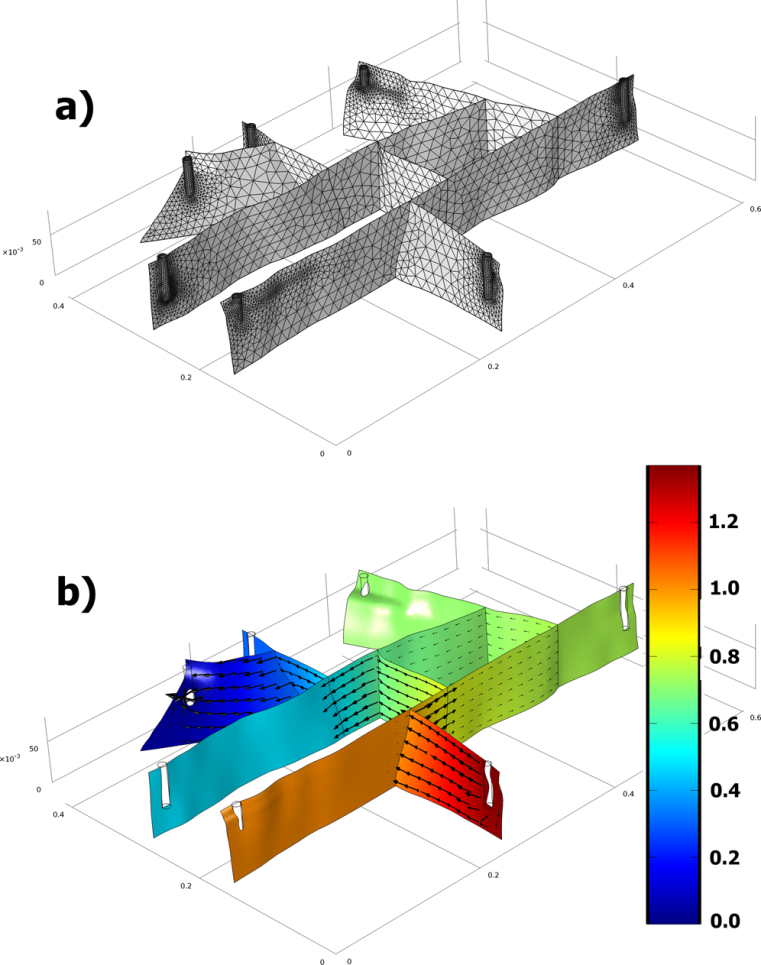


Figure 7 a) Finite element mesh of numerical model b) a result of a simulation, color scale indicates the hydraulic head, black arrow represents the specific discharge.

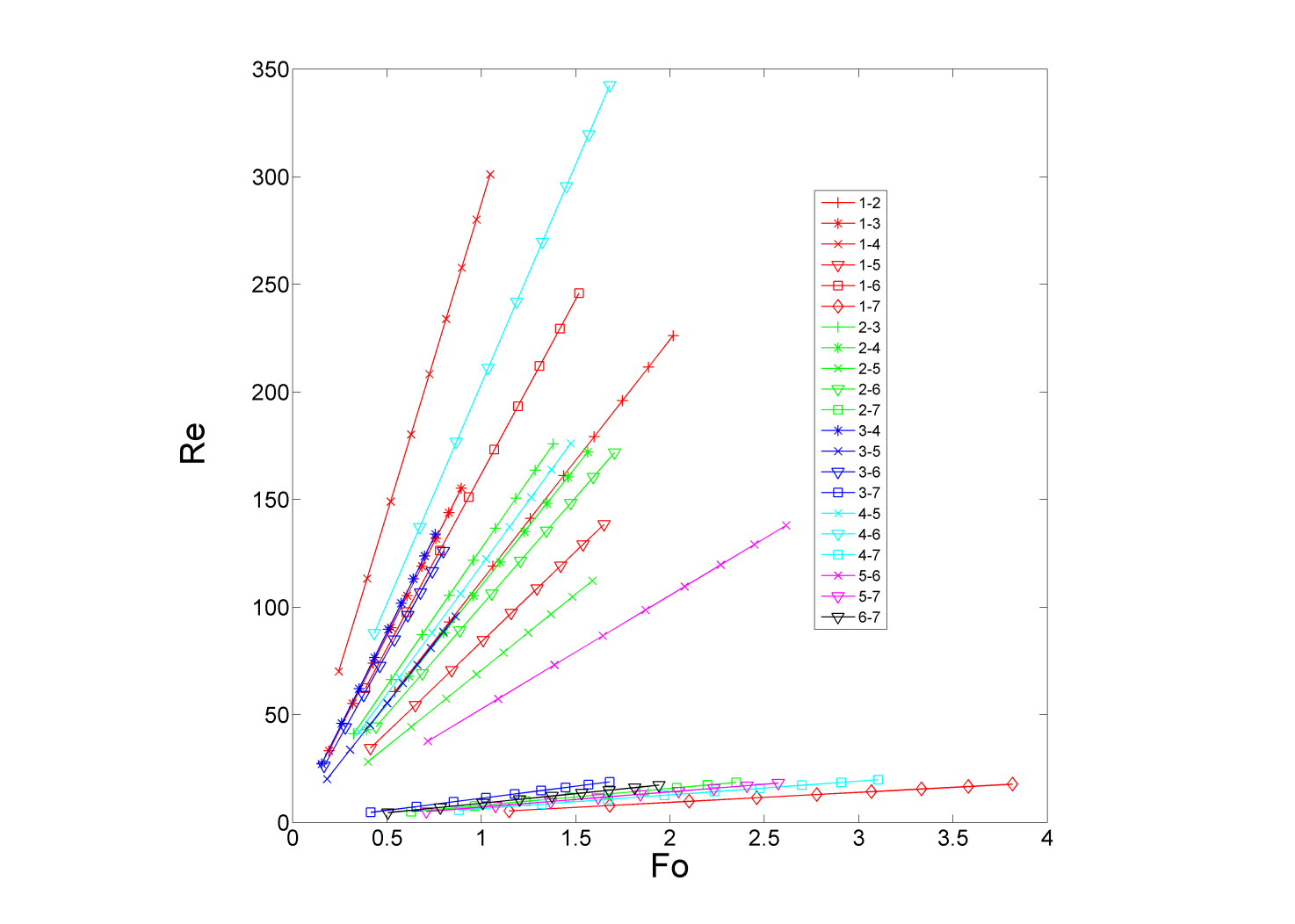


Figure 8 Reynolds number versus Forchheimer number for all paths.



Figure 9. Fractured block. The fracture in correspondence of the hole 7 is showed in the lower left side.

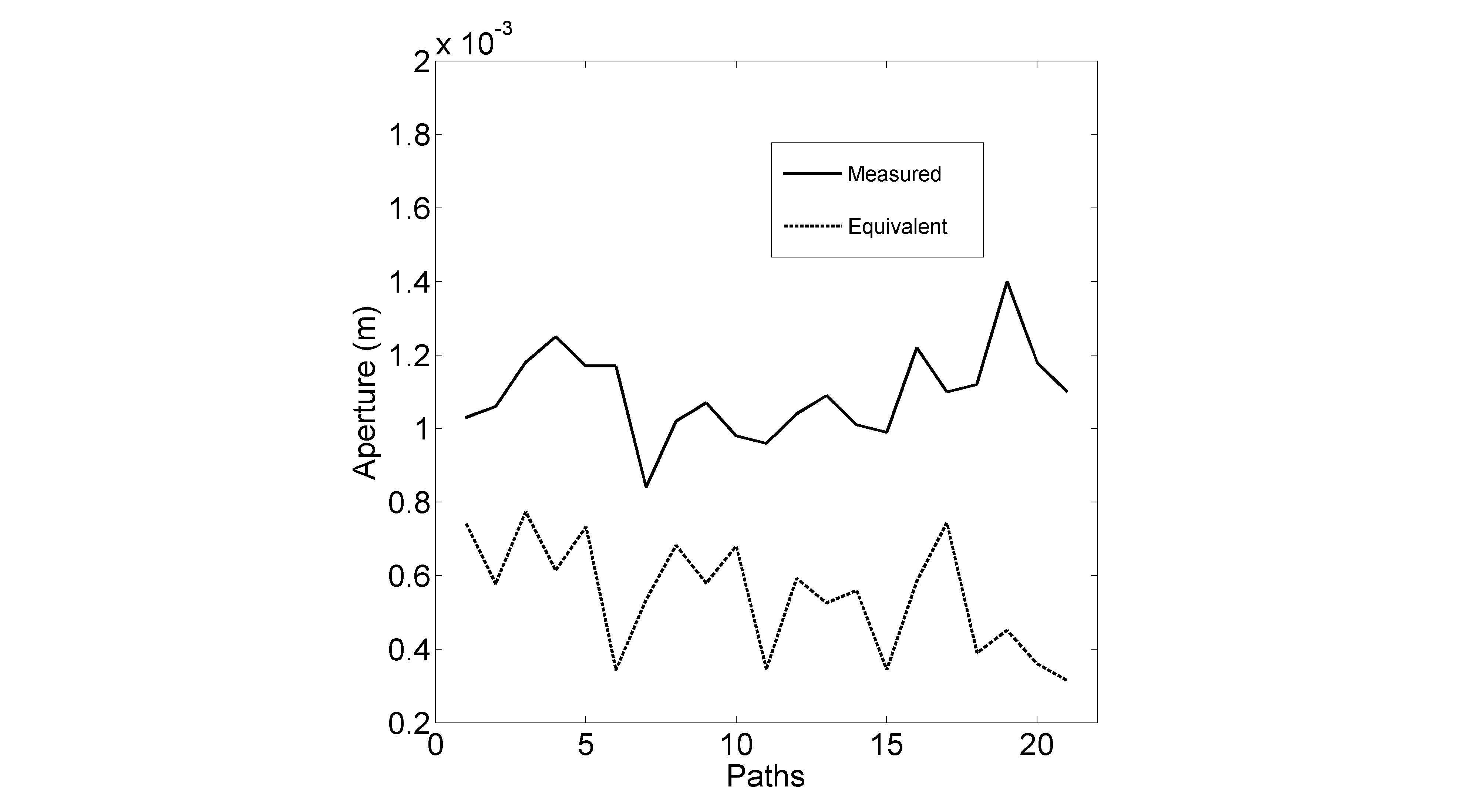


Figure 10. Average measured aperture (continuous line) and equivalent aperture (dashed line) for each path.

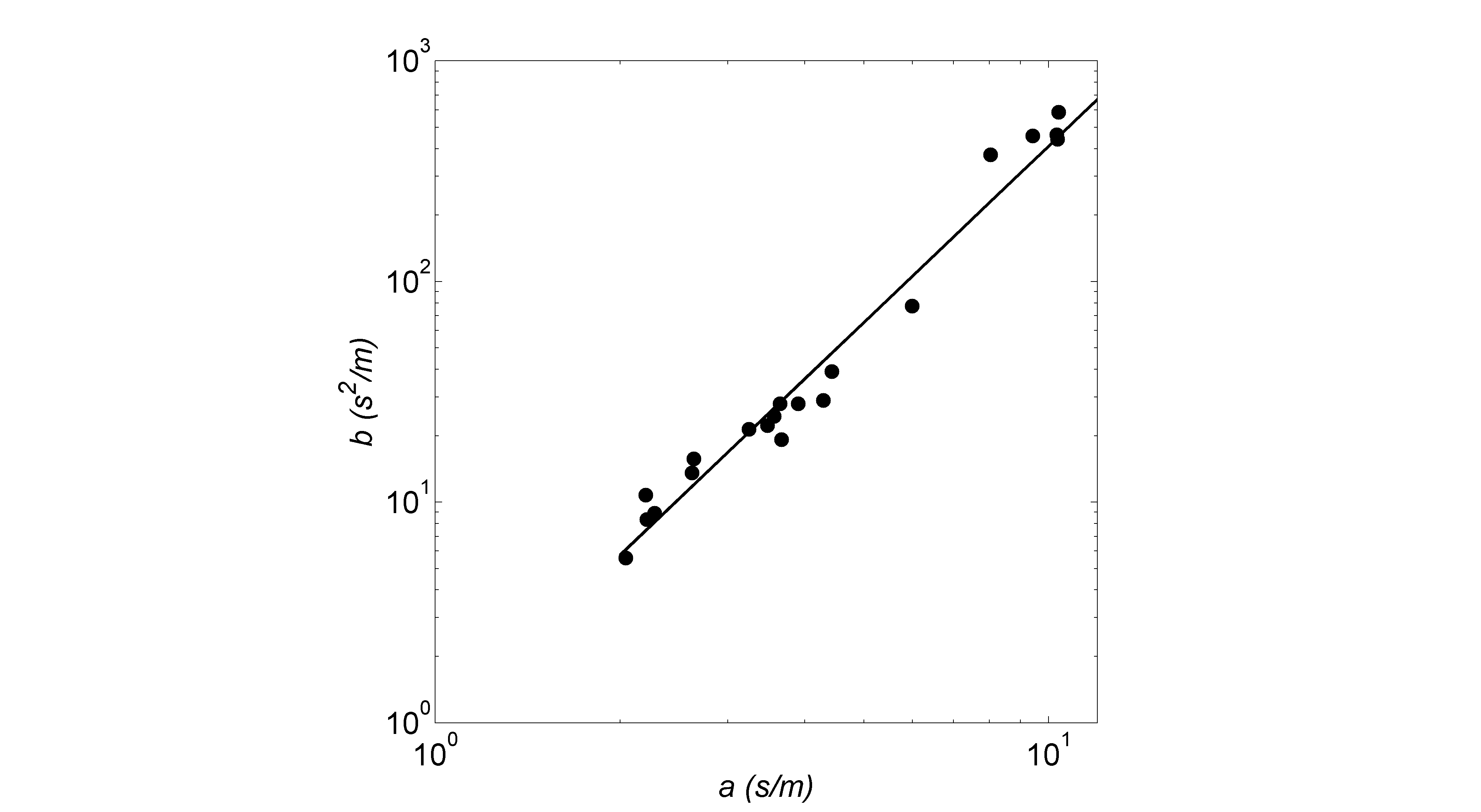




Figure 11. Linear term *a* versus non linear term *b*.

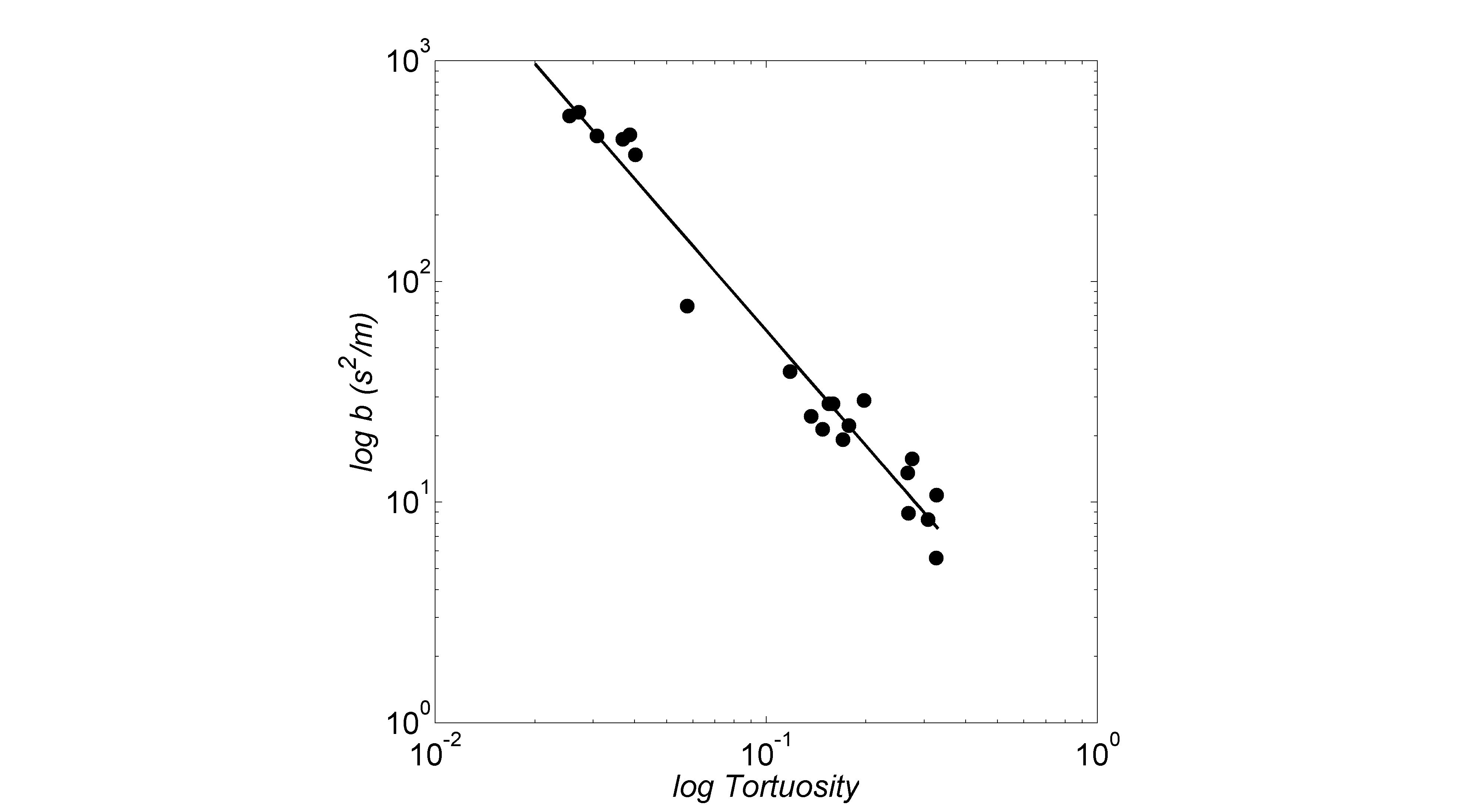




Figure 12. Tortuosity factor *τ* versus non linear term *b*.

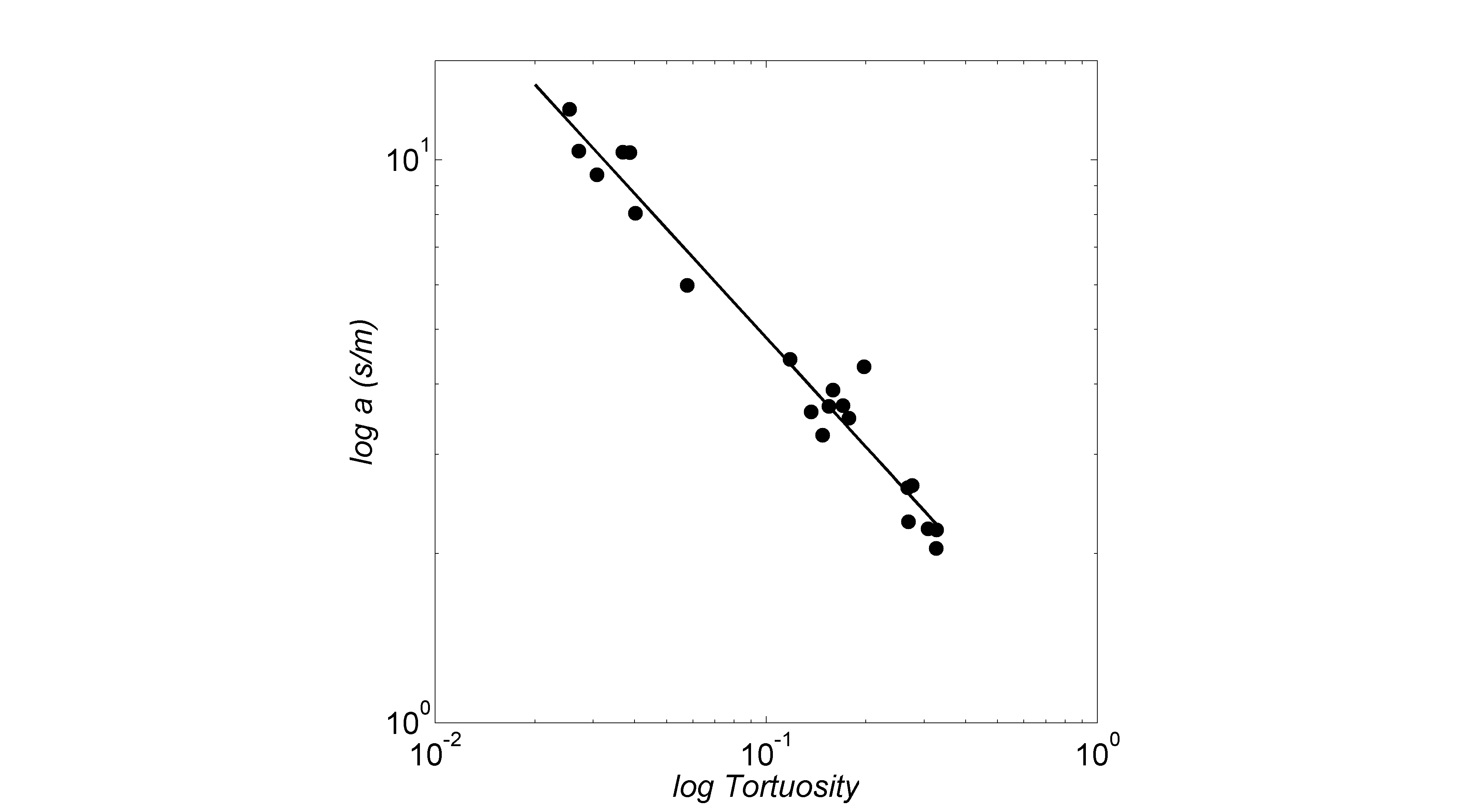




Figure 13. Tortuosity factor *τ* versus linear term *a*.