Reviewer 2:

1) I do have really a problem with applying Richards flow models with Van Genuchten models to karst systems. These models were derived for porous media, actually soils, and this is when they are valid. I agree, that the fissured and slow system of a karst aquifer can be simulated using such an approach. However, for the conduit system, I can see no justification for this. A conduit is not a porous medium, and even as an upscaled system of conduits does not behave like a porous medium, so that this approach does not work. The authors give no justification for their approach (as actually the article by Kaufmann does not also), they just mention that this approach "strictly speaking does not apply". However, to me this is not a discussion on "strictly speaking", but a very basic flaw. Actually, later in the article the authors provide themselves the problems associated with this approach, as they have to introduce minimum relative permeabilities for the conduit system, as the high value of van Genuchten alpha otherwise makes the conduit system actually impermeable (quite contrary to the obvious facts). So I do not see the point in using and applying a model, which is obviously not based on the correct physics.
We agree that the application of the Richards equation to gravity driven flow in highly permeable karst conduits for simulation of rapid percolation is not appropriate. For that reason we did apply rapid recharge at the bottom of the karst conduit (p. 7, line 201-202). Geyer et al. (2008) showed that the hydraulic responses of spring discharge occurs already a few hours after a precipitation event. Because of the daily time steps of the applied HGS model, we think this procedure is appropriate (p. 7, line 202-205). The unsaturated conduit continuum now only serves for slow percolation of water. The calibrated unsaturated conduit parameters are therefore pure calibration parameters to maintain model stability.

2) The second major problem I have with the manuscript is the use of a 2D vertical model. As can be clearly seen form the field site map, the catchment area focuses laterally towards the spring, so that the 2D approach chosen is obviously wrong due to the large horizontal flow components perpendicular to the main axis of the model area and the 2D cross section chosen by the authors. I think that this matters, because thus the spring discharge is not correctly connected to the precipitation, which basically drives the time variant behavior of the model, and thus the parameter identification is wrong. As the authors claim to find representative parameter sets, I can not agree here. And I see no obstacle to a 3D approach here, which would provide correct flow rates in the model and at the spring. Then, the parameters would actually be physically meaningful and characteristic of the system. So to me, a 3D model approach is clearly required.
The main problem for the simulation of flow and transport in karst systems is a sparse database and the strong heterogeneity of the system (chapter Introduction). Very often, karst spring responses are therefore described by reservoir modeling approaches, which do not involve any geometry of the system. Therefore we wanted to apply an approach, which accounts for double porosity flow, is able to simulate geometrical features (e.g. an inclined aquifer base) in a simplified way, and works with a limited database. The focus in our work lies in the sensitivity of the model

parameters (feedbacks, etc.) and model limitations, especially with respect to unsaturated flow.

We agree that a three-dimensional domain would represent the flow field better, but for most karst aquifer systems, there is no information about the location of the rapid and diffuse recharge area and the three-dimensional hydraulic parameter field. (added to chapter Conclusions p. 12, line 376)

For these reasons, we did test the 2D-vertical flow model, which lumps horizontal flow components and employs some calibration parameters (e.g. exchange coefficient). The applied hydraulic parameters are in the range of available field observations reported in literature (Table 1).

3) It also does not give proper references to the large amount of work done on simulating karst spring responses, as well as to simulating exchange in a dual flow system approach. Here I think also the literature form karst genesis should be taken into consideration, as the basic approaches and processes are identical. Here I think the second author can provide the adequate literature references.

We added the following citations to the introduction (p. 2, line 38):

Dreybrodt, W., Gabrovsek, F., Romanov, D. (2005): Processes of speleogensis: A modeling approach. Carsologica.

Birk, S, Geyer, T., Liedl, R., Sauter, M.: Process-Based Interpretation of Tracer Tests in Carbonate Aquifers. Ground Water, 43

4) The method section is poorly written. Please use the correct symbol for partial derivatives in eq. 1 and 2. Eq. 5 seems wrong to me, as this should be the total water content, not porosity. This is actually important for the later scenarios, so clarification is required. In line 12 on page 1519, the indexing is wrong, as one index appears twice. In line 1 of page 1520, the authors set residual saturations to 1, which should probably just saturations.

p. 3, line: 70+71: corrected partial derivatives

p. 3, line 80-86: corrected total porosity

p. 4, line 92: corrected indexing, change k_rm to k_rc

p. 4, line 100: removed "residual"

5) The formulas eq 8, 9, 10 are wrong. This is not the van Genuchten parameterization! Please give the correct formulas. Also in eq. 11 through 13, the same index nu appears for both systems and the exchange term. Please check, if this is actually the model used.

p. 4, Line 94-96: replaced alpha_m+psi with alpha_m*psi

p. 4, eq. 102-104: will correct index nu

6) I do not understand the explanation to equation 24. The authors state, that the root mean square error is calculated from the difference of "the spring discharge derived by the model" and the "calibrated model value"? As far as I can see, there is measured data available, so you could compare to measured data. And maybe you mean that you compare to the spring discharge simulated using the calibrated *model? Also, please state on which basis you calculate this error, i.e. daily, weekly, monthly ?*

As we intended to present a parameter study rather then achieving a perfect match to the field data the RMSE for the sensitivity analyses has been calculated with respect to the calibrated model value. We will add the RMSE for the calibrated value with respect to the field data to Figure 5.

The error has been calculated on daily basis, the information has been added to the text (p. 6, line 144).

7) Remove Fig.1 – this is not needed. It also has a wrong legend (2)

We believe that the figure is useful to describe the concept of linear and non-linear interparameter dependencies. Please see also our answer to question 15.

8) Why is recharge added "at the bottom of the conduit continuum" (page 1524, Line 17). Please justify.

The conduit recharge has been added to the bottom of the domain because fast percolation is not to simulate with the Richards equation. Geyer et al. (2008) showed that rapid recharge in the catchment area Gallusquelle occurs already some hours after a rainfall event, which is negligible with respect to the daily time steps applied in the model (p. 7, line 202-205).

9) When describing your model results in section 4 for the spring discharge comparison, please use dates to refer to individual events. I am not clear about the peaks the authors refer to, as their time is not given.

Both events have been labeled in Fig. 5 now as first and second peak.

10) Please also show the water tables during the simulation. I suggest you show water tables with time at an observation well i.e. at -8000m. This would allow a comparison.

The maximum and minimum water tables during a discharge event are given in Figure 6.

11) I do not understand why there are non-vertical flow paths in the unsaturated zone of the matrix continuum. (Figure 2). Why is this? This is an unusual behaviour for such a large scale porous medium.

This is caused by the coupling to the conduit continuum which imposes a internodal hydraulic gradient over the unsaturated matrix continuum by using calibrated threshold values (k_rminc, minimum relative conductivity). We describe this in Section 4.1 (p. 9, lines 248-253) and have added this behavior.

Theoretically, we could couple the conduit continuum with the matrix continuum only for the saturated zone for stationary simulations. However, for transient flow this is not possible, as the water table in both continua varies.

12) The comments on applying van Genuchten parameters should be moved to the introduction or model section, and put into perspective there.

We did add to the introduction (p. 2, lines 54-56):
"Flow simulations are based on the Richards' equation and respective parameters are described via the van Genuchten parametric model. The application and limitations of the approach for flow simulation in karst systems are discussed."

13) In section 5, page 1527, line 23, the authors argue that a similar behavior as for K_c can be seen for theta_c. I do not agree, the discharges for theta_c look very similar in Fig. 7. Clarify!

Also I do not understand what you really vary. Theta is the water content, so what do you really vary when you vary theta_m?

Also you state in this section that K_m is not sensitive. Again I do not agree, as in Table 2 this is one of the highest values. However, because I did not understand table two at all, I may be wrong here. This is actually the only reference to the table, and it is neither explained what is shows, not is it discussed. Here explanations are required and an exemplary discussion of a few cases.
It is true that theta_c does only show a similar behavior regarding its influence on the discharge but does not as strongly affect the discharge. We clarified this (p.10, lines 295-298).

Theta_m has been replaced with Theta_sm which denotes the saturated water content and is assumed to be the effective porosity. We added this also to the methods part to avoid confusion (p. 3, line 75).

We agree that Table 2 has been referenced slightly out of context and does not explicitly show that K_m is insensitive. K_m is insensitive and has therefore not be shown in a figure like Fig. 7 and 8. We now explained Table 2 two in more detail in section 5.3 (p.12, line 354) instead and accordingly placed a reference in this section. Furthemore we added Table 2 showing the RMSE values and recession coefficients to complement Fig. 7 and 8.

14) Section 5.3 just describes the Figures in Fug. 9. However, no explanation is given, why the model results becomes sensitive on some parameters only for certain parameter combinations. This is a very interesting point, and should be explained with detail and care.
We agree that this is an important finding of the work, which should be considered when performing sensitivity analysis with complex model systems, i.e. it shows the uncertainty of a sensitivity analysis. However, the interpretation of single parameter combinations is out of range of this work, also with respect to the length of the article.

15) The authors write of non-linear RMSE in the case of two parameters, but this is actually not true. Also for single parameter variations the results is non-linear, so this has nothing to do with varying two parameters at a time.

We believe that this is a misunderstanding and explained our definition of "linear" and "non-linear" in this context in more detail and reference to Fig 1 (p. 6, lines 157-161).

16) The authors use a very simple model to represent the spring catchment (2D, homogeneous, steady state in the saturated zone). They should discuss, how this affects the parameter identification and the parameter space investigated.

The model is transient for both continua. With respect to parameters, we did check that the calibrated physical parameters are in the range of field investigations (Table 1). The influence of pure calibration parameters on the simulated discharge curve is given by the sensitivity analyses.

17) On page 1531, line 10, you state that the model could "successfully" simulate the spring response. You have to justify this statement. Please compare the modeled spring response to prior work (Sauter, Birk et al.) and state, where the improvements are. When looking at Fig. 5, it douse not look convincingly like a good fit. Please also state why you regard this as a successful fit.

Our intention was to produce the best fit possible with the reported physical parameters and the given modeling approach.

Sauter (1992) did get a comparable fit with a double continuum model for the saturated zone. The author did apply a function for the transfer of water from the soil zone to the groundwater surface, which is not necessary for our model. In the presented model here, unsaturated flow is added as additional process. We added this to Chapter 4 (Result and discussion, p. 8, lines 231-234). Birk et al. 2005 did not simulate a recharge event for the Gallusquelle spring.

18) In the introduction, the authors state that this manuscript is aided at providing hints for a better characterization of a karst aquifer. However, many of the parameters (even for the homogeneous model used) can not be measured. So please discuss, how this work might contribiute to characterization effeorts, and what is to be learned. The Conclusions section is rather a summary than a conclusion.

As we state in Table 1, most of the chosen parameter values are reported in literature. Therefore the combination of reported values to simulate discharge events is a challenge. We added this to the conclusions (p. 13, line 387).

19) Table 1 is not complete. Please provide values for the total porosity used, as well as for theta_c. Units for alpha_c are probably wrong, as well as the indexing of the footnotes. Footnotes should be integrated into the Table heading.

Values total porosity are now provided. (Table 1)

The units of alpha_c and the indexing have been corrected (Table 1).

We believe that that footnotes are easily readable below the table.

20) Fig. 3 gives a 3D impression of the study area. This is correct, if a 3D model is used. For a 2D model, show the geology along the cross section actually used.

We added a 2D cross section of the area to Fig. 2 and removed the figure showing the three dimensional model.

21) In Fig. 5. Why is spring discharge given in mm/d? Units should be m3/d, so probably this is normalized. Please explain how spring discharge is normalized.

The spring discharge is normalized to the catchment area. This information has beeen added (p. 8, line 228-229).

This procedure has the advantage that recharge and discharge can be compared with the same units, and helps, for example, to compare the shape of discharge curves of different springs.

22) Fig. 6. The upper part of Fig. 6 is not used – it is not referred to in the text, and it is not explained in the legend what is shown. Also, label sub-figures, so that clear referencing is possible – this will also improve readability of the text. Show saturations form 0 to 1, nnot just 0.4 to 0.9. In the Figure caption you state, that the water table height is nearly equal, but this is not shown in the Figure.

The upper part of figure 6 has been removed and water tables have been integrated into the lower figures. Differences in water table heights of both continua can not be shown visually in the figure as they are below the average line resolution.

23) Fig. 7: Why did you chose the parameter ranges shown? Tab 1 e.g. gives different values for ranges? Please explain your choices. Also, it would be good to give the RMS of the cases shown, so that the reader gets an idea of what the RMSE means.

Parameter ranges were chosen based on field data if available. Depending on the calibrated model value, ranges were varied on linear or log-scale, therefore the maximum and minimum ranges do not perfectly match available field data (p.6, line 149-150).

A table (Tab. 2) has been added showing the RMSE values of the sensitivity analyses (see also question 13).

Manuscript prepared for Hydrol. Earth Syst. Sci.
with version 4.2 of the LaTeX class copernicus.cls.
Date: 28 June 2012

Simulation of saturated and unsaturated flow in karst systems at catchment scale using a double continuum approach

J. Kordilla[1], M. Sauter[1], T. Reimann[2], and T. Geyer[1]
[1]Geoscientific Centre, University of Göttingen, Göttingen, Germany
[2]Institute for Groundwater Management, TU Dresden, Dresden, Germany
Correspondence to: J. Kordilla (jkordil@gwdg.de)

Abstract. The objective of this work is the simulation of saturated and unsaturated flow in a karstified aquifer using a double continuum approach. The HydroGeoSphere code (Therrien et al., 2006) is employed to simulate spring discharge with the Richards equations and van Genuchten parameters to represent flow in the (1) fractured matrix and (2) conduit continuum coupled by a linear exchange term. Rapid vertical small-scale flow processes in the unsaturated conduit continuum are accounted for by applying recharge boundary conditions at the bottom of the saturated model domain. An extensive sensitivity analysis is performed on single parameters as well as parameter combinations. The transient hydraulic response of the karst spring is strongly controlled by the matrix porosity as well as the van Genuchten parameters of the unsaturated matrix, which determine the head dependent inter-continuum water transfer when the conduits are draining the matrix. Sensitivities of parameter combinations partially reveal a non-linear dependence over the parameter space. This can be observed for parameters not belonging to the same continuum as well as combinations, which involve the exchange parameter, showing that results of the double continuum model may depict a certain degree of ambiguity.

1 Introduction

Discharge dynamics in karst aquifers are determined by superposition of several effects: (1) water infiltration into soil, (2) water percolation through the unsaturated zone, (3) groundwater flow in highly conductive karst conduits and interaction with (4) groundwater flow in the low-conductive fissured and fractured rock matrix. These different effects, without even having considered the variability of precipitation and evapotranspiration, are a result of the particular properties of the individual compartments: soil-epikarstic zone, vadose zone, and phreatic zone. Each of these compartments is, in turn, characterized by two coupled flow systems: a highly permeable one with low storage and a less permeable one with high storage. Therefore, different individual (rapid, slow) flow components with characteristic temporal distributions are induced. Accordingly, the final spring discharge is then a function of the individual flow contributions of each of these compartments (Smart and Hobbs, 1986), which makes the inverse analysis of spring discharge a major challenge, requiring elaborate modeling tools and a large spectrum of data to constrain the model. The simulation of coupled saturated and unsaturated flow is still a challenge in hydrogeology in particular in fractured (Therrien and Sudicky, 1996) and karstified systems (Reimann et al., 2011a). This is predominantly a result of the data scarcity respecting the hydraulic parameter field of real karst systems. Therefore, flow in karst systems is often simulated with lumped parameter modeling approaches, which translate precipitation signals to discharge hydrographs by applying simple transfer functions (Dreiss, 1989). Generally, this type of approach is appropriate for situations in which predicted system states are expected to range between already observed events. The simulation of natural karst systems with distributed parameter models is reported only in a few studies (e.g. Jeannin, 2001; Hill and Polyak, 2010). However, distributive modeling approaches incorporate flow laws and, therefore, are adequate for the process based simulation of karst hydraulics (e.g. Birk et al., 2006; Reimann et al., 2011b) and transport problems (e.g. Dreybrodt et al., 2005; Birk et al., 2005) . Teutsch and Sauter (1991) demonstrate in how far the different mathematical model approaches are suitable for different types of problems (flow, transport, regional, local). An approach that takes into account the limited information about aquifer geometry and still allows the simulation of the dynamics of the karst system at an event basis, i.e. considers the dual flow behavior of karst systems is the double continuum approach (e.g. Teutsch and Sauter, 1991; Sauter et al., 2006). The approach was introduced by Barenblatt et al. (1960) and applied for simulation of karst hydraulics on catchment scale by Teutsch (1988) and Sauter (1992). It yields equations for simulation of slow and diffuse flow in the fissured matrix and the discrete rapid underground drainage by solution conduits in karst systems. Here, we want to assess the relative importance of individual factors and parameter combinations on the discharge behavior of a karst spring without detailed knowledge about the hydraulic parameter field of an aquifer system. This type of information is of major importance to focus characterization efforts in catchment based karst studies. Furthermore, the importance of infiltration dynamics, i.e. the temporal distribution of the rapid and the slow flow component on the discharge dynamics is to be determined. We employ the integrated saturated-unsaturated double-continuum approach HydroGeoSphere (Therrien et al., 2006) to simulate recharge and discharge dynamics in a karst aquifer with a thick unsaturated zone. Flow simulations are based on the Richards equation and respective parameters are described via the van Genuchten parametric model. The application and limitations of the approach for flow simulation in karst systems are discussed. A comprehensive parameter study was conducted in order to elucidate sensitive and important model parameters as well as parameter dependencies, and to reduce the model ambiguity to assist in focused karst characterization.

2 Methods

2.1 Modeling approach

The application of the double-continuum approach requires two sets of flow equations, one for the matrix (primary) and one for the conduit (secondary) continuum, solved consecutively at the same node and coupled with an exchange term that defines the hydraulic interface and controls the inter-continuum exchange flow. The applied HydroGeoSphere model (Therrien et al., 2006) is a non-commercial code available to the interested user under http://hydrogeosphere.org/. The model has been extensively used for various studies involving dual porosities such as Mclaren et al. (2000), Rosenbom et al. (2009) and Schwartz et al. (2010). The governing equation in the applied model is the Richards equation (Richards, 1931), which is slightly modified to account for inter-continuum water exchange:

$$-\nabla w_m(q_m) + \Gamma_{ex} \pm R_m = w_m \frac{\partial}{\partial t}(\theta_{sm} S_{wm}) \tag{1}$$

$$-\nabla w_c(q_c) + \Gamma_{ex} \pm R_c = w_c \frac{\partial}{\partial t}(\theta_{sc} S_{wc}), \tag{2}$$

where w_m and w_c are the volumetric fractions of each continuum of the total porosity, such that $w_m = 1.0 - w_c$. S_{wm} and S_{wc} are the water saturations of the respective continuum and R_m and R_c denote a volumetric fluid flux per unit volume (source/sink term) for each continuum. The saturated water content of the matrix and conduit system are assumed equal to the the effective matrix porosity θ_{sm} and conduit porosity θ_{sc} and are related to the water content of the matrix θ_m and of the conduit θ_c according to

$$\theta_m = S_{wm}\theta_{sm} \tag{3}$$

$$\theta_c = S_{wc}\theta_{sc} \tag{4}$$

The conduit and total porosity are given as

$$\theta_{total} = \theta_{sm}(1 - w_c) + \theta_{sc} w_c = \theta_{sm}(w_m) + \theta_{sc} w_c. \tag{5}$$

i.e. the whole simulation domain consists of nodes with primary porosity θ_{sm} with a volumetric fraction of $w_m = 1.0 - w_c$ and secondary porosity θ_{sc} with a volumetric fraction of w_c. Given that the local conduit porosity is chosen to be 1.0 and that both continua cover the whole domain the overall conduit porosity can simply be evaluated as:

$$\theta_{sc} \overset{\triangle}{=} \theta_{sc(local)} w_c. \tag{6}$$

The fluxes q_m and q_c are obtained from

$$
\begin{aligned}
q_m &= -K_m k_{rm} \nabla(\psi_m + z) && (7)\\
q_c &= -K_c k_{rc} \nabla(\psi_c + z), && (8)
\end{aligned}
$$

where K_m and K_c denote hydraulic conductivity, ψ_m and ψ_c are the pressure heads in each continuum and z is the elevation head.

In the unsaturated zone, the relative permeabilities k_{rm}, k_{rc} and k_{ri} (interface) depend on the water saturation which in turn is related to the pressure head according to van Genuchten (1980):

$$
\begin{aligned}
S_{wm} &= S_{wrm} + (1 - S_{wrm}) \left[1 + |\alpha_m \psi_m|^{\beta_m}\right]^{-\nu_m} && (9)\\
S_{wc} &= S_{wrc} + (1 - S_{wrc}) \left[1 + |\alpha_c \psi_c|^{\beta_c}\right]^{-\nu_c} && (10)\\
S_{wi} &= S_{wri} + (1 - S_{wri}) \left[1 + |\alpha_i \psi_i|^{\beta_i}\right]^{-\nu_i} && (11)
\end{aligned}
$$

for $\psi < 0$, where S_{wrm}, S_{wrc} and S_{wri} are the residual saturations, α_m, α_c and α_i denote the inverse air-entry pressure head, β_m, β_c and β_i are the pore-size distribution indices of each continuum and the interface. Note that the evaluation of the interface relative conductivity is based on the pressure head of the matrix. In the saturated zone where $\psi \geq 0$ the saturations are $S_{wm} = S_{wc} = S_{wi} = 1$. The relative permeability is given by:

$$
\begin{aligned}
k_{rm}(S_{wm}) &= S_{em}^{(lp)} \left[1 - \left(1 - S_{em}^{1/\nu_m}\right)^{\nu_m}\right]^2 && (12)\\
k_{rc}(S_{wc}) &= S_{ec}^{(lp)} \left[1 - \left(1 - S_{ec}^{1/\nu_c}\right)^{\nu_c}\right]^2 && (13)\\
k_{ri}(S_{wi}) &= S_{ei}^{(lp)} \left[1 - \left(1 - S_{ei}^{1/\nu_i}\right)^{\nu_i}\right]^2 && (14)
\end{aligned}
$$

with l_p being the pore connectivity parameter (equals 0.5 after Mualem, 1976), S_e the effective saturation

$$
\begin{aligned}
S_{em} &= \frac{S_{wm} - S_{wrm}}{1 - S_{wrm}} && (15)\\
S_{ec} &= \frac{S_{wc} - S_{wrc}}{1 - S_{wrc}} && (16)\\
S_{ei} &= \frac{S_{wi} - S_{wri}}{1 - S_{wri}} && (17)
\end{aligned}
$$

and ν is defined as:

$$
\nu_m = 1 - \frac{1}{\beta_m} \qquad (18)
$$

$$\nu_c = 1 - \frac{1}{\beta_c} \tag{19}$$

$$\nu_i = 1 - \frac{1}{\beta_i} \tag{20}$$

for $\beta > 1$. In the saturated zone the storage terms on the right-hand side of Eq. (1) and Eq. (2) are replaced by:

$$S_{wm} S_{sm} \frac{\partial \psi_m}{\partial t} + \theta_{sm} \frac{\partial S_{wm}}{\partial t} \tag{21}$$

$$S_{wc} S_{sc} \frac{\partial \psi_c}{\partial t} + \theta_{sc} \frac{\partial S_{wc}}{\partial t}, \tag{22}$$

where S_{sm} and S_{sc} are the specific storage coefficients. Water release by compaction of the porous medium is neglected in the unsaturated zone. The term Γ_{ex} in Eq. (1) and Eq. (2) describes the volumetric fluid exchange rate per unit volume between primary and secondary continuum and is given as:

$$\Gamma_{ex} = \alpha_{ex} K_i k_{ri} (\psi_c - \psi_m), \tag{23}$$

where K_i is the hydraulic conductivity of the interface (e.g. sediments) and k_{ri} the relative interface permeability (Barenblatt et al., 1960). The exchange parameter α_{ex} is determined by calibration and defined as (Gerke and Van Genuchten, 1993):

$$\alpha_{ex} = \frac{\beta}{\alpha^2} \gamma_w, \tag{24}$$

where β is a geometry factor (3 for rectangular matrix blocks, 15 for spheres), a is the distance between the center of a matrix block and the adjacent fracture or conduit and γ_w is an empirical coefficient usually set to 0.4. Strictly speaking, the van Genuchten approach, adopted in HydroGeoSphere does not apply to fractured and karstified rock materials. The highly heterogeneous flow field and preferential flow paths associated with such media and the consequently greater size of an REV compared to porous media are rendering the parameter determination by laboratory experiments impractical. Still, the van Genuchten parameters reflect properties of an unsaturated porous material and can be considered as an adequate parameter set to describe transient infiltration processes if they are treated as calibration parameters in order to upscale from the Darcy-scale averaging volume to the field scale.

2.2 Sensitivity analysis

An extensive sensitivity analysis is performed to determine the influence of the calibrated parameters on the computed flow. The root-mean-square error (RMSE) is chosen to rate the accuracy of fit and calculate deviations from the calibrated model. The RMSE is defined as (Bamberg et al., 2007):

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (m_i - f_i)^2}, \tag{25}$$

where n is the total number of data points, m_i denotes the simulated spring discharge derived by the sensitivity analyses and f_i is the calibrated model value. For sensitivity analyses the RMSE has been evaluated using daily data and documented parameter ranges were employed if possible. However, for some variables, in particular the van Genuchten parameters for the unsaturated zone of hard rocks, only few data and estimates can be found in the literature, e.g. Weiss (1987), Sauter (1992), Contractor and Jenson (2000) and Roulier et al. (2006). Consequently, the sensitivity of these parameters was determined systematically to evaluate the degree of ambiguity of the model. Depending on field observations parameter ranges were varied on linear or log-scale. Parameter spaces were assigned to cover at least the reported values from field experiments. In order to provide a further quantitative comparison recession coefficients are given for one important recharge event during March 1988 (α_1) and the following recession until beginning of April 1988 (α_2). Due to the complex flow model it is likely that some parameters do not show a linear correlation and sometimes the simulated discharge curve is only influenced by specific parameter combinations. The final analysis of parameter sensitivity on an idealized example is subdivided into: (1) insensitive parameters, (2) one sensitive parameter and (3) both parameters are sensitive (see Fig. 1, from left to right). In the latter case parameter A may be more sensitive for a certain range of parameter B. Given a constant parameter B$_n$ and B$_{n+dn}$ where n denotes the parameter value and dn a change in the parameter the parameter combination is refered to as non-linear if the change in RMSE over the whole range of parameter A is different for B$_n$ and B$_{n+dn}$ that is if

$$f(A,B_n) \neq f(A,B_{n+dn}) \qquad (26)$$

This case is particularly important if a properly calibrated model can be achieved for two different parameter combinations where at least one parameter is a pure calibration parameter, i.e. its range is difficult to estimate by field observations. Therefore, more than 1000 model runs were performed and the influence of parameter combinations on the simulated discharge curve analyzed.

3 Case study

3.1 Description of the field site

The HydroGeoSphere model is employed to simulate flow in the catchment area of the Gallusquelle spring from February 1988 to January 1990. The Gallusquelle spring is situated in Southwest Germany on the Swabian Alb, a northeast-southwest striking Jurassic carbonate plateau. The catchment area has been studied extensively by several authors including aquifer characterization (Birk et al., 2005; Sauter, 1992), speleology (Abel et al., 2002; Gwinner, 1976), and flow processes (Geyer et al., 2008). The size of the catchment is about $45\,\mathrm{km}^2$. It is delineated by a water divide in the northwest and the River Fehla in the northeast (see Fig. 2). In the south the catchment is bounded by the northeastern fault zone of the Hohenzollerngraben, which was found to be impermeable by tunneling works in the 1960's. After Sauter (1992) the base of the aquifer is formed by Kimmeridge marls (ki1). With a 70 m to 90 m thickness, this unit is rather consistent in the northwestern parts of the project area and consists of calcareous marls and occasional limestone intercalations. In the southeastern area no borehole information about the lower boundary of the unit are available. The uppermost catchment is made up of a sequence of Kimmeridgian limestones (ki2-5) from which the majority is developed as algal-sponge facies and, therefore, belongs to the Unterer Massenkalk or Oberer Massenkalk. The largest part of the catchment comprises limestones belonging to the Unterer Massenkalk (ki2 and ki3). The whole Jurassic sequence dips southeast at an angle of 1.2 degrees.

3.2 Geometry of the flow model

Based on the geological model, a vertical two-dimensional model domain of the catchment area was set up. The length of the domain is 10 km with a vertical thickness of 225 m. It reflects a cross section parallel to the direction of flow in the Gallusquelle catchment (see Fig. 2, lower figure). The model domain is represented by two continua reflecting flow in the low permeability fissured matrix (*matrix continuum*) and the highly permeable conduits (*conduit continuum*). The top of the model domain is set to 775 m a.s.l., which is an average elevation between ca. 910 m a.s.l. in the north-western part of the catchment and ca. 640 m a.s.l. in the south-eastern catchment and higher than the maximum groundwater head in the catchment. Every continuum is spatially discretized into 50 columns with a length of 200 m and a width of 1 m and 44 layers with a thickness of 5 m.

3.3 Boundary conditions

The lateral sides of the matrix continuum and of the conduit continuum, as well as the top of the conduit continuum are defined as no flow boundaries (see Fig. 3). A constant head boundary is applied to the right side of the conduit continuum at 634 m a.s.l. to represent the spring and allow discharge. A specified flux boundary is set at the top of the matrix continuum to account for diffuse recharge. Daily data of total recharge was estimated by Geyer (2008) for the simulation period on the Gallusquelle catchment. The applied water balance approach accounts for canopy storage, snow storage and soil-moisture storage before water entering the bedrock. A second specified flux boundary is set on the bottom of the whole conduit continuum to add rapid recharge in the aquifer. The location of the boundary condition considers that the transit time of the rapid recharge component through the unsaturated zone is below one day (Geyer et al., 2008) and, therefore, negligible with regard to the daily time steps. The simulation of rapid water percolation from the top of the conduit continuum to the groundwater surface is physically not possible with the van Genuchten approach, because it does not consider gravity driven flow processes like film and droplet flow. The initial head distribution for transient discharge simulations is computed with a steady state simulation. The applied total recharge for the simulation is 1.5 mm/d, which corresponds to the average recharge across the catchment area during the year 1988. Ten percent of the total recharge is employed as rapid recharge component at the bottom of the conduit continuum. The amount is in the range of the rapid recharge component estimated by Sauter (1997) from event analysis using oxygen isotopes in precipitation and Gallusquelle spring water to differentiate between different flow components.

3.4 Parameterization

For the model calibration, known parameters are only varied within reasonable ranges that agree with actual field observations (Tab. 1). Unknown model parameters are investigated by an extensive sensitivity analysis. The specific storage coefficients for matrix and conduits are negligible since the aquifer is unconfined; hence, water released due to compaction in the saturated zone is irrelevant. As there are no documented values for the hydraulic properties of the interface available, the van Genuchten parameters α_i, β_i, S_{wri} and the interface hydraulic conductivity K_i were set to values equal to the surrounding fissured matrix. Accordingly, inter-continuum water exchange is solely controlled by adjusting the exchange parameter α_{ex}. Model calibration is accomplished by fitting the observed and simulated discharge curves. Finally, the flow model contains 21 adjustable parameters for the fissured matrix and the conduit continuum.

4 Results and discussion

4.1 Model calibration

The calibrated model shows a good fit with most of the specific characteristics of the discharge hydrograph during the period between Feb./16th/1988 and Jan./20th/1990 (see Fig. 4). Please note that the discharge has been normalized to to catchment area (45 km^2). Calibrated values for all varied parameters are comparable to values documented in the literature (Tab. 1). The observed discharge curve shows less sharp peaks and is smoother than the simulated curve. Sauter (1992) did get a comparable fit with a double continuum model for the saturated zone. The author did apply a function for the transfer of water from the soil zone to the groundwater surface, which is not necessary for our model.

During the time period investigated, two strong discharge events occurred, caused by major snowmelts which are refered to as first and second peak (see Fig. 4). The discharged water volume agrees well with the simulated data during the time period of the first peak. During the second discharge event (second peak) the simulated peak height is overestimated. It is not possible to change the relative peak height difference between the first and second peak with the available calibration parameters. The recession curve slopes after discharge events show a good fit, except during low flow conditions between July and October 1989. This behavior could be attributed to the simplified geometry of the numerical model, which does not include the documented slightly inclined aquifer base and the geometry of different karstified zones in the karst system. The hydraulic heads in the matrix continuum and the conduit continuum are nearly identical during the simulation period with a difference of a few centimeters. Above the water table the matrix saturation drops to 0.35 near the surface (see Fig. 5). Flow paths in the unsaturated matrix continuum and conduit continuum are slightly inclined towards the spring, whereas flow in the saturated zone is laterally oriented towards the outlet, i.e. the karst spring. The flow paths of the unsaturated matrix continuum, which would be expected to be vertical for such a large scale porous medium, are caused by the strong influence of the conduit continuum which imposes a strong hydraulic gradient all over the matrix continuum. This behavior cannot be prevented unless the secondary continuum would be restricted to cover only the saturated zone. However, this is not an adequate solution considering the transient behavior of the system, i.e. the variation of water levels within both continua. The saturation in the conduit continuum is close to zero and has a very sharp transition along the water table. In this model, unsaturated flow in the conduits is also calibrated by the k_{rminc} parameter (minimum relative permeability of the conduits). Without this parameter, the relative permeability of the conduit continuum is a function of the residual saturation, i.e. setting of k_{rminc} simply overrides Eq. (13). This is the case for most of the unsaturated conduit continuum, where saturation declines very quickly (below 0.05) above the water table for the given van Genuchten parameters. Therefore, with the applied van Genuchten parameters only, water flow in the unsaturated conduit continuum is extremely small such that exchange from the matrix into the conduit system is nearly completely prevented and a proper model calibration is impossible due to numerical insufficiencies. However, Tokunaga and Wan (1997) showed that gravity driven film flow processes occur on unsaturated fracture walls, which contribute to water percolation along surfaces and may act as an interface from the conduit system to the matrix system, thus giving a physical meaning to the k_{rminc} parameter. As the original van Genuchten model relies on a uni-modal distribution of the pore space the hydraulic response of such flow processes cannot be expected to be fully resolved by the model. Attempts to refine the original van Genuchten approach and include hydraulic features of fractures into a continuum model have been made for example by Ross and Smettem (1993), Durner (1994) and Brouyère (2006) by constructing a continuous bi-modal retention curve.

An important role for the water exchange in the double continuum approach plays the exchange parameter α_{ex}. It determines the ability of water to move in and out of the conduit continuum and lumps geometrical and hydraulic properties of the karst matrix system. The surface-volume ratio, for example, is higher for a dendritic system than for a single conduit with the same conduit volume. The exchange parameter in the calibrated model is set to a high value such that it does not act as an additional barrier for water transfer between both continua and water transfer is mainly controlled by the hydraulic properties of the two continua.

5 Sensitivity analysis

5.1 Single variation of hydraulic parameters for saturated flow conditions

Tab. (2) gives an overview of the recession coefficients and RMSE values obtained for the sensitive parameters. A parameter has been discarded as insensitive if the maximum RMSE is below 0.05 $\mathrm{mm/d}$. The recession coefficients have been measured at the first strong recharge event beginning of March 1988 (α_1) and during the low flow recession beginning of April 1988 (α_2). The calibrated values are $\alpha_1 = 0.23$ and $\alpha_2 = 0.03$ which is close to what has been reported by Sauter (1989) for a conduit dominated recession ($\alpha = 0.25$) for the same recharge event. Figure 6 (upper two graphs) shows the computed spring discharge for several model runs with varying hydraulic conductivity K_c and porosity θ_c in the conduit continuum. These parameters strongly influence the simulated spring discharge. Figure 7 (upper two graphs) additionally shows the respective recession coefficients. An increased conduit conductivity K_c results in higher α_1 recession coefficients and lower base flow levels indicated by the strong decrease of α_2. A decreased conduit conductivity K_c favors a slow recession and decreases α_1 to 0.06 which according to Sauter (1989) already indicates a mixed system (fractured matrix + conduits) response. Discharge peaks are broadened and the base flow is higher. In case the conduit drains the matrix system an increase of K_c enhances the exchange process between matrix and conduits by decreasing the hydraulic gradient in the conduit continuum and consequently increasing the hydraulic gradient between matrix and conduits. The conduit porosity θ_{sc} follows a similar pattern, i.e. an increase will enhance the exchange process, however, the impact on the discharge curves is far less pronounced within the given ranges and both recession coefficients are all in the same order of magnitude indicating a conduit dominated recession. A contrasting behavior is observed by a variation in matrix porosity θ_{sm}. With an increase in the parameter the water transfer between the continua during recharge events is decreased because of the lower head difference between conduit and matrix and the discharge curve is smoothened accordingly (see Fig. 6). The recession coefficient α_1 and α_2 are consequently slightly lower while for a very low matrix porosity of 1.2% it is apparent that recessions coefficients represent a strongly conduit dominated system $\alpha_1 = 0.33$ and $\alpha_2 = 0.066$. K_m displays a low sensitivity within the given parameter range which can be attributed to the high exchange parameter of the calibrated model of $\alpha_{ex} = 1.0$. The high value leads to an immediate equalization of heads between conduit and matrix such that water will not be restrained within the matrix system when total heads are slightly higher than within the conduit system. The matrix system always depends on the hydraulic state of the conduit continuum, which discharges water rapidly to the spring. However, in a three-dimensional karst system, flow velocities within the matrix will be little influenced by the conduit system with increasing distance to the conduit. The exchange parameter α_{ex} is sensitive only for strong reductions on the order of three to four magnitudes. A reduction to 0.001 lowers the peak height of both main peaks while decreasing recession curve slopes α_1 to 0.06 and slightly increasing base levels. Further reduction to

0.0001 drastically decreases peak heights and increases the base levels. The resulting α_2 coeffcients are very low (0.002) and recharge events show no more pronounced peaks. An exchange reduction to 0.1 or 0.01 has no significant influence on the discharge curves which indicates a sensitive interval between 0.001 and 0.0001.

5.2 Single variation of unsaturated zone parameters

Variations of the sensitive van Genuchten parameters for the vadose zone are shown in Fig. (8) and the corresponding recession coefficients in Fig. (9). The decrease of α_m and β_m results in a strong rise of peak heights and increase of recession slopes ($\alpha_1 = 0.36$ and 0.26 respectively). The influence of the van Genuchten α_m parameter on the discharge curve is connected to the inter-continuum water exchange process. Lowering the parameter increases the capillary rise, i.e. the matrix has higher saturation (and relative permeability) above the water table. Consequently the increased permeability leads to a stronger and earlier exchange of water from the matrix into the conduit continuum, such that recharge events affect spring discharge a lot earlier (pronounced event peaks). The opposite can be observed for a value of 0.365 where the saturation fringe declines very quickly with lower saturations above the water table. This reduces the main exchange interface to a smaller area above the water table. Thus during high recharge events, peak heights are reduced since water will remain longer in the matrix continuum and the α_2 recession coefficient becomes slightly lower (0.025) reflecting the delayed discharge via the conduit system. The van Genuchten parameter β_m can be considered insensitive compared to α_m. The conduit van Genuchten parameters α_c and β_c are as well insensitive for the shown simulations. In the range of chosen values, the conduits do not produce a strong capillary rise, i.e. the unsaturated zone above the conduit water table always displays a sharp transition from saturated to strongly unsaturated. As mentioned earlier the application of the van Genuchten parameters to a highly conductive and discrete flow system such as a conduit implies a general abstraction of the physically based van Genuchten parameter set in order to create an upscaled continuum system with a characteristic infiltration behavior and travel time distribution as well as an exchange interface in the unsaturated zone. In this work the exchange process in the unsaturated zone can be controlled by the α_c parameter in order to increase the capillary rise in the conduit continuum and enhance inter-continuum water transfer. However, such an approach also introduces a spatial information (i.e. the thickness of the conduit capillary fringe in vertical direction) which is not known in real karst systems. As described before the k_{rminc} parameter is used instead to maintain a constant water exchange in the unsaturated zone independent of the hydraulic state of the conduit system if saturations are too low. The residual water saturation of the matrix S_{wrm} and the minimum relative permeability of the conduits k_{rminc} both show a similar behavior regarding parameter variations. Increasing the parameters yields an enhanced exchange from the matrix to the conduit continuum due to a higher relative conductivity. Consequently recharge events are transmitted faster to the model outlet, i.e. the spring.

5.3 Combined parameter variations

The above presented sensitivity analyses imply only one single parameter varied at a time. However, a further important observation is that certain parameter combinations may show non-linear behavior with respect to their sensitivity, i.e. the influence of one parameter on the RMSE is not linear over the whole range of a second parameter. Tab. (3) shows maximum RMSE obtained for each parameter combination and if a non-linear relationship can be observed (bold RMSE values). For example, the simultaneous variation of the matrix van Genuchten parameter α_m and the conduit conductivity K_c displays a pronounced sensitivity for low α_m values (Fig. 10). While for the calibrated α_m value of 0.0365 m^{-1} the conduit conductivity K_c is almost insensitive in the range of 1-10 $\mathrm{m/s}$ a lower α_m value of 0.00365 m^{-1} yields a high RMSE of 1.6 $\mathrm{mm/d}$ already at a K_c value of 10 $\mathrm{m/s}$, i.e. $\partial\text{RMSE}(\alpha_m = 0.00365)/\partial K_c$ is much higher. A similar behavior can be shown for a combination of matrix porosity θ_{sm} and the conduit conductivity where lower porosities yield higher RMSE values with an increase in conduit conductivity to 100 $\mathrm{m/s}$ whereas for rather high matrix porosities of 0.102 the increase in RMSE is less pronounced. The conduit conductivity K_c exhibits a higher sensitivity for the calibrated exchange parameter $\alpha_{ex} = 1.0$ (see Fig. 10) such that a high conductivity value (100 $\mathrm{m/s}$) results in RMSE values of ca. 1.4 $\mathrm{mm/d}$ while for a lower exchange parameter the same conductivity yields a deviation of only 0.4 $\mathrm{mm/d}$. The exchange parameter has a higher sensitivity for high conductivity values of the conduit system while it is nearly insensitive for low values (1 $\mathrm{m/s}$). In sum, the variation of the exchange parameter influences the discharge curve depending on the combination with other parameters. This behavior can also be observed for the combination of matrix porosity θ_{sm} and exchange parameter α_{ex}. Here the exchange parameter has a higher sensitivity for matrix porosities between 0.032 and 0.102 while at the lower limit (0.012 - 0.022) this sensitivity vanishes.

6 Conclusions

The applied two-dimensional double continuum approach lumps the horizontal flow components of a karst system but accounts physically-based for the dual flow in the subsurface. The advantage of the approach is that only limited informations about the geometry of the aquifer system and recharge area are necessary. Due to their large volume, vertical conduits in a karst unsaturated system would act as flow barriers if simulated by the Richards equation. However, flow in vertical shafts is not controlled by matrix potential and capillary forces but rather flow processes dominated by gravitational forces such as film flow (Tokunaga and Wan, 1997; Tokunaga et al., 2000), turbulent film flow (Ghezzehei, 2004), droplet flow (Doe, 2001; Dragila and Weisbrod, 2004) and rivulet flow (Su et al., 2001; Dragila et al., 2004; Su et al., 2004). In order to be able to use a consistent modeling approach, boundary conditions were modified and conduit recharge was directly injected at the bottom of the saturated conduit system. This procedure allows the simulation of rapid recharge with the given modeling code. Slow percolation of water through the unsaturated zone was simulated with the van Genuchten parametric model. The approach is successfully employed to simulate the discharge curve of the karst system Gallusquelle for a period of two years with hydraulic parameter ranges reported in literature. Because of the high amount of model parameters of the saturated-unsaturated flow model, a comprehensive sensitivity analysis was performed. The analysis shows that the simulated discharge curve displays high sensitivity to a variation of a number of model parameters. The sensitivity study demonstrates that the simulation of karst hydraulics requires a-priori knowledge about parameter ranges of model variables to reduce ambiguity of the model. However, especially for unsaturated zone parameters in double continuum karst systems, only little information about the parameter ranges is documented and further research is needed. Furthermore, the analysis shows that the sensitivity of a parameter depends to a large degree on the other calibrated model parameters. Therefore, sensitivity analyses should simultaneously take into account parameters of both continua in order to detect deviations from a linear behavior if both parameters are sensitive. It also means that conclusions about parameter sensitivity change from model to model and are not simply transferable. The fissured matrix porosity as well as van Genuchten parameters of the matrix continuum are the most important parameters for an appropriate flow simulation. The conduit system drains the fissured matrix and can, due to its high hydraulic conductivity, effectively discharge varying quantities of water transferred from the matrix continuum. It should be noted that the double-continuum approach assumes Darcian flow for the matrix as well as the conduits. Considering the high flow velocities in the conduits it is apparent that strictly Darcian flow will underestimate the heads (no energy loss due to friction) and consequently the exchange from matrix to conduits when the conduit continuum is draining the matrix system. More realistic results may be obtained by evaluating these influences for example by applying turbulent flow in the conduit continuum (Shoemaker, 2008; Reimann et al., 2011b). The van Genuchten parameters of the matrix are the most crucial property in terms of sensitivity, uncertainty and model limitations. The exchange process between matrix and conduit continuum is mainly controlled by differences in hydraulic properties. The α_{ex} parameter was set to a rather high value during the calibration, i.e. exchange is not limited by a too low exchange coefficient. According to Gerke and Van Genuchten (1993) the parameter is defined to express the interface connectivity on a rather small scale, e.g. between a porous medium and macropores. On catchment scale it might implicitly correspond to the type of conduit system (i.e. dendritic vs. large single conduits). If the parameter is used to calibrate the model by limiting water transfer between continua, attention should be paid to the non-linear behavior of certain parameter combinations and their resulting sensitivities. The application of van Genuchten parameters to fractured aquifer systems treats them as upscaled calibration parameters. Local scale flow processes, e.g. film and droplet flow along fracture surfaces, are not physically represented. Additionally, the dual-continuum approach lumps the geometrical features of the conduit system and the fissured matrix blocks, respectively, in the saturated and unsaturated zone.

References

Abel, T., Sauter, M., and Hinderer, M.: Karst genesis of the Swabian Alb, south Germany, since the Pliocene, Acta Geol. Pol., 52, 43–54, 2002.

Bamberg, G., Baur, F., and Krapp, M.: Statistik, Oldenbourg, München, 2007.

Barenblatt, G. I., Zheltov, I. P., and Kochina, I. N.: Basic Concepts in the Theory of Seepage of Homogenous Liquids in Fissured Rocks Strata, J. Appl. Math. Mech., 24, 1286–1303, 1960.

Birk, S., Geyer, T., Liedl, R., and Sauter, M.: Process-Based Interpretation of Tracer Tests in Carbonate Aquifers, Ground Water, 43, 381–388, 2005.

Birk, S., Liedl, R., and Sauter, M.: Karst spring responses examined by process-based modeling., Ground Water, 44, 832–836, 2006.

Brouyère, S.: Modelling the migration of contaminants through variably saturated dual-porosity, dual-permeability chalk, J. Contam. Hydrol., 82, 195 – 219, 2006.

Contractor, D. N. and Jenson, J. W.: Simulated effect of vadose infiltration on water levels in the Northern Guam Lens Aquifer, J. Hydrol., 229, 232–254, 2000.

Doe, T.: What do drops do? Surface wetting and network geometry effects on vadose-zone fracture flow, in: Conceptual models of flow and transport in the fractured vadose zone, chap. 8, pp. 243–270, National Academy Press, Washington D.C., 2001.

Dragila, M. I. and Weisbrod, N.: Flow in Menisci Corners of Capillary Rivulets, Vadose Zone J., 3, 1439–1442, 2004.

Dragila, M. I., Weisbrod, N., and Council, N. R.: Fluid motion through an unsaturated fracture junction, Water Resour. Res., 40, 1–11, 2004.

Dreiss, S. J.: Regional scale transport in a karst aquifer: 1. Component separation of spring flow hydrographs, Water Resour. Res., 25, 117–125, 1989.

Dreybrodt, W., Gabrovšek, F., and Romanov, D.: Processes of Speleogenessis: A Modeling Approach, Karst Research Institute at ZRC SAZU, ZRC Publishing, Ljubljana, 2005.

Durner, W.: Hydraulic conductivity estimation for soils with heterogeneous pore structure, Water Resour. Res., 30, 211, 1994.

Gerke, H. H. and Van Genuchten, M.: Evaluation of a first-order water transfer term for variably saturated dual-porosity flow models, Water Resour. Res., 29, 1225–1225, 1993.

Geyer, T.: Process-based characterisation of flow and transport in karst aquifers at catchment scale, Ph.D. thesis, University of Göttingen, 2008.

Geyer, T., Birk, S., Liedl, R., and Sauter, M.: Quantification of temporal distribution of recharge in karst systems from spring hydrographs, J. Hydrol., 348, 452–463, 2008.

Ghezzehei, T. A.: Constraints for flow regimes on smooth fracture surfaces, Water Resour. Res., 40, 1–14, 2004.

Gwinner, M. P.: Origin of the Upper Jurassic limestones of the Swabian Alb (Southwest Germany), E. Schweizerbart, 1976.

Hill, C. and Polyak, V.: Karst hydrology of Grand Canyon, Arizona, USA, Journal of Hydrology, 390, 169–181, 2010.

Jeannin, P. Y.: Modeling flow in phreatic and epiphreatic karst conduits, Water Resour. Res., 37, 191–200, 2001.

Mclaren, R. G., Forsyth, P. A., Sudicky, E. A., Vanderkwaak, J. E., Schwartz, F. W., and Kessler, J. H.: Flow and transport in fractured tuff at Yucca Mountain: numerical experiments on fast preferential flow mechanisms, J. Contam. Hydrol., 43, 211–238, 2000.

Mualem, Y.: A new model for predicting the hydraulic conductivity of unsaturated porous media, Water Resour. Res., 12, 513–522, 1976.

Reimann, T., Geyer, T., Shoemaker, W. B., Liedl, R., and Sauter, M.: Effects of dynamically variable saturation and matrix-conduit coupling of flow in karst aquifers, Water Resour. Res., 47, 1–19, 2011a.

Reimann, T., Rehrl, C., Shoemaker, W. B., Geyer, T., and Birk, S.: The significance of turbulent flow representation in single-continuum models, Water Resour. Res., 47, 1–15, 2011b.

Richards, L. A.: Capillary Conduction of Liquids Through Porous Mediums, Physics, 1, 318 – 333, 1931.

Rosenbom, A., Therrien, R., and Refsgaard, J.: Numerical analysis of water and solute transport in variably-saturated fractured clayey till, J. Contam. Hydrol., 104, 137–152, 2009.

Ross, P. J. and Smettem, K. R. J.: Describing soil hydraulic properties with sums of simple functions, Soil Sci. Soc. Am. J., 57, 26–26, 1993.

Roulier, S., Baran, N., Mouvet, C., Stenemo, F., Morvan, X., Albrechtsen, H.-J. r., Clausen, L., and Jarvis, N.: Controls on atrazine leaching through a soil-unsaturated fractured limestone sequence at Brévilles, France., J. Contam. Hydrol., 84, 81–105, 2006.

Sauter, M.: Quantification and Forecasting of Regional Groundwater Flow and Transport in a Karst Aquifer (Gallusquelle, Malm, SW. Germany), Tübinger Geowissenschaftliche Arbeiten, 1992.

Sauter, M.: Differentiation of flow components in a karst aquifer using the δ^{18}O signature, in: Tracer Hydrology, edited by Kranjc, A., pp. 435–441, Balkema, 1997.

Sauter, M., Geyer, T., Kovacs, A., and Teutsch, G.: Modellierung der Hydraulik von Karstgrundwasserleitern Eine Übersicht, Grundwasser, 3, 143–156, 2006.

Schwartz, F., Sudicky, E., and McLaren, R.: Ambiguous hydraulic heads and C^{14} activities in transient regional flow, Ground Water, 48, 366–379, 2010.

Shoemaker, W. B.: Documentation of a conduit flow process (CFP) for MODFLOW-2005, US Dept. of the Interior, US Geological Survey, 2008.

Smart, P. L. and Hobbs, S. L.: Characterisation of carbonate aquifers: A conceptual base, in: Proceedings of the Environmental Problems in Karst Terranes and Their Solutions Conference, Bowling Green, Ky. Published by NWWA, pp. 17–31, 1986.

Su, G. W., Geller, J. T., Pruess, K., and Hunt, J. R.: Solute transport along preferential flow paths in unsaturated fractures, Water Resour. Res., 37, 2481–2491, 2001.

Su, G. W., Geller, J. T., Hunt, J. R., and Pruess, K.: Small-Scale Features of Gravity-Driven Flow in Unsaturated Fractures, Vadose Zone J., 3, 592–601, 2004.

Teutsch, G.: Grundwassermodelle im Karst: Praktische Ansätze am Beispiel zweier Einzugsgebiete im Tiefen und Seichten Malmkarst der Schwäbischen Alb, Ph.D. thesis, Universität Tübingen, 1988.

Teutsch, G. and Sauter, M.: Groundwater modeling in karst terranes: Scale effects, data acquisition and field validation, in: Proc. Third Conf. Hydrogeology, Ecology, Monitoring, and Management of Ground Water in Karst Terranes, Nashville, TN, pp. 17–35, 1991.

Therrien, R. and Sudicky, E. A.: Three-dimensional analysis of variably-saturated flow and solute transport in discretely-fractured porous media, J. Contam. Hydrol., 3542, 1–44, 1996.

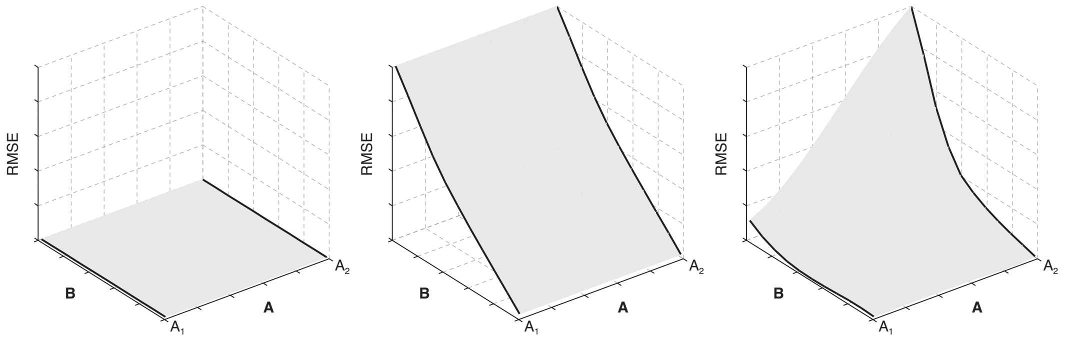
Therrien, R., McLaren, R., Sudicky, E., and Panday, S.: HydroGeoSphere: A three-dimensional numerical model describing fully-integrated subsurface and surface flow and solute transport, Manual (Draft), HydroGeoLogic Inc., Herndon, VA, 2006.

Tokunaga, T. K. and Wan, J.: Water film flow along fracture surfaces of porous rock, Water Resour. Res., 33, 1287, 1997.

Tokunaga, T. K., Wan, J., and Sutton, S. R.: Transient film flow on rough fracture surfaces, Water Resour. Res., 36, 1737–1746, 2000.

van Genuchten, M. T.: A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils, Soil Sci. Soc. Am. J., 44, 892–898, 1980.

Weiss, E. G.: Porositäten, Permeabilitäten und Verkarstungserscheinungen im mittleren und oberen Malm der südlichen Frankenalb, Ph.D. thesis, Universität Erlangen, 1987.



$$\frac{\partial RMSE(A_1)}{\partial B} = \frac{\partial RMSE(A_2)}{\partial B} = 0 \qquad \frac{\partial RMSE(A_1)}{\partial B} = \frac{\partial RMSE(A_2)}{\partial B} \neq 0 \qquad \frac{\partial RMSE(A_1)}{\partial B} \neq \frac{\partial RMSE(A_2)}{\partial B} \neq 0$$

Fig. 1. Examples for inter-parameter dependencies. From left to right: (1) both parameters insensitive, (2) both parameters sensitive with linear dependency, (3) both parameters sensitive but non-linear dependency.

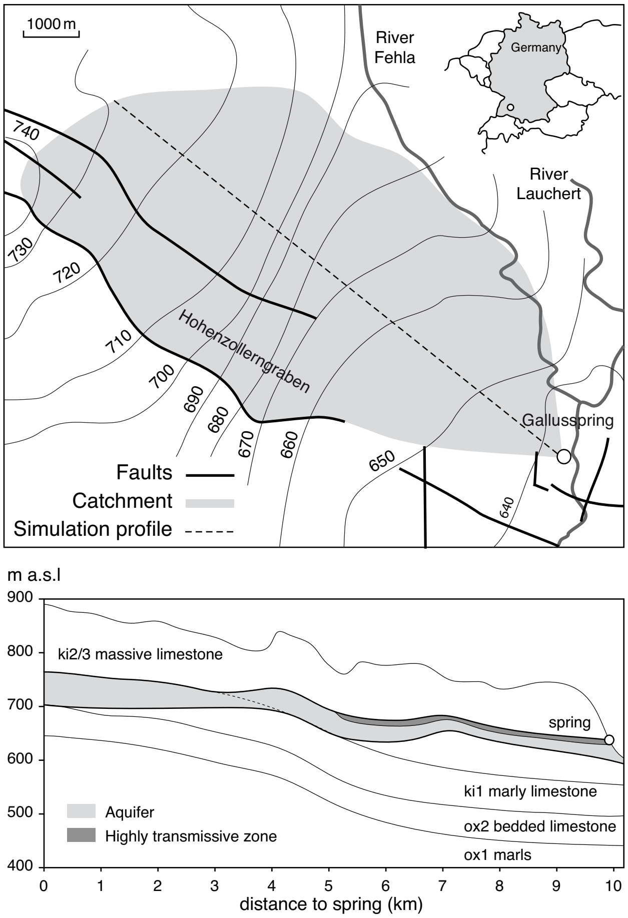


Fig. 2. Gallusquelle catchment and crosssection used for the simulation.

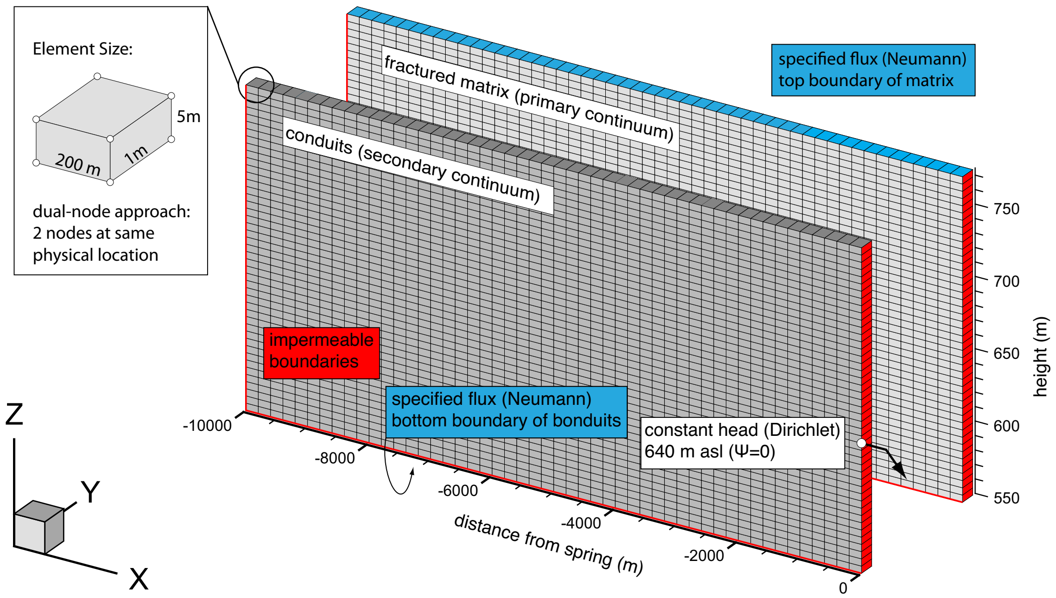


Fig. 3. Model grid of the two-dimensional model. A constant head boundary is applied at 640 m in the second continuum. Recharge is applied along the top boundary in both continuas. Every node in the primary continuum has its counterpart in the secondary continuum at the same physical location.

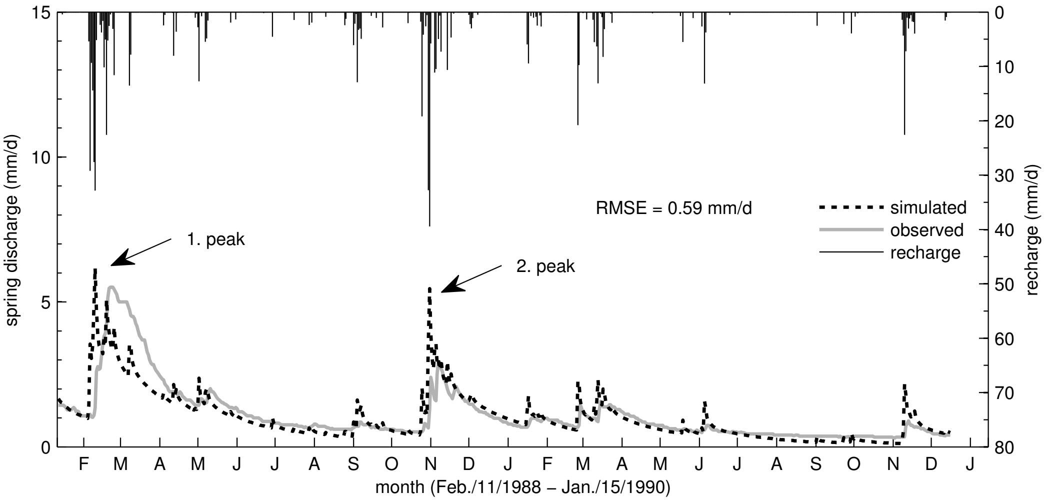


Fig. 4. The calibrated curve shows an acceptable fit. The secondary peak height is overestimated by the model. The first recharge event (first peak) reaches the spring too early. Recession curve slopes show a good overall correlation.

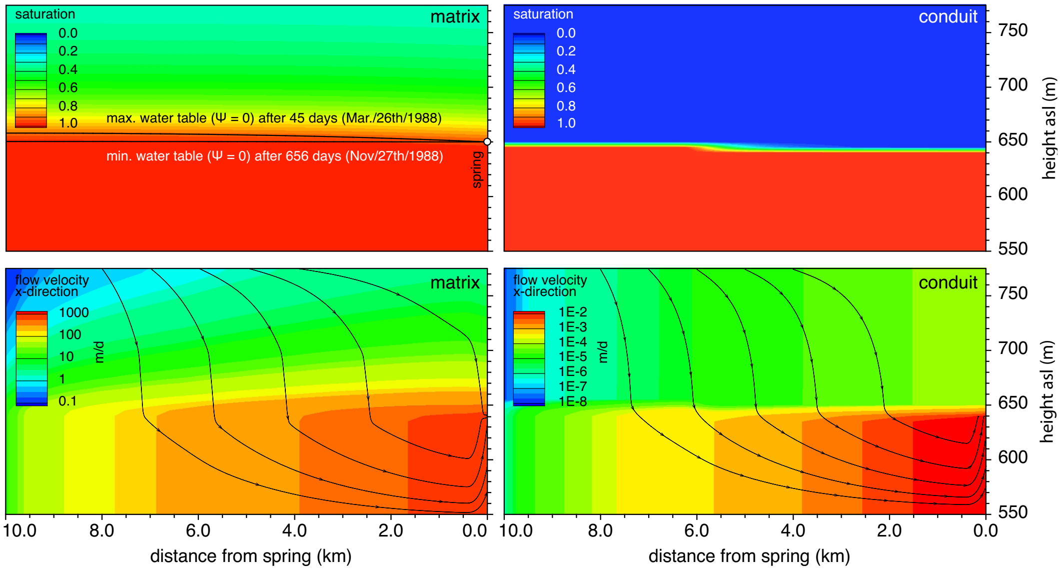


Fig. 5. Results of the transient flow model (Mar./26th/1988). Shown water tables apply for both continua as the height is nearly equal and differs 1 cm at most. The van Genuchten parameter α_m leads to a strong difference between the continuas. The capillary rise is more pronounced in the matrix system, where saturation is ca. 0.35 near the surface. Saturation in the conduit system is lower than 0.1 above the water table and below 0.0001 near the surface. Flow velocities (only x-direction vector, note the different scaling for matrix and conduits) are apparently higher in the conduit continuum.

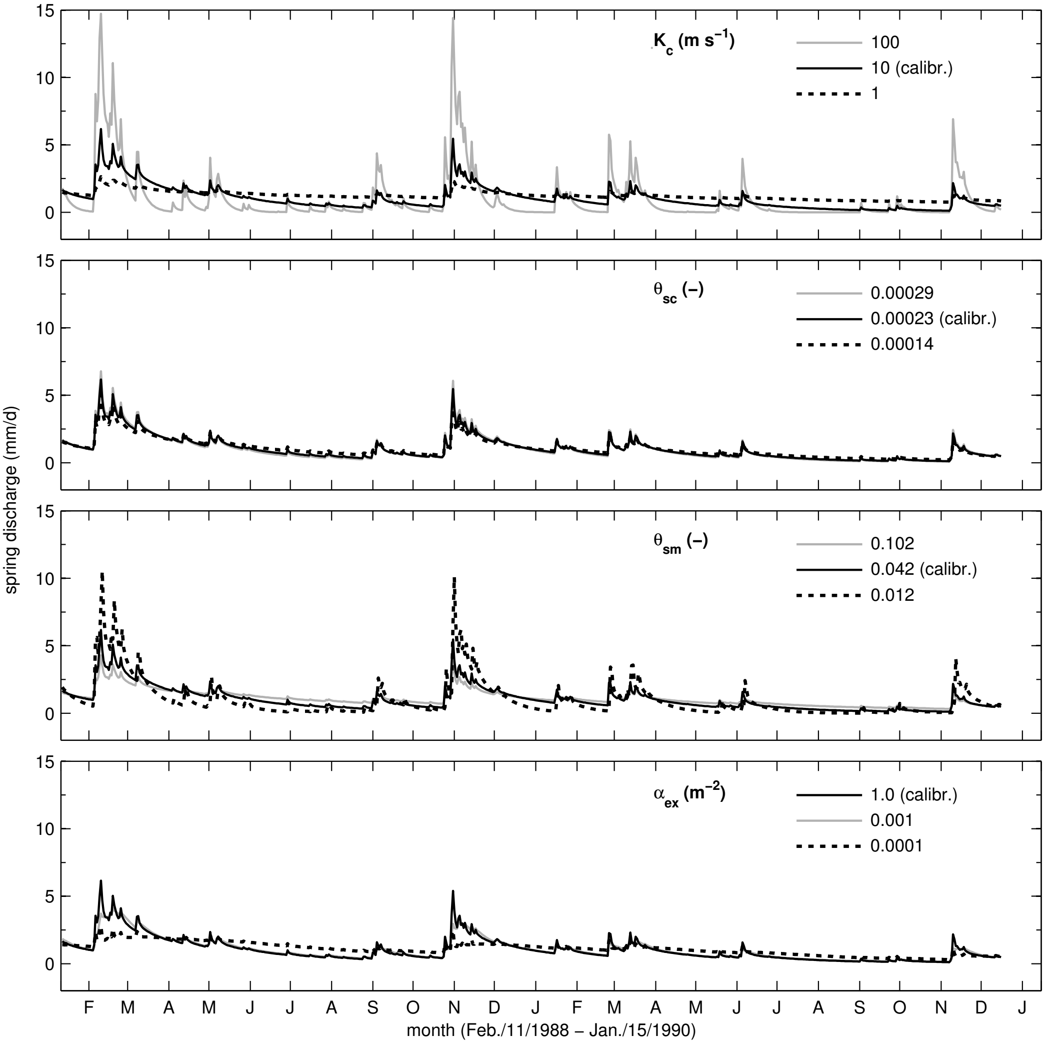


Fig. 6. Variation of hydraulic parameters (K_c, θ_{sc}, θ_{sm}) and the exchange parameter (α_{ex}).

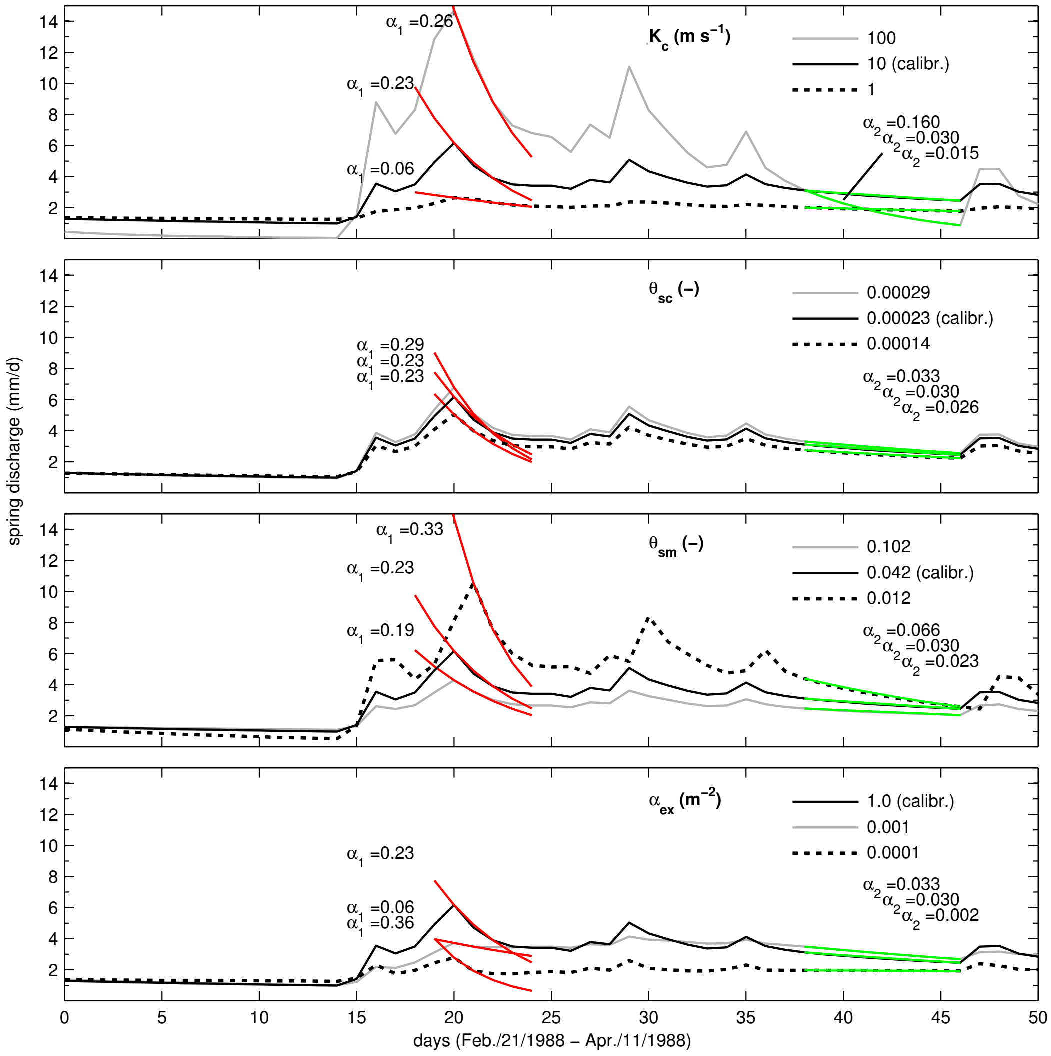


Fig. 7. Variation of hydraulic parameters (K_c, θ_{sc}, θ_{sm}) and the exchange parameter (α_{ex}).

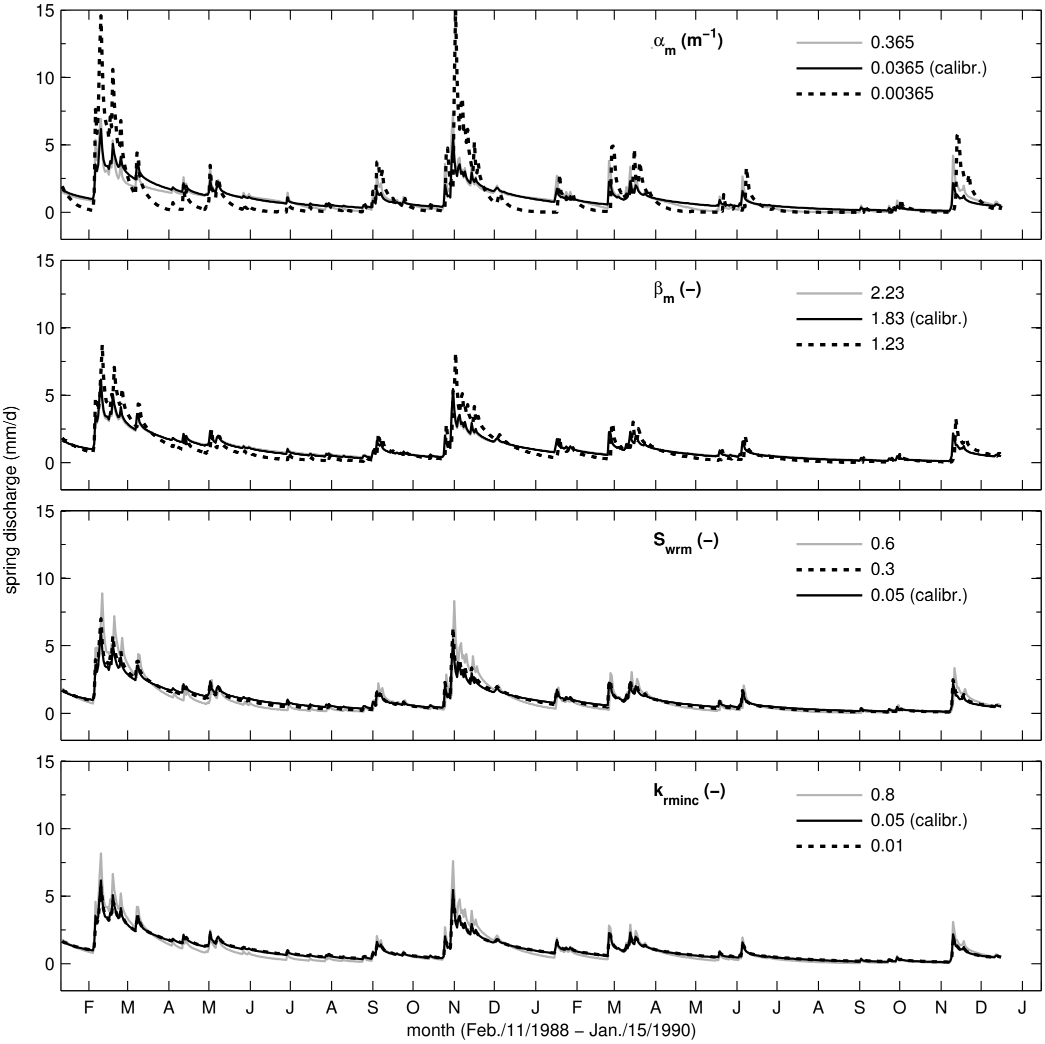


Fig. 8. Variation of the van Genuchten Parameters (α_m, β_m, S_{wrm}, k_{rminc}).

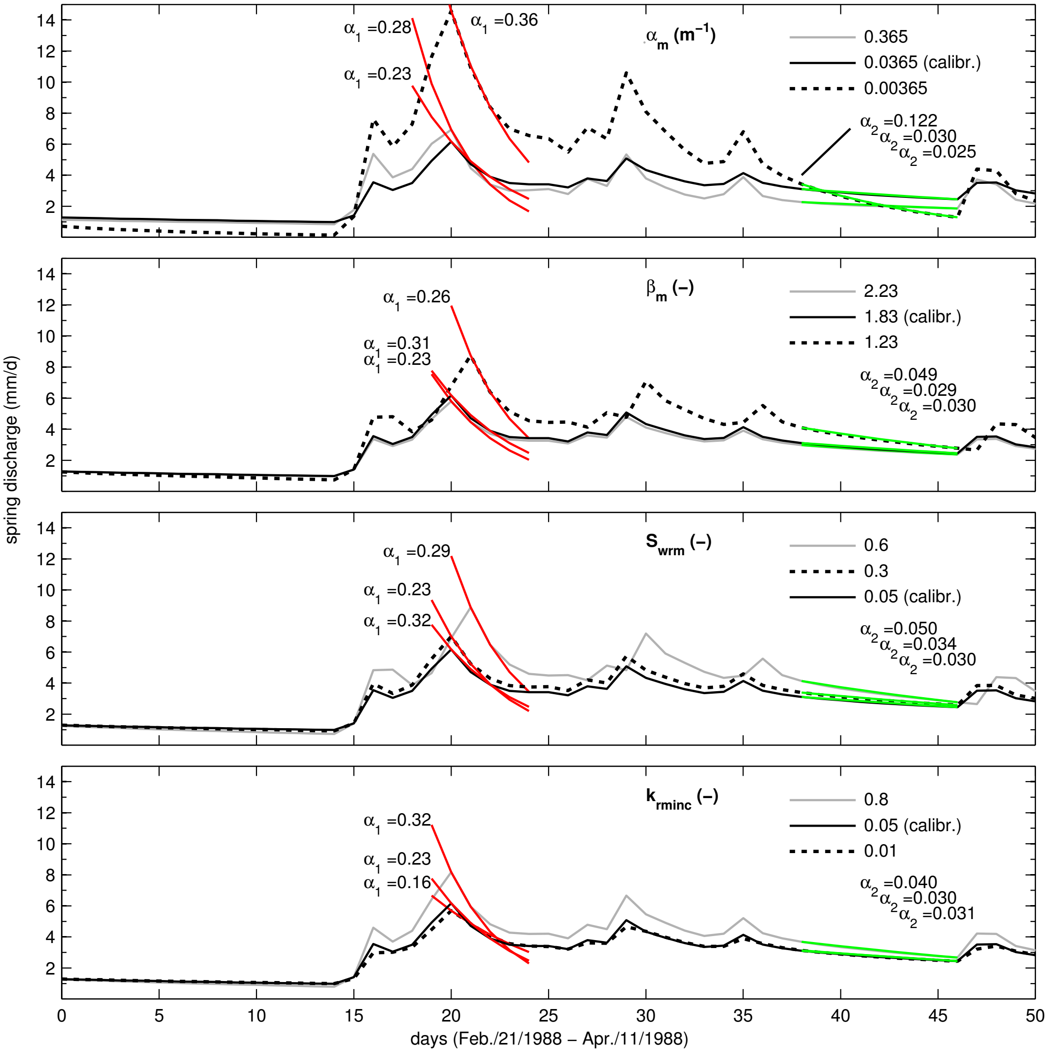


Fig. 9. Variation of the van Genuchten Parameters (α_m, β_m, S_{wrm}, k_{rminc}).

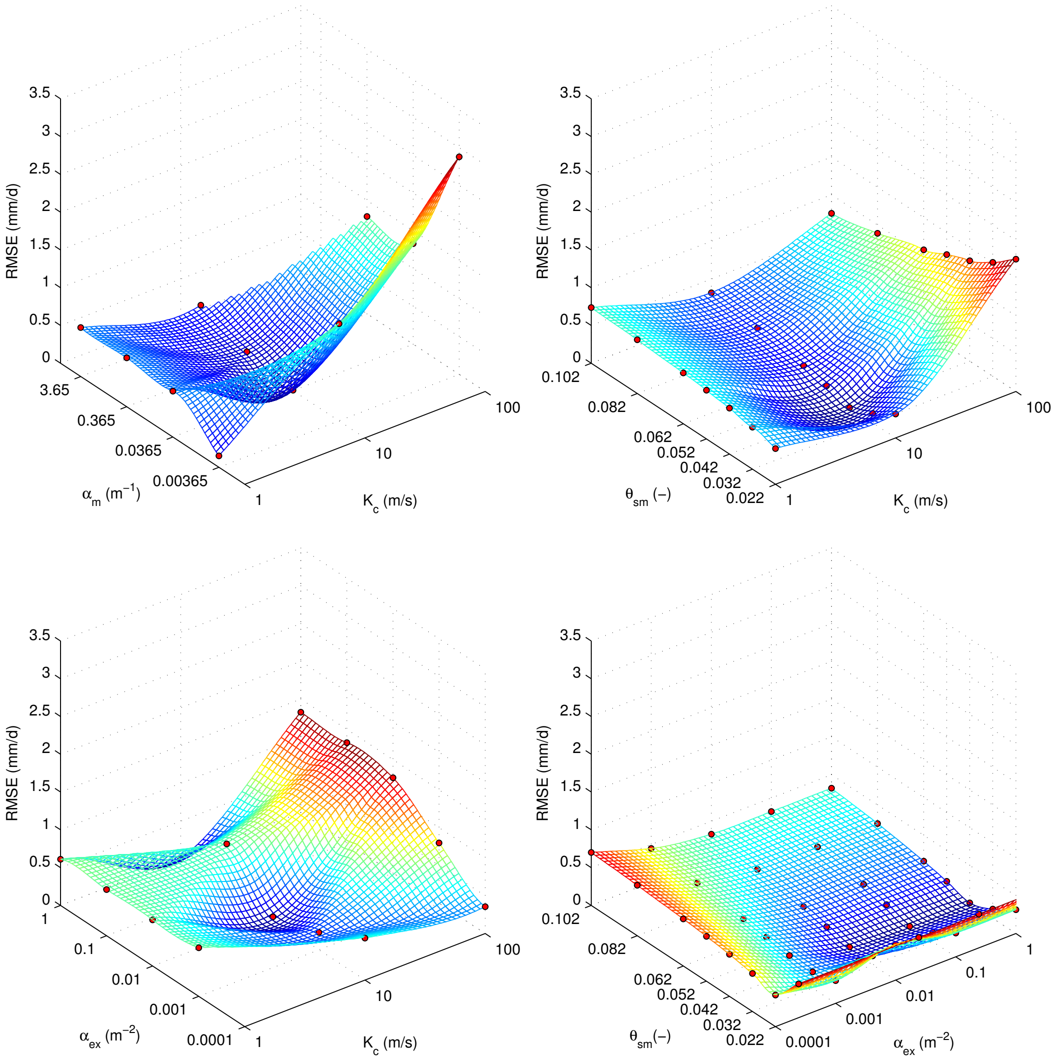


Fig. 10. Sensitivity matrices showing nonlinear inter-parameter dependencies.

Table 1. Estimated values for the flow model derived from the model calibration and value ranges reported in the literature. Subscript m and c denote the matrix resp. the conduit continuum.

Parameter	Unit	Value	Literature Values	Reference[2]
K_c	$(\mathrm{m/s})$	10.0	3.0 - 10.0	1
K_m	$(\mathrm{m/s})$	2.9×10^{-6}	1.0×10^{-6} - 1.0×10^{-4}	1
θ_{sc}[1]	(-)	0.00023	0.00016 - 0.00064	1
θ_{sm}	(-)	0.042	0.007 - 0.025, >0.0 - 0.12	1,2
α_{ex}	(m^{-2})	1.0	-	-
Q	(%)	90	-	-
α_m	(m^{-1})	0.0365	0.0365, 0.0328 - 0.623	3,4
β_m	(-)	1.83	1.83, 0.01 - 3.0	3,4
S_{wrm}	(-)	0.05	0.01 - 0.05	4
α_c	(-)	5.1	5.1	3
β_c	(-)	2.56	2.56	3
S_{wrc}	(-)	0.0	-	-
k_{rminc}	(-)	0.05	-	-

[1] The local conduit continuum porosity is 1.0 i.e. $w_c \theta_{sc(local)} = \theta_{total} - w_m \theta_{sm}$ implicitly gives the total conduit porosity such that $\theta_{sc} \overset{\wedge}{=} w_c \theta_{sc(local)}$.The total porosity is $\theta_{total} = 0.0422$.

[2] References: 1 - Sauter (1992); 2 - Weiss (1987); 3 - Roulier et al. (2006); 4 - Contractor and Jenson (2000).

Table 2. RMSE values and recession coeffcients for all sensitive parameters. Parameters with a maximum RMSE below 0.05 $\mathrm{mm/d}$ have been considered as insensitive. Subscript m and c denote the matrix resp. the conduit continuum.Bold numbers denote the calibrated value.

Parameter	Value	Recess. coeff. (1/d)		RMSE (mm/d)
		α_1	α_2	
K_c	100	0.26	0.160	1.329
	10	**0.23**	**0.030**	-
	1	0.06	0.015	0.645
θ_{sc}	0.00029	0.29	0.033	0.188
	0.00023	**0.23**	**0.030**	-
	0.00014	0.23	0.026	0.108
θ_{sm}	0.102	0.33	0.066	0.318
	0.042	**0.23**	**0.030**	-
	0.012	0.19	0.023	0.317
α_{ex}	**1.0**	**0.23**	**0.033**	-
	0.001	0.06	0.030	0.231
	0.0001	0.36	0.002	0.555
α_m	0.365	0.28	0.025	0.329
	0.0365	**0.23**	**0.030**	-
	0.00365	0.36	0.122	1.285
β_m	2.23	0.31	0.029	0.064
	1.83	**0.23**	**0.300**	-
	1.23	0.26	0.049	0.563
S_{wrm}	0.6	0.29	0.050	0.54
	0.3	0.32	0.034	0.142
	0.05	**0.23**	**0.030**	-
k_{rminc}	0.8	0.32	0.040	0.334
	0.05	**0.23**	**0.030**	-
	0.01	0.16	0.031	0.068

Table 3. Parameter combinations used for the sensitivity analyses and highest RMSE (mm/d) observed. Bold RMSE values denote a non-linear inter-parameter dependence.

	Range														
	Low	High	K_c	K_m	θ_{sc}	θ_{sm}	α_{ex}	Q	α_m	β_m	S_{wrm}	α_c	β_c	S_{wrc}	k_{rminc}
K_c	1.0	100.0	-												
K_m	2.89×10^{-7}	2.89×10^{-4}	1.34	-											
θ_{sc}	1.40×10^{-4}	2.90×10^{-4}	1.48	0.19	-										
θ_{sm}	0.022	0.102	**2.23**	0.83	0.89	-									
α_{ex}	1.00×10^{-4}	1.0	**1.33**	0.56	0.61	**0.77**	-								
Q	30	95	1.33	0.06	0.23	0.77	0.57	-							
α_m	0.365×10^{-3}	3.65	**3.20**	$\cdots$	$\cdots$	$\cdots$	1.29	$\cdots$	-						
β_m	1.23	2.23	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.58	$\cdots$	1.58	-					
S_{wrm}	0.05	0.6	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	1.87	$\cdots$	-				
α_c	3.1	8.1	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.56	$\cdots$	$\cdots$	$\cdots$	$\cdots$	-			
β_c	2.16	3.16	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.56	$\cdots$	$\cdots$	0.56	$\cdots$	0.25	-		
S_{wrc}	0.0	0.4	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.56	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.01	$\cdots$	-	
k_{rminc}	0.01	0.8	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.56	$\cdots$	$\cdots$	$\cdots$	$\cdots$	0.33	$\cdots$	$\cdots$	-